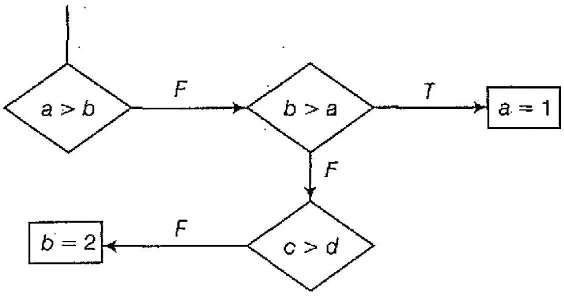
BHU MCA

Solved Paper 2012

1. Consider the following flow chart



Which of the following is not equivalent to the above flow chart?

(a) if $(a > b)$
if $(b > c)$
$a = 1$;
else if $(c > d)$
$b = 2$;

(b) if $(a <= b)$
if $(b > c)$
$a = 1$;
else if $(c <= d)$
$b = 2$;

(c) if $(a > b)$
; else if $(b > c)$
$a = 1$;
else if $(c <= d)$
; $b = 2$;

(d) if $(a > b)$
; else if $(b > c)$
$a = 1$;
else if $(c > d)$
; else $b = 2$;

2. The following program fragment,

if (a = 0)
printf ("a is zero") ;
else
printf ("a is not zero") ;

results in the printing of

(a) a is zero
(b) a is not zero
(c) nothing
(d) garbage

3. If y is of integer type, then the expressions

$$3*(y-8)/9 \text{ and } (y-8)/9*3$$

yield the same value, if

(a) y is an even number
(b) y is an odd number
(c) $y-8$ is an integral multiple of 9
(d) $y-8$ is an integral multiple of 3

4. If the integer needs two bytes of storage, then maximum value of an unsigned integer is

(a) $2^{16}-1$
(b) $2^{15}-1$
(c) 2^{16}
(d) 2^{15}

5. The minimum number of temporary variables needed to swap the contents of two variables is

(a) 1
(b) 2
(c) 3
(d) 0

6. Which is true of conditional compilation?

(a) It is taken care of by the compiler
(b) It is setting the compiler option conditionally
(c) It is compiling a program based on a condition
(d) It is operation taken by the compiler

7. The basic arithmetic operation performed by a computer is

(a) addition
(b) mulitiplication
(c) subtraction
(d) division

8. The base of the binary number system is

(a) 2
(b) 16
(c) 8
(d) 10

9. Main memory unit of computer

(a) performs arithmetic
(b) stores a small amount of data and instructions
(c) stores bulk of data and instructions
(d) supervises the working of all the units

10. A Central Processing Unit (CPU) consist of

(a) input, output unit
(b) memory unit
(c) arithmetic and logical unit, central unit
(d) keyboard, printer

Directions (Q.Nos. 11-12) *On prepositions may be taken as a drill in the use of prepositions. Select the most appropriate preposition carefully and then put your cross against the right answer.*

11. He was heart broken... her indifference ... him.

(a) at ,to (b) by, for
(c) by, to (d) at , on

12. Do not look down... the poor.

(a) do (b) upon
(c) at (d) in

Directions (Q.Nos. 13-15) *In the following questions you have passages, with questions following passages. Read passage carefully and choose the best answer to each question and mark it in the Answer Sheet.*

But alas in 1964, when I was nine, my young life shattered into pieces once again. My mother passed away after an illness of just 15 days. She had been the anchor of my life. And how I missed her but little children are resilient. My maternal grandmother was staying with us and my mother's widowed younger sister came to help out with us. So, well did my aunt fit into our household that in two years my father had married her. She become our mother and her only daughter become our sister.

13. The tone of the passage is

(a) gloomy (b) humorous
(c) ironical (d) lyrical

14. The writer's response to his father marrying again is that of

(a) indifference (b) disdain
(c) approval (d) disapproval

15. The writer's life got 'shattered into pieces' when

(a) his father married again
(b) he lost his mother
(c) he fell seriously ill
(d) a sister was born to him

16. In which of the following countries did the decimal system of numbers originate?

(a) England (b) France
(c) India (d) Greece

17. A place which has 2 as the first digit in its PIN Code must be situated in which of the following states?

(a) Uttar Pradesh (b) Maharashtra
(c) Gujarat (d) Andhra Pradesh

18. Which of the following has been found useful in keeping the level of cholesterol down?

(a) Serpentina (b) Tulsi
(c) Turmeric (d) Garlic

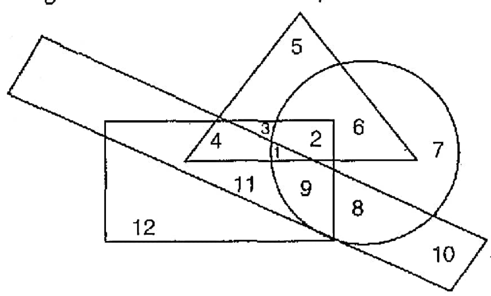
19. The southernmost point of Indian territory is in which of the following States/Union Territories?

(a) Tamil Nadu
(b) Lakshadweep
(c) Kerela
(d) Andaman and Nicobar Islands

20. Which of the following diseases usually spreads through air?

(a) Plague (b) Tuberculosis
(c) Typhoid (d) Cholera

Directions (Q.Nos. 21-25) *These questions are based on the following diagram in which circle stands for the educated,the square for hard working, the triangle for urban and the rectangle for honest, the diagram are numbered from 1 to 12. Study the diagram and answer each question.*



21. Hard working and non- urban people who are neither educated nor honest are indicated by

(a) 7 (b) 8
(c) 10 (d) 12

22. Uneducated, urban, hard working and honest people are indicated by

(a) 2 (b) 3
(c) 4 (d) 5

23. Urban hard working who are neither educated nor honest are indicated by

(a) 2 (b) 3
(c) 4 (d) 5

24. Non- urban educated hard working and honest people are indicated by

(a) 7 (b) 8 (c) 9 (d) 10

25. Non-urban people who are honest and hard working but not educated are indicated by

(a) 11 (b) 10 (c) 9 (d) 3

26. People should drink more milk because

(a) it would help the farmers
(b) it does not keep fit for a long time
(c) it is sold in many schools
(d) it is a good food

27. Motorists must have driving mirrors in their cars so that
(a) their lady passengers can use them
(b) they can see the traffic behind them
(c) other motorists can see them
(d) their head lamps will not dazzle oncoming traffic

28. Glass is used for window's because
(a) glass is cheap
(b) you can see through glass
(c) a broken window is easily replaced
(d) glass is easily cut to the right size

29. Many people send Christmas cards to their friends because
(a) they are pretty
(b) it is a cheaper to send them, throught post
(c) it is the custom to send good wises to friends at Christmas time in that way
(d) it provides the work for postman

30. A, B, C and D are standing at the corners of a square field. They walk along the sides of the square in the clockwise direction. They stop after covering four sides which one of the following statement is true?
(a) C is North - East of B (b) D is East - West of A
(c) A is West of B (d) B is East - South of D

31. A's mother is sister of B and has a daughter C. How is A related to B?
(a) Niece (b) Uncle
(c) Daughter (d) Father

32. A is brother of B and C; D is C's mother. D is B's sister and E is B's sister. How is C related to E?
(a) Niece (b) Cousin
(c) Aunt (d) Mother

33. Out of five friends A is shorter than B but taller than E. C is tallest and D is little shorter than A. Which one is the shortest?
(a) A (b) E (c) C (d) D

34. In a row, at a bus stop A is 7th from the left and B is 9th from the right. They both interchange their positions. A becomes 11th from the left. How many people are there in the row?
(a) 18 (b) 19 (c) 20 (d) 21

35. A man drove his car straight towards East for 5 km. Then, he turn right and drove for 3 km and then he turned to his South and drove for 3 km. How far was he from the starting point?
(a) 6 km (b) 5 km (c) 7 km (d) 8 km

Directions (Q.Nos. 36-40) *Read the following informatior carefully to answer the questions that follow.*

A and B are good at hockey and volleyball. C and A are good at hockey and baseball. D and B are good at cricket and volleyball. D and E are good as football and baseball.

36. Who is good of the largest number of games?
(a) E (b) D
(c) C (d) B

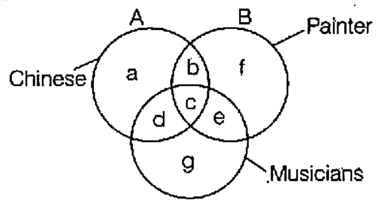
37. Who is good at cricket, hockey and volleyball?
(a) E (b) D
(c) C (d) B

38. Who is good at baseball, volley ball and hockey?
(a) E (b) D
(c) A (d) B

39. Who is good at cricket, baseball and volleyball?
(a) E (b) D
(c) C (d) B

40. Who of the following is good at all the four games?
(a) E (b) D
(c) C (d) B

Directions (Q.Nos. 41-44) *In the figure given below, there are three intersecting circles each representing certain section of people. Different regions are marked a-g. Read the statements in each of the following questions and choose the letter of the region which correctly represents the statemet.*



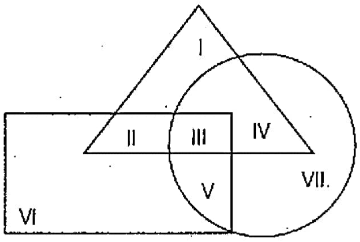
41. Chinese who are painters as well as musicians
(a) a (b) b
(c) c (d) d

42. Chinese who are musicians but not painters
(a) d (b) c
(c) b (d) a

43. Painters who are neither Chinese nor musicians
(a) b (b) c
(c) f (d) g

44. Chinese who are painters but not musicians
(a) b
(b) c
(c) d
(d) g

45. The triangle, square and circle shown below respectively represent the urban hard working and educated people. Which one of the areas marked I-VII is represented by the urban educated people who are not hard working?



(a) II (b) I (c) IV (d) III

Directions (Q.Nos. 46-50) *Read the following information carefully to answer the questions that follow*

Vijay starts from his home with his wife at 9:30 am. On his scooter. He goes 1 km East, drop his wife at her office, turns left goes another kilometre. Then, turns right, and after a kilometre reaches the bank where he spends five minutes. Then, he turn towords North and reaches hospital which is 1 km from the bank. After taking to a doctor friend for 5 min he turns towards West and reaches his office, which is 2 km from the hospital at 9:58 am.

46. At what time did Vijay reach the bank?

(a) 9:42 am (b) 9:39 am
(c) 9:45 am (d) 9:36 am

47. What is the average speed?

(a) 18 km/h (b) 25 km/h
(c) 30 km/h (d) 20 km/h

48. How much time would Vijay had taken in reaching his office by the same route, if he had not stopped at the bank and hospital?

(a) 20 min (b) 23 min
(c) 18 min (d) 13 min

49. How far is the hospital from his wife's office?

(a) 3 km (b) 4 km (c) $4\frac{1}{2}$ km (d) 1 km

50. How far is Vijay's office from his home as the crow flies?

(a) 3 km (b) 4 km
(c) 2 km (d) 5 km

51. If α and β be two directions of projection to hit a given point (h, k), then

(a) $\cos(\alpha+\beta) = -\frac{h}{k}$ (b) $\sin(\alpha+\beta) = -\frac{h}{k}$
(c) $\cot(\alpha+\beta) = -\frac{h}{k}$ (d) $\tan(\alpha+\beta) = -\frac{h}{k}$

52. A shot is fixed at an angle α to the horizontal up an inclined plane of inclination β. It will strike the plane horizontally, if

(a) $\tan\alpha = \tan\beta$ (b) $2\tan\alpha = \tan\beta$
(c) $\tan\alpha = 2\tan\beta$ (d) $4\tan\alpha = \tan\beta$

53. If at any instant the velocity of projectile be u and its direction of motion α to the horizon, then it will be moving at right angles to this direction after time

(a) $\frac{u}{g}\sin\alpha$ (b) $\frac{u}{g}\cos\alpha$
(c) $\frac{u}{g}\operatorname{cosec}\alpha$ (d) $\frac{u}{g}\sec\alpha$

54. The simple harmonic motion is the motion of a particle which moves in a straight line, so that the acceleration is always directed towards a fixed points on the line and varies as the

(a) distance from the fixed point
(b) square of the distance from the fixed point
(c) reciprocal of the distance from the fixed point
(d) reciprocal of the square of the distance from the fixed point

55. The singular solution of the differential equation $y = xp + a\sqrt{1+p^2}\left(p = \frac{dy}{dx}\right)$ is a

(a) parabola (b) hyperbola
(c) circle (d) straight line

56. A mass of 10 kg falls from rest, and is, then brought to rest by penetrating 1 m into some sand; the average thrust of the send on it is (taking $g = 10\text{m/s}^2$)

(a) 800 N (b) 900 N
(c) 1000 N (d) 1100 N

57. If a particle moves along a plane curve, then its velocity along the normal at every point is

(a) zero (b) unity
(c) finite (d) infinite

58. If a particle moves on a cycloid, then the motion is

(a) linear (b) simple harmonic
(c) simple (d) parabolic

59. If a particle moves along $x = a(2t + \sin 2t)$ and $y = a(1 - \cos 2t)$, then acceleration is

(a) constant
(b) variable
(c) unknown
(d) known

60. The displacement has
(a) only magnitude
(b) only direction
(c) both magnitude and direction
(d) constant negative quantity

61. Weights W, ω and W are attached to points B, C, D respectively of a light string AE, where B, C and D divide the string into 4 equal lengths. If the string hangs in the form of 4 consecutive sides of a regular octagon with the ends A and E attached to points on the same level, then
(a) $W = 2\omega$
(b) $W = (\sqrt{2}+1)\omega$
(c) $W = (\sqrt{3}+1)\omega$
(d) $W = 4\omega$

62. A uniform rod of weight W rests with its ends in contact with two smooth planes, inclined at angles α and β respectively to the horizon and intersecting in a horizontal line. The inclination θ of the rod to the vertical is given by
(a) $2\tan\theta = \tan\beta - \tan\alpha$
(b) $2\tan\theta = \tan\alpha - \tan\beta$
(c) $2\cot\theta = \cot\beta - \cot\alpha$
(d) $2\cot\theta = \cot\alpha - \cot\beta$

63. Let P, Q, R be the sum of the components of various forces acting at a point, in three mutually perpendicular directions. Then, the forces are in equilibrium if
(a) $P = Q = R$
(b) $P + Q = Q + R = R + P = 0$
(c) $P = Q = R = 0$
(d) $P + Q + R = 0$

64. The moments of a system of coplanar forces (not in equilibrium) about three collinear points A, B and C in the plane are G_1, G_2 and G_3. Then,
(a) $G_1 \cdot AB + G_2 \cdot BC + G_3 \cdot CA = 0$
(b) $G_1 \cdot BC + G_2 \cdot CA + G_3 \cdot AB = 0$
(c) $G_1 \cdot CA + G_2 \cdot AB + G_3 \cdot BC = 0$
(d) $G_1 \cdot G_2 + CA \cdot AB + G_3 \cdot BC = 0$

65. If six forces, of relative magnitudes 1, 2, 3, 4 and 5 and 6 act along the sides of a regular hexagon,taken in order, then the single equivalent force is of relative magnitude
(a) 1
(b) 3
(c) 5
(d) 6

66. $ABCD$ ... is a polygon of n sides, and forces act at a point parallel and proportional to AB, $2BC$ and $3CD$, etc. If O be the centroid of all the points B, C, D, ... including A, then their resultant is parallel and proportional to
(a) $(n-2)OA$
(b) $(n+1)\ OA$
(c) nOA
(d) $(n-1)\ OA$

67. Two forces given in magnitude at each through a fixed point, and are inclined at a constant angle θ. If θ varies, then the locus of A is
(a) a straight line
(b) a circle
(c) a parabola
(d) an ellipse

68. If three forces acting at a point are represented in magnitude and direction by three sids of a triangle, taken in order, they are in equilibrium. This condition is
(a) only necessary and not sufficient
(b) only sufficient but not necessary
(c) both necessary as well as sufficient
(d) neither necessary nor sufficient

69. The resultant of P and Q is R. If Q is doubled, R is also doubled. If Q is reversed, R is again doubled. Then $P^2 : Q^2 : R^2$ is given by
(a) 1 : 1 : 1
(b) 2 : 2 : 3
(c) 2 : 3 : 2
(d) $3 : 2 : \sqrt{2}$

70. In the Linear Programming Problem (LPP)
Maximize $Z = 4x + y$
Subject to constraints

$$3x + 5y \le 15$$
$$5x + y \le 15$$
$$-x + y \le 2$$
$$4x + 5y \le 20\ ]$$
$$x, y \ge 0 \text{ has}$$

(a) no solution
(b) one solution
(c) infinite solution
(d) finite solution

71. Fit a straight line regression of Y of X from the following table

X	0	1	2	3	4	5	6
Y	2	1	3	2	4	3	5

(a) $Y = 0.35 + 1.578\,X$
(b) $Y = 1.578 + 0.35\,X$
(c) $Y = 1.357 + 0.5\,X$
(d) $Y = 0.5 + 1.357\,X$

72. The value of the correlation coefficient between two variables lies between
(a) 0 and ∞
(b) $-\infty$ and ∞
(c) 0 and 1
(d) -1 and 1

73. In the method of least square of curve fitting, if n are constants, then the normal equations are
(a) n^2
(b) n
(c) $n-1$
(d) $n+1$

74. For a normal curve the greatest ordinate is
(a) $\frac{1}{\sigma\sqrt{2\pi}}$
(b) $\frac{1}{\sqrt{2\pi\sigma}}$
(c) $\sigma\sqrt{2\pi}$
(d) $2\pi\sigma$

75. In case of Poisson distribution
(a) mean > variance
(b) mean < variance
(c) mean = variance
(d) mean and variance are not related

76. The variance of Binomial distribution is
(a) np (b) $1-np$
(c) npq (d) $\pm\sqrt{n-pq}$

77. For a frequency distribution standard deviation is computed by using the formula
(a) $\sigma = \frac{\Sigma f(x-\bar{x})}{\Sigma f}$ (b) $\sigma = \frac{\sqrt{\Sigma f(x-\bar{x})^2}}{\Sigma f}$
(c) $\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$ (d) $\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})}{\Sigma f}}$

78. If the sizes in the frequency distribution are given in an ascending order of magnitude, then the median is calculated by
(a) $M_d = l + \left(\frac{\frac{N}{2}+C}{f}\right) \times i$ (b) $M_d = l + \left(\frac{\frac{N}{2}-C}{f}\right) \times i$
(c) $M_d = l - \left(\frac{\frac{N}{2}+C}{f}\right) \times i$ (d) $M_d = l - \left(\frac{\frac{N}{2}-C}{f}\right) \times i$

79. The average of n numbers $x_1, x_2, x_3, \ldots, x_n$ is M. If x_1 is replaced by x', then the new average is
(a) $M - x_n + x'$ (b) $\frac{M - x_n + x'}{n}$
(c) $\frac{(n-1)M - x_n + x'}{n}$ (d) $\frac{nM - x_n + x'}{n}$

80. In a box containing 100 bulbs, 10 are defective. What is the probability that out of a sample of 5 bulbs, none is defective?
(a) 10^{-5} (b) $\left(\frac{1}{2}\right)^5$
(c) $\left(\frac{9}{10}\right)^5$ (d) $\frac{9}{10}$

81. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel, is
(a) $\frac{3}{11}$ (b) $\frac{4}{11}$ (c) $\frac{2}{11}$ (d) 0

82. In a single throw with two dice, the chances of throwing eight is
(a) $\frac{7}{36}$ (b) $\frac{1}{18}$ (c) $\frac{1}{9}$ (d) $\frac{5}{36}$

83. The probability that A, B and C can solve problem is $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ respectively they attempt independently, then the probability that the problem will solved is
(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

84. The value of $\cos^{-1}\sqrt{\frac{2}{3}} - \cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}}$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

85. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to
(a) 0 (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

86. The solution of the equation
$$3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$$ is
(a) $x = \sqrt{3}$ (b) $x = \frac{1}{\sqrt{3}}$
(c) $x = 1$ (d) $x = 0$

87. If the angles of elevation of the top and bottom of a flag staff fixed at the top of a tower at a point distant a from the foot of a tower are α and β, then height of the flag staff is
(a) $a(\sin\alpha - \sin\beta)$ (b) $a(\cos\alpha - \cos\beta)$
(c) $a(\cot\alpha - \cot\beta)$ (d) $a(\tan\alpha - \tan\beta)$

88. If the angle of elevation of a cloud at a height h above the level of water in a lake is α and the angle of depression of its image in the lake is β, then the height of the cloud above the surface of the lake is not correct
(a) $\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$ (b) $\frac{h\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$
(c) $\frac{h(\cot\alpha + \cot\beta)}{\cot\alpha - \cot\beta}$ (d) $\frac{h\cos(\alpha+\beta)}{\sin(\beta-\alpha)}$

89. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is
(a) 1 (b) 0
(c) $\frac{x-1}{x+1}$ (d) $\frac{x+1}{x-1}$

90. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$ is

(a) -1 (b) 1 (c) -2 (d) 2

91. The maximum value of $\frac{\log_e x}{x}$ is

(a) 1 (b) $\frac{2}{e}$ (c) e (d) $\frac{1}{e}$

92. If $A + B + C = \pi$ and $x = \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$, then

(a) $x \geq \frac{1}{8}$ (b) $x \leq \frac{1}{8}$
(c) $x \geq \frac{1}{2}$ (d) $x \leq \frac{1}{2}$

93. The value(s) of $\cos\frac{\pi}{7}\cos\frac{4\pi}{7}\cos\frac{5\pi}{7}$ is/are

(a) $-\frac{1}{8}$ (b) $-\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

94. If $\operatorname{cosec} A + \cot A = \frac{5}{2}$, then $\tan A$ is equal to

(a) $\frac{4}{9}$ (b) $\frac{3}{5}$
(c) $\frac{15}{16}$ (d) $\frac{20}{21}$

95. The point of intersection of the lines $\mathbf{r} \times \mathbf{a} = \mathbf{b} \times \mathbf{a}$ and $\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$ is

(a) $\mathbf{a}$ (b) $\mathbf{b}$
(c) $\mathbf{a} + \mathbf{b}$ (d) $\mathbf{a} - \mathbf{b}$

96. What is the value of $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) + (\mathbf{a} \times \mathbf{c}) \times (\mathbf{d} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{d}) \times (\mathbf{b} \times \mathbf{c})$?

(a) $2[\mathbf{adc}]\mathbf{b}$ (b) $2[\mathbf{bdc}]\mathbf{a}$
(c) $2[\mathbf{acd}]\mathbf{b}$ (d) $2[\mathbf{bcd}]\mathbf{a}$

97. If **a**, **b** and **c** are vectors from the origin to the point A, B and C, then $(\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a})$

(a) perpendicular to the plane ABC
(b) parallel to the plane ABC
(c) lies in the plane ABC
(d) is null vector

98. The volume of the parallelopiped whose edges are represented by $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ is

(a) 10 (b) 7 (c) 6 (d) 5

99. Which of the following is correct?

(a) $(\mathbf{a} \times \mathbf{b}) = (\mathbf{a} \cdot \mathbf{b})^2 + \mathbf{a}^2\,\mathbf{b}$
(b) $(\mathbf{a} \times \mathbf{b}) = (\mathbf{a} \cdot \mathbf{b})^2 - \mathbf{a}^2\,\mathbf{b}^2$
(c) $(\mathbf{a} \times \mathbf{b})^2 + (\mathbf{a} \cdot \mathbf{b})^2 = \mathbf{a}^2\,\mathbf{b}^2$
(d) $(\mathbf{a} \times \mathbf{b})^2 + (\mathbf{a} \cdot \mathbf{b})^2 + \mathbf{a}^2\,\mathbf{b}^2 = 0$

100. If **a**, **b** and **c** be three non-coplanar vectors, then the cross product is

(a) distributive over scalar product of vectors
(b) not distributive over scalar product of vectors
(c) distributive over addition of vectors
(d) not distributive over addition of vectors

101. The angle between two non-zero vectors **a** and **b** is given by

(a) $\sin^{-1}\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{ab}|}$ (b) $\cos^{-1}\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a} \cdot \mathbf{b}|}$
(c) $\sin^{-1}\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$ (d) $\cos^{-1}\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

102. The point of intersection of the line $\mathbf{r} = \mathbf{a} + \mathbf{b} + t\mathbf{c}$ and the plane $\mathbf{r} = \mathbf{a} - \mathbf{b} + t_1(\mathbf{a} + \mathbf{b} - \mathbf{c}) + t_2(\mathbf{a} - \mathbf{b} + \mathbf{c})$ is

(a) $2\mathbf{a} - 3\mathbf{b} + 4\mathbf{c}$ (b) $2\mathbf{a} - 3\mathbf{b} - 4\mathbf{c}$
(c) $2\mathbf{a} + 3\mathbf{b} + 4\mathbf{c}$ (d) $2\mathbf{a} + 3\mathbf{b} - 4\mathbf{c}$

103. The area of the triangle having vertices (1, 3, 2), (2, – 1, 1) and (– 1, 2, 3) is

(a) $\frac{1}{2}\sqrt{107}$ (b) $\frac{1}{2}\sqrt{155}$ (c) $\frac{1}{2}\sqrt{165}$ (d) $\frac{1}{2}\sqrt{187}$

104. The system of vectors are

(a) Never closed under addition and multiplication
(b) Closed under addition and in a restricted sense in multiplication
(c) Closed under addition and multiplication
(d) Closed under addition only

105. Which of the following differential equations can be reduced to homogeneous form?

(a) $y(e^x + x^2y)\,dx - e^x\,dy = 0$
(b) $x^3\frac{dy}{dx} - x^2y + y^4\cos x = 0$
(c) $(4x + 6y + 5)\,dx = (2x + 3y + 4)\,dy$
(d) $(1 + y^2)\,dx + (x - \sin y)\,dy = 0$

106. Which of the following differential equation is linear?

(a) $(1 + y)\frac{dy}{dx} + \cos x = 0$
(b) $x + y\frac{dy}{dx} = 0$
(c) $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + (1 + x)\,y = e^x$
(d) $(1 + y)\frac{d^2y}{dx^2} + xy = e^x + x$

107. The differential equation $y = px + f(p)$ is called of

(a) Clairaut's form
(b) newtonian form
(c) Bernoulli's form
(d) Euler's form

108. If r is radius and k is thickness of a frustum of a sphere, then its curved surface of frustum is

(a) $\frac{1}{2}\pi rk$ (b) πrk (c) $2\pi rk$ (d) $4\pi rk$

109. If h is height and r_1, r_2 are the radius of the end of the frustum of a cone, then the volume of the frustum is

(a) $\frac{\pi h}{3}(r_1^2 - 3r_1r_2 + r_2^2)$ (b) $\frac{\pi h}{3}(r_1^2 + 3r_1r_2 + r_2^2)$

(c) $\frac{\pi h}{3}(r_1^2 - r_1r_2 + r_2^2)$ (d) $\frac{\pi h}{3}(r_1^2 + r_1r_2 + r_2^2)$

110. The value of $\int_0^{\pi/2} \log(\tan x)\,dx$ is equal to

(a) 0 (b) $\frac{x}{4}$ (c) $\frac{x}{2}$ (d) π

111. The value of $\int \frac{x + \sin x}{1 + \cos x}\,dx$ is

(a) $x \cot \frac{x}{2}$ (b) $x \tan \frac{x}{2}$

(c) $x \sin \frac{x}{2}$ (d) $x \cos \frac{x}{2}$

112. The value of $\int \frac{xe^x}{(x+1)^2}\,dx$ is

(a) $\frac{1}{x+1}e^x + C$ (b) $(x-1)^2 e^x + C$

(c) $(x+1)e^x + C$ (d) $e^x + C$

113. If x and y be two real variables, such that $x > 0$ and $xy = 1$, then the minimum value of $x + y$ is

(a) 1 (b) -1

(c) 2 (d) -2

114. The condition that the curve $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ should intersect orthogonally is that

(a) $a + b = a' + b'$ (b) $a - b = a' - b'$

(c) $\frac{1}{a} + \frac{1}{b} = \frac{1}{a'} + \frac{1}{b'}$ (d) $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$

115. The equation of tangent at (2, 2) of the curve $xy^2 = 4(4 - x)$ is

(a) $x - y = 4$ (b) $x + y = 4$

(c) $x - y = 2$ (d) $x + y = 2$

116. The derivative of $\sin^{-1}\frac{1-x^2}{1+x^2}$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

(a) -1 (b) 0

(c) $\frac{1}{x}$ (d) x

117. The function $f(x)$ defined by

$$f(x) = x\left[1 + \frac{1}{3}\sin(\log x^2)\right], \quad x \neq 0$$
$$f(x) = 0, \quad x = 0$$

, then

(a) $f(x)$ is continuous at $x = 0$

(b) $f(x)$ has discontinuity of first kind at $x = 0$

(c) $f(x)$ has discontinuity of second kind at $x = 0$

(d) $f(x)$ has removable discontinuity at $x = 0$

118. $\lim_{n \to \infty} (1 + x)^{1/x}$ is equal to

(a) 0 (b) 1

(c) e (d) $\frac{1}{e}$

119. For the conic $\frac{l}{r} = 1 + e\cos\theta$, the sum of reciprocals of the segments of any focal chord is equal to

(a) l (b) $2l$

(c) $\frac{1}{l}$ (d) $\frac{2}{l}$

120. If the line $lx + my = n$, touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if

(a) $a^2l^2 - b^2m^2 = n^2$

(b) $al - bm = n$

(c) $a^2l^2 + b^2m^2 = n^2$

(d) $al + bm = n$

121. The straight line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

(a) $p^2 = a^2\cos^2\alpha - b^2\sin^2\alpha$

(b) $p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$

(c) $p^2 = a^2\sin^2\alpha - b^2\cos^2\alpha$

(d) $p^2 = a^2\sin^2\alpha + b^2\cos^2\alpha$

122. The focal distance of a point on the parabola $y^2 = 8x$ is 4. Its ordinates are

(a) ± 1 (b) ± 2

(c) ± 3 (d) ± 4

123. The radius of the circle on which the four points of intersection of the lines $(2x - y + 1)(x - 2y + 3) = 0$ with the axes lie, is

(a) 5 (b) $\frac{5}{\sqrt{2}}$

(c) $\frac{5}{2\sqrt{2}}$ (d) $\frac{5}{4\sqrt{2}}$

124. The equation of circle passing through (–1, 2) and concentric with $x^2 + y^2 - 2x - 4y - 4 = 0$ is

(a) $x^2 + y^2 - 2x - 4y + 1 = 0$

(b) $x^2 + y^2 - 2x - 4y + 2 = 0$

(c) $x^2 + y^2 - 2x - 4y + 4 = 0$

(d) $x^2 + y^2 - 2x - 4y + 8 = 0$

125. The angle between the two straight line represented by the equation $6x^2 + 5xy - 4y^2 + 7x + 13y - 3 = 0$ is

(a) $\tan^{-1}\frac{3}{5}$ (b) $\tan^{-1}\frac{5}{3}$

(c) $\tan^{-1}\frac{2}{11}$ (d) $\tan^{-1}\frac{11}{2}$

126. The equation of the straight line passing through the point of intersection of $4x + 3y - 8 = 0$ and $x + y - 1 = 0$ and the point (– 2, 5) is

(a) $9x + 7y - 17 = 0$ (b) $4x + 5y + 6 = 0$

(c) $3x - 2y + 19 = 0$ (d) $3x - 4y - 7 = 0$

127. If m is the mid-point of the side BC of the triangle ABC, then

(a) $AB^2 + AC^2 = AM^2 + BM^2$

(b) $AB^2 + AC^2 = 2AM^2 + 2BM^2$

(c) $AM^2 + MB^2 = 2AC^2$

(d) $2AB^2 + 2AC^2 = AM^2 + MB^2$

128. The straight line passes through the point $P(2, \sqrt{3})$ and makes an angle of 60° with the x-axis. The length of the intercept on it between the point P and the line $x + \sqrt{3}\, y = 12$ is

(a) 1.5 (b) 2.5

(c) 3.5 (d) 4.5

129. Let H be a sub-group of a group G and $a, b \in H$. Let $a \sim b$ iff $a \equiv b$ mod H, then which of the following is true?

(a) '~' is a reflexive relation

(b) '~' is symmetric relation

(c) '~' is a transitive relation

(d) All of the abvoe

130. Let P be a probability function on $S = (l_1, l_2, l_3, l_4)$ such that $P(l_2) = \frac{1}{3}$, $P(l_3) = \frac{1}{6}$, $P(l_4) = \frac{1}{9}$. Then, $P(l_1)$ is

(a) $\frac{7}{18}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{2}$

131. If U = {Natural numbers}, A = {Multiples of 3} and B = {Multiples of 5}, then $A - B$ equals to

(a) $\overline{A} \cap B$ (b) $A \cap \overline{B}$

(c) $\overline{A} \cup \overline{B}$ (d) $\overline{A \cap B}$

132. Which of the following statement is true?

(a) For any two sets A and B either $A \subseteq B$ or $B \subseteq A$

(b) $\{a\} \in \{a, b, c\}$

(c) $\{a, b, a, b, a, b, \ldots\}$ is an infinite set

(d) $\{a, b, c\} = \{c, a, b\}$

133. State which of the following statements is correct?

(a) Every set has a proper subset

(b) Every subset of a finite set is finite

(c) Every subset of an infinite set is infinite

(d) The set $\{x : x + 8 = 8\}$ is the null set

134. If the matrices $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 4 & 1 \end{bmatrix}$, then AB is equal to

(a) $\begin{bmatrix} -3 & -1 \\ -9 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 1 \\ 9 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 \\ -9 & 3 \end{bmatrix}$

135. If ω is cube root of unity, then the value of determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is equal to

(a) – 1 (b) 1 (c) 0 (d) 2

136. The expression $\frac{12}{3 + \sqrt{5} + 2\sqrt{2}}$ is equal to

(a) $1 - \sqrt{5} + \sqrt{2} + \sqrt{(10)}$

(b) $1 - \sqrt{5} - \sqrt{2} + \sqrt{(10)}$

(c) $1 + \sqrt{5} - \sqrt{2} + \sqrt{(10)}$

(d) $1 + \sqrt{5} + \sqrt{2} - \sqrt{(10)}$

137. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then the value of r is

(a) 1 (b) 2 (c) 3 (d) 4

138. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero. Then, $\frac{a}{b}$ is equal to

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

139. The sum of 16 terms of the following series $\frac{1^3}{1}+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\ldots$ 16 term is

(a) 446 (b) 816
(c) 464 (d) 460

140. If $x = 1 + a + a^2 + a^3 + \ldots \infty\ (a < 1)$, $y = 1 + b + b^2 + b^3 + \ldots \infty\ (b < 1)$, then the value of $1 + ab + a^2b^2 + a^3b^3 + \ldots \infty$ is equal to

(a) $\frac{xy}{x+y+1}$ (b) $\frac{xy}{x+y-1}$
(c) $\frac{x-y}{x+y+1}$ (d) $\frac{x-y}{x+y-1}$

141. If the sum of first p terms of an AP is q and the sum of the first q terms is p, then the sum of the first $(p+q)$ terms is

(a) $p+q+1$ (b) $-(p+q+1)$
(c) $-(p+q)$ (d) $p+q$

142. The nth term of the series $2\frac{1}{2}+1\frac{7}{13}+1\frac{1}{9}+\frac{20}{23}+\ldots$ is

(a) $\frac{20}{5n^2+3}$ (b) $20(5n+3)$
(c) $\frac{2}{5n-3}$ (d) $\frac{20}{5n+3}$

143. The value of $7\log\frac{16}{15}+5\log\frac{25}{24}=3\log\frac{81}{80}$ is

(a) log 2 (b) zero
(c) unity (d) 0.2

144. The value of complex number $\left(\frac{1+i}{\sqrt{2}}\right)^8+\left(\frac{1-i}{\sqrt{2}}\right)^8$ is

(a) 2 (b) -2
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

145. If the sum of three consecutive odd natural numbers is 153, then the numbers are

(a) 47, 49, 51 (b) 49, 51, 53
(c) 51, 53, 55 (d) 53, 55, 57

146. The solution of the following equation

$$\frac{x-1}{3}-\frac{4x+1}{4}=\frac{1}{12}$$

is

(a) -1 (b) 1
(c) -2 (d) 2

147. The factorisation of the expression $36x^2-12x+1-25y^2$ is

(a) $(5x-6y+1)(5x+6y+1)$
(b) $(6x-5y+1)(6x+5y+1)$
(c) $(5x-6y-1)(5x+6y-1)$
(d) $(6x-5y-1)(6x+5y-1)$

148. If $\frac{\log x}{b-c}=\frac{\log y}{c-a}=\frac{\log z}{a-b}$, then $x^a y^b z^c$ equals to

(a) 0 (b) 1
(c) xyz (d) None of these

149. Which of the following decimal number is in the form $\frac{p}{q}$?

(a) $\frac{60}{7}$ (b) $\frac{68}{9}$
(c) $\frac{63}{4}$ (d) $\frac{62}{5}$

150. Which of the following is the decimal representation of $\frac{22}{7}$?

(a) $3.\overline{142857}$
(b) $3.\overline{142867}$
(c) $3.\overline{14957}$
(d) $3.\overline{14967}$

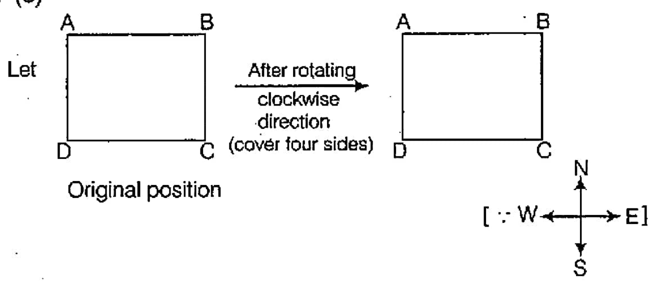
Answer *with* Explanations

1. (b) Since the condition under if $(a <= b)$ is not given in the flowchart.
2. (c) Compiler gives 'L value required' due to condition $(a = 0)$ under, if. So, the given program fragment nothing to be print.
3. (c) If $(y - 8)$ is an integral multiple of 9 both will yield the same value.
4. (a) If the integer needs two bytes of storage, then maximum value of an unsigned integer is $2^{16} - 1$.
5. (a) The minimum number of temporary variables needed to swap the contents of two variables is 1.

e.g.,

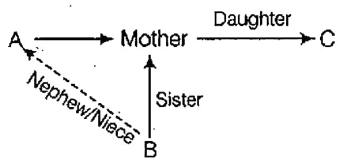
```
int x = 10, y = 10, z;
main ()
{
    z = x;
    x = y;
    y = z;
}
```

6. (c) Conditional compilation is compiling a program based on a condition.
7. (a) The basic arithmetic operation performed by a computer is 'addition'.
8. (a) The base of the binary number system is 2 because it has only two number 0 and 1.
9. (b) Main memory unit of computer stores a small amount of data and instructions.
10. (c) A Central Processing Unit (CPU) consists of Arithmetic and Logical Unit (ALU), Central Unit (CU).
11. (b) He was heart broken by her indifference for him.
12. (c) Do not look down at the poor.
13. (a) The tone of the passage is **gloomy**.
14. (c) The writer's response to his father marrying again is that of approval.
15. (b) The writer's life got 'shattered into pieces' when he lost his mother.
16. (c) India, originate the decimal system of numbers.
17. (a) Uttar Pradesh is only state which has 2 as the first digit in its PIN code.
18. (a) Serpentina has been found useful in keeping the level of holesterol down.
19. (d) Andaman and Nicobar Islands is the Southern most point of Indian territory.
20. (b) Tuberculosis usually spreads through air.
21. (d) Number 12 indicated hard working and non-urban people who are neither educated non-honest people.
22. (c) Numebr 4 indicated uneducated, urban, hard working and honest people.
23. (b) Number 3 indicated urban hard working neither educated nor honest people.
24. (c) Number 9 indicated non-urban educated hard working and honest people.
25. (a) Number 11 indicates non-urban people who are honest and hard working but not educated.
26. (d) People should drink more milk because it is a good food.
27. (b) Motorists must have driving mirrors in their cars, so that they can see the traffic behind them.
28. (b) Glass is used for window's because you can see through glass.
29. (c) Many people send Christmas cards to their friends because it is the custom to send good wises to friends at christmas time in that way.
30. (c)



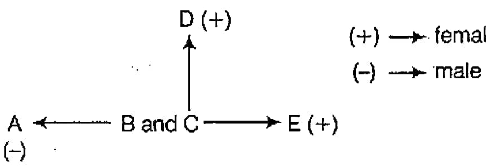
Hence, A is West of B is definitely true.

31. (a)



Hence, A is Nephew/Neice of B.

32. (b)



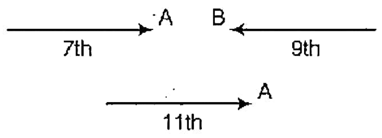
So, C may be the cousin of E.

33. (b) According to the question,

$$C > B > A > D > E$$

Hence, E is the shortest among them.

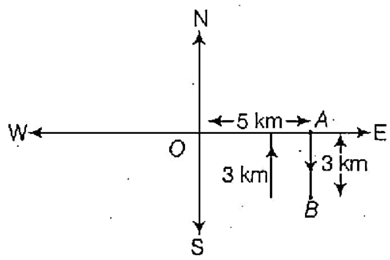
34. (b)



Hence, number of people in the row

$$= 11 + 9 - 1 = 19$$

35. (b) When he turned to his south and drove for 3 km then he reaches the point A.



So, the distance of final position from the starting point is $OA = 5$ km.

Solutions (Q.Nos. 36-40)

	Hockey	Volleyball	Baseball	Cricket	Football
A	✓	✓	✓		
B	✓	✓		✓	
C	✓		✓		
D		✓	✓	✓	✓
E			✓		✓

36. (b) D is good in largest number of games.

37. (d) B is good at cricket, hockey and volleyball.

38. (c) A is good at baseball, volleyball and hockey.

39. (b) D is good at cricket, baseball and volleyball.

40. (b) D is good at all the four games.

41. (c) 'c' represents painters as well as musicians.

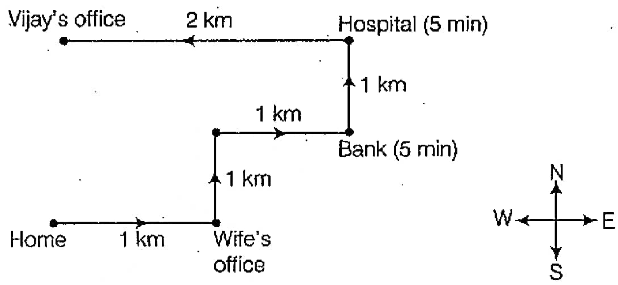
42. (a) 'd' represents chinese who are musicians but not painters.

43. (c) f represents painters who are neither chinese nor musicians.

44. (a) 'b' represents chinese who are painters but not musicians.

45. (c) Area IV represents the urban educated people who are not hard working.

Solutions (Q.Nos. 46-50)



Total distance $= (1 + 1 + 1 + 1 + 2)$

$$= 6 \text{ km}$$

Total time duration $= (9.58 - 9.30) - (5 + 5)$

$$= 28 \text{ min} - 10 \text{ min}$$

$$= 18 \text{ min}$$

$\therefore$ Time taken to cover 1 km distance

$$= \frac{18}{6} \text{ min}$$

$$= 3 \text{ min}$$

46. (b) Required time $= 9.30 + (3 + 3 + 3)$ min

$$= 9.30 + 9 \text{ min}$$

$$= 9{:}39 \text{ am}$$

47. (d) Required average speed $= \dfrac{6}{18/60}$ km/h

$$= \frac{6 \times 60}{18} \text{ km/h}$$

$$= 20 \text{ km/h}$$

48. (c) Required time $= 28 - (5 + 5) = 28 - 10$

$$= 18 \text{ min}$$

49. (a) Required distance $= (1 + 1 + 1) = 3$ km

50. (c) Required distance $= (1 + 1) = 2$ km

51. (c)

52. (c)

53. (c) $\tan(\alpha - 90°) = \dfrac{(u \sin\alpha - gt)}{u \cos\alpha}$

$$\Rightarrow \quad -\cot\alpha = \tan\alpha - \frac{gt}{u\cos\alpha}$$

$$\therefore \quad t = \frac{u\cos\alpha}{g}[\tan\alpha + \cot\alpha]$$

$$= \frac{u}{g}\operatorname{cosec}\alpha$$

54. (a) The simple harmonic motion is the motion of a particle which moves in a straight line. So, that the acceleration is always directed towards a fixed point on the line and varies as the distance from the fixed point.

55. (c) Given differential equation is,

$$y = Px + a\sqrt{1+P^2} \qquad \left(\because P = \frac{dy}{dx}\right)$$

which is in Clairauts form, $[y = Px + f(P)]$

So, its general solution is

$$y = cx + a\sqrt{1+c^2} \qquad \ldots\text{(i)}$$

$\Rightarrow \quad a\sqrt{1+c^2} = (y - cx)$

On squaring both sides, we get

$\Rightarrow \quad a^2(1+c^2) = (y-cx)^2$

$\Rightarrow \quad a^2 + a^2c^2 = y^2 + c^2x^2 - 2cxy$

$\Rightarrow \quad (a^2 - x^2)c^2 + 2xyc + (a^2 - y^2) = 0 \qquad \ldots\text{(ii)}$

The envelope of the family of curves (ii) is the singular solution.

From Eq. (ii) c-discriminant relation is

$$(2xy)^2 - 4(a^2 - x^2)(a^2 - y^2) = 0$$

$\Rightarrow \quad 4x^2y^2 - 4(a^4 - a^2x^2 - a^2y^2 + x^2y^2) = 0$

$\Rightarrow \quad 4x^2y^2 - 4a^4 + 4a^2x^2 + 4a^2y^2 - 4x^2y^2 = 0$

$\Rightarrow \quad x^2 + y^2 = a^2$

which represents a circle.

56. (b)

57. (a) If a particle moves along a plane curve, then its velocity along the normal at every point is zero.

58. (b) If a particle moves on a cycloid, then the motion is simple harmonic.

59. (d) Given curve,

$$x = a(2t + \sin 2t) \text{ and } y = a(1 - \cos 2t)$$

Now, $\quad \dfrac{dx}{dt} = a(2 + 2\cos 2t)$

and $\quad \dfrac{dy}{dx} = 2a \sin 2t$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{dy}{dt} \cdot \dfrac{dt}{dx} = 2a\sin 2t \times \dfrac{1}{a(2 + 2\cos 2t)}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{\sin 2t}{(1+\cos 2t)} = \dfrac{2\sin t \cdot \cos t}{1 + 2\cos^2 t - 1} = \dfrac{\sin t}{\cos t}$

$\Rightarrow \quad \dfrac{dy}{dx} = \tan t$

Again, $\quad \dfrac{d^2y}{dx^2} = \sec^2 t \cdot \dfrac{dt}{dx} = \sec^2 t \times \dfrac{1}{2a(1+\cos^2 t)}$

$$= \frac{\sec^2 t}{2a \cdot 2\cos^2 t} = \frac{1}{4a}\sec^4 t$$

$\therefore \quad \text{Acceleration} = \dfrac{1}{4a}\sec^4 t$

So, the acceleration is known.

60. (c) The displacement has both magnitude and direction.

61. (b)

62. (d)

63. (d)

64. (b) Here, $\quad G_1 = F \cdot OA \sin\theta$

$G_2 = F \cdot OB \sin\theta$

$\Rightarrow \quad G_3 = F \cdot OC \sin\theta$

$\therefore \quad G_1 \cdot BC + G_2 \cdot CA + G_3 \cdot AB$

$= F\sin\theta\,(OA \cdot BC + OB \cdot CA + OC \cdot AB)$

$\Rightarrow F\sin\theta\,[OA\,(OC - OB) + OB\,(OA - OC) + OC\,(OB - OA)] = 0$

$\Rightarrow (F \cdot OA \sin\theta)(BC) + (F \cdot OB \sin\theta)(CA) + (F \cdot OC \sin\theta)(AB) = 0$

$\Rightarrow G_1 \cdot BC + G_2 \cdot CA + G_3 \cdot AB = 0$

65. (d)

66. (d)

67. (c)

68. (c)

69. (c) Let θ be the angle between P and Q, then

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta \qquad \ldots\text{(i)}$$

$$(2R)^2 = P^2 + (2Q)^2 + 4PQ\cos\theta \qquad \ldots\text{(ii)}$$

and $\quad (2R)^2 = P^2 + Q^2 + 2PQ\cos(\pi - \theta)$

$\Rightarrow \quad (2R)^2 = P^2 + Q^2 - 2PQ\cos\theta \qquad \ldots\text{(iii)}$

From Eqs. (i) and (iii) we get

$$5P^2 = 2P^2 + 2Q^2$$

$\Rightarrow \quad 2P^2 + 2Q^2 - 4R^2 = 0 \qquad \ldots\text{(iv)}$

From Eqs. (ii) and (iii), we get

$$4R^2 = P^2 + 2Q^2$$

$\Rightarrow \quad P^2 + 2Q^2 - 4R^2 = 0 \qquad \ldots\text{(v)}$

From Eqs. (iv) and (v), we get

$$\frac{P^2}{2} = \frac{Q^2}{3} = \frac{R^2}{2}$$

Hence, $P^2 : Q^2 : R^2 = 2 : 3 : 2$

70. (b) Given, LPP is

Max $z = 4x + y$

Subject to constraints;

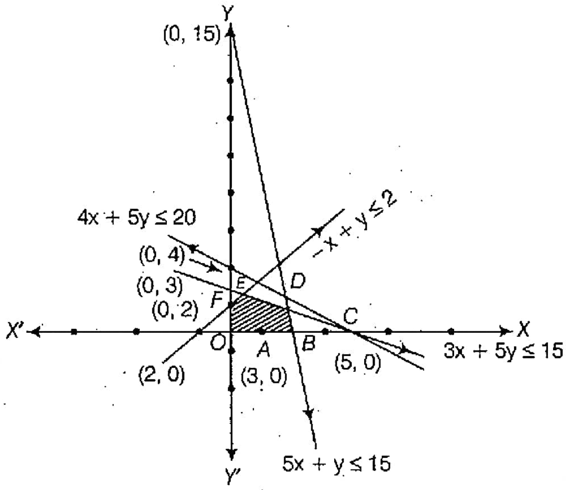
$$3x + 5y \le 15$$
$$5x + y \le 15$$
$$-x + y \le 2$$
$$4x + 5y \le 20$$
$$x, y \ge 0$$

First we assume all the inequalities into equations, we get

Equations	Corresponding points
$3x + 5y = 15$	(5, 0), (0, 3)
$5x + y = 15$	(3, 0), (0, 15)
$-x + y = 2$	(−2, 0), (0, 2)
$4x + 5y = 20$	(5, 0), (0, 4)

Now, we plot all these stationary points in plotting paper, we get

(Let 2 square = 1 unit)



From the above graph we observe that the points $O(0, 0), B(3, 0), F(0, 2), E\left(\frac{5}{8}, \frac{21}{8}\right)$ and $D\left(\frac{30}{11}, \frac{15}{11}\right)$ form a convex polygon OBDEF.

Now, we apply Corner point method

Points	Max $z = 4x + y$
$O(0, 0)$	$4 \times 0 + 0 = 0$
$B(3, 0)$	$4 \times 3 + 0 = 12$
$D\left(\frac{30}{11}, \frac{15}{11}\right)$	$4 \times \frac{30}{11} + \frac{15}{11} = \frac{135}{11}$ (max)
$E\left(\frac{5}{8}, \frac{21}{8}\right)$	$4 \times \frac{5}{8} + \frac{21}{8} = \frac{41}{8}$
$F(0, 2)$	$4 \times 0 + 2 = 2$

Hence, the given LPP have only one solution *i.e.*, $x = \frac{30}{11}$

and $y = \frac{15}{11}$

Hence, its maximum value is $\frac{135}{11}$.

71. (c) The straight line regression of y on x is given by

$$y - \bar{y} = byx\,(x - \bar{x})$$

where $byx = r \cdot \frac{\sigma y}{\sigma x} = \frac{\text{Cov}(x, y)}{\sigma^2 x}$

$$= \frac{n\,\Sigma\, x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

Here, $\bar{y} = \frac{20}{7}\; \bar{x} = \frac{21}{7} = 3$

$$byx = \frac{8\,(74) - 20 \times 21}{7\,(91) - (21)^2}$$

$$= \frac{518 - 420}{637 - 441} = \frac{22}{49}$$

So, $y - \frac{20}{7} = \frac{22}{49}(x - 3)$

which is equivalent as

$$y = 0.5\,x + 1.357$$

72. (d) The value of the correlation coefficient between two variables lies between −1 and 1.

i.e., $-1 \le r \le 1$

73. (d)

74. (a) For a normal curve the greatest ordinate is $\frac{1}{\sigma\sqrt{2\pi}}$.

(by property of normal distribution)

75. (c) In case of Poission distribution;

Mean = λ and variance = λ

i.e., mean = variance.

76. (c) The variance of binomial distribution is npq, where $p + q = 1$.

77. (c) Standard deviation from continuous series is

$$\sigma = \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f}}$$

78. (b) If the sizes in the frequency distribution are given in an ascending order of magnitude, then the median is calculated by;

$$M_d = l + \frac{\left(\frac{N}{2} - C\right)}{f} \times i$$

where, l = lower limit of the median class

f = frequency of the median class

N = the sum of all frequencies

i = the width of the median class

C = the cumulative frequency of the class preceding to median class.

79. (d) $\mu = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

$$nM = x_1 + x_2 + x_3 + \ldots + x_{n-1} + x_n$$

i.e., $nM - x_n = x_1 + x_2 + \ldots + x_{n-1}$

$$\frac{nM - x_n + x'}{n} = \frac{x_1 + x_2 + \ldots + x_{n-1} + x'}{n}$$

$\therefore$ New average $= \frac{nM - x_n + x'}{n}$

80. (c) Let P (non-defective bulb) $= \frac{90}{100} = \frac{9}{10} = P$

P (defective bulb) $= \frac{10}{100} = \frac{1}{10} = q$

$n = 5, \quad r = 5$

So, the probability that none is defective

$$= {}^5C_5 \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^{(5-5)}$$

$$= 1 \cdot \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^0$$

$$= 1 \cdot \left(\frac{9}{10}\right)^5 \cdot 1 = \left(\frac{9}{10}\right)^5$$

81. (b) The sample space = {P, R, O, B, A, B, I, L, I, T, Y}

Total number of sample space,

$n(S) = 11$

Favourable events = Vowel letters

= {O, A, I, I}

Total number of favourable events,

$n(E) = 4$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{4}{11}$

82. (d) Total number of sample space

$n(S) = 6 \times 6 = 36$

Favourable events = {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)}

Total number of favourable events,

$n(E) = 5$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{5}{36}$

83. (*) P (Problem will solved)

$= 1 - P$ (Problem will not solved)

$$= 1 - \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{3}\right)$$

$$= 1 - \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

$$= 1 - \frac{8}{27}$$

$$= \frac{19}{27}$$

84. (d) $\cos^{-1} \sqrt{\frac{2}{3}} - \cos^{-1} \frac{\sqrt{6}+1}{2\sqrt{3}}$

$$= \cos^{-1} \left[\sqrt{\frac{2}{3}} \cdot \frac{\sqrt{6}+1}{2\sqrt{3}} + \sqrt{1 - \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2} \cdot \sqrt{1 - \left(\frac{\sqrt{6}+1}{2\sqrt{3}}\right)^2} \right]$$

$$= \cos^{-1} \left[\frac{\sqrt{6}+1}{3\sqrt{2}} + \sqrt{1 - \frac{2}{3}} \cdot \sqrt{1 - \frac{7+2\sqrt{6}}{12}} \right]$$

$$= \cos^{-1} \left[\frac{\sqrt{6}+1}{3\sqrt{2}} + \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{5-2\sqrt{6}}}{2\sqrt{3}} \right]$$

$$= \cos^{-1} \left[\frac{\sqrt{6}+1}{3\sqrt{2}} + \frac{\sqrt{5-2\sqrt{6}}}{6} \right]$$

$$= \cos^{-1} \left[\frac{\sqrt{6}+1}{3\sqrt{2}} + \frac{\sqrt{(\sqrt{3}-\sqrt{2})^2}}{6} \right]$$

$$= \cos^{-1} \left[\frac{2\sqrt{6}+2+\sqrt{6}-2}{6\sqrt{2}} \right]$$

$$= \cos^{-1} \left(\frac{3\sqrt{6}}{6\sqrt{2}} \right) = \cos^{-1} \left(\frac{1}{2} \times \sqrt{3} \right) = \cos^{-1} \left(\cos \frac{\pi}{6} \right) = \frac{\pi}{6}$$

85. (c) $\sin^{-1} x + \cot^{-1} \left(\frac{1}{2}\right) \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1} x = \tan^{-1} \left(\frac{1}{2}\right)$

or $\quad x = \frac{1}{\sqrt{5}}$

86. (b) $3 \sin^{-1} \left(\frac{2x}{1+x^2}\right) - 4 \cos^{-1} \left(\frac{1-x^2}{1+x^2}\right) + 2 \tan^{-1} \left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

$\Rightarrow 3 \cdot 2 \cdot \tan^{-1} x - 4 \cdot 2 \tan^{-1} x + 2 \cdot 2 \tan^{-1} x = \frac{\pi}{3}$

$\left\{ \because 2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2}\right) = \cos^{-1} \left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1} \left(\frac{2x}{1-x^2}\right) \right\}$

$\Rightarrow \quad 6 \tan^{-1} x - 8 \tan^{-1} x + 4 \tan^{-1} x = \frac{\pi}{3}$

$\Rightarrow \quad 2 \tan^{-1} x = \frac{\pi}{3}$

$\Rightarrow \quad \tan^{-1} x = \frac{\pi}{6}$

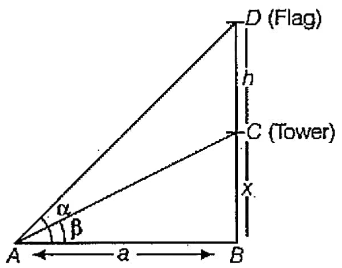
$\Rightarrow \quad x = \tan \left(\frac{\pi}{6}\right)$

$\therefore \quad x = \frac{1}{\sqrt{3}}$

87. (d) Let $CD = h$ and $BC = x$

Now, in ΔABD,

$$\tan \alpha = \frac{h+x}{a} \quad \ldots(i)$$



In ΔABC;

$$\tan\beta = \frac{x}{a} \Rightarrow x = a\tan\beta$$

On putting this value in Eq. (i), we get

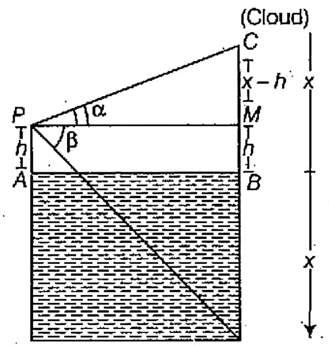
$$\tan\alpha = \frac{h + a\tan\beta}{a}$$

$$\Rightarrow \quad a\tan\alpha = h + a\tan\beta$$

$$\therefore \quad h = a(\tan\alpha - \tan\beta)$$

88. (b) Let $BC = BD = x$

$\angle CPM = \alpha$ and $\angle MPD = \beta$



and $\quad BM = AP = h$

$\therefore \quad MC = x - h$

and $\quad DM = x + h$

Now, in ΔCPM,

$$PM = MC\cot\alpha = (x-h)\cot\alpha \quad \ldots(i)$$

and in ΔDPM,

$$PM = DM\cot\beta = (x+h)\cot\beta \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$(x-h)\cot\alpha = (x+h)\cot\beta$$

$$\Rightarrow x(\cot\alpha - \cot\beta) = h(\cot\alpha + \cot\beta)$$

$$\therefore \quad x = \frac{h(\cot\alpha + \cot\beta)}{(\cot\alpha - \cot\beta)} = \frac{h(\tan\alpha + \tan\beta)}{(\tan\beta - \tan\alpha)}$$

$$= \frac{h(\cos\alpha\cdot\sin\beta + \sin\alpha\cdot\cos\beta)}{(\cos\alpha\cdot\sin\beta - \cos\beta\cdot\sin\alpha)}$$

$$= h\frac{\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$$

89. (b) $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$$y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$$\left\{\because \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right\}$$

$$y = \frac{\pi}{2}$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 0$$

90. (b) Given that,

$$\sin x + \sin^2 x = 1$$

$$\Rightarrow \quad \sin x = 1 - \sin^2 x = \cos^2 x \quad \ldots(i)$$

Now, $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$

$= (\cos^2 x)^6 + 3(\cos^2 x)^5 + 3(\cos^2 x)^4 + (\cos^2 x)^3$

$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x$ [from Eq. (i)]

$= \sin^3 x\{\sin^3 x + 3\sin^2 x + 3\sin x + 1\}$

$= \sin^3 x(1 + \sin x)^3$

$= (\sin x + \sin^2 x)^3$

$= (1)^3$ [from Eq. (i)]

$= 1$

91. (d) Let $y = \frac{\log_e x}{x}$

Now, differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{x\cdot\frac{1}{x} - \log_e x\cdot 1}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1 - \log_e x}{x^2}$$

Now, maximum or minimum value of y,

Put $\quad \frac{dy}{dx} = 0$

$$\Rightarrow \quad \frac{1 - \log_e x}{x^2} = 0 \quad (\because x \neq 0)$$

$$\Rightarrow \quad \log_e x = 1 = \log_e e$$

$$\Rightarrow \quad x = e$$

Now, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{x^2\cdot\left(-\frac{1}{x}\right) - (1 - \log_e x)\cdot 2x}{x^4}$$

$$= \frac{-x - 2x + 2x\log_e x}{x^4}$$

$$= \frac{-3 + 2\log_e x}{x^3}$$

$\frac{d^2y}{dx^2}$ at $(x=e) = \frac{-3+2\log_e e}{e^3} = \frac{-3+2}{e^3}$

$= -\frac{1}{e^3} < 0$ (Max)

So, y is maximum at $x = e$.

$\therefore$ Maximum value of $y = \frac{\log_e e}{e}$ (at $x = e$)

$= \frac{1}{e}$

92. (b)

93. (c) $\cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{5\pi}{7}$

$= \cos\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\left(\pi-\frac{5\pi}{7}\right)$

$= -\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}$

$= -\cos(2)^{\circ}\left(\frac{\pi}{7}\right)\cdot\cos(2)^1\left(\frac{\pi}{7}\right)\cdot\cos(2)^2\left(\frac{\pi}{7}\right)$

$= -\frac{\sin 2^3\cdot\left(\frac{\pi}{7}\right)}{2^3\cdot\sin\frac{\pi}{7}}$

$\left(\because \cos A\cdot\cos 2A\cdot\cos 4A\ldots\cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A}\right)$

$= \frac{-\sin\frac{8\pi}{7}}{8\cdot\sin\frac{\pi}{7}} = -\frac{\sin\left(\pi+\frac{\pi}{7}\right)}{8\cdot\sin\frac{\pi}{7}}$

$= -\left\{\frac{-\sin\frac{\pi}{7}}{8\cdot\sin\frac{\pi}{7}}\right\} = \frac{1}{8}$

94. (d) $\operatorname{cosec} A + \cot A = \frac{5}{2}$

$\Rightarrow \quad \frac{1}{\sin A} + \frac{\cos A}{\sin A} = \frac{5}{2}$

$\Rightarrow \quad 2(1+\cos A) = 5\sin A$

On squaring on both sides, we get

$\Rightarrow 4(1+\cos A)^2 = 25\sin^2 A = 25(1-\cos^2 A)$

$\Rightarrow 4 + 4\cos^2 A + 8\cos A = 25 - 25\cos^2 A$

$\Rightarrow 29\cos^2 A + 8\cos A - 21 = 0$

$\cos A = \frac{-8\pm\sqrt{64+84\cdot 29}}{58}$

$= \frac{-8\pm\sqrt{64+2436}}{58} = \frac{-8\pm\sqrt{2500}}{58}$

$= \frac{-8\pm 50}{58} = \frac{40}{58}, \frac{-58}{58}$

$= \frac{21}{29}, -1$

$\therefore \quad \tan A = \sqrt{\sec^2 A - 1} = \sqrt{\left(\frac{1}{\cos A}\right)^2 - 1}$

$= \sqrt{\left(\frac{29}{21}\right) - 1} = \sqrt{\frac{841-441}{(21)^2}} = \sqrt{\frac{400}{441}}$

$= \frac{20}{21}$, when $\left(\cos A = \frac{21}{29}\right)$

95. (c)

96. None of the options.

97. (a)

98. (b) Let $\mathbf{a} = 2\mathbf{i} - B\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{c} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

Now, $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$

$= 2(4-1) + 3(2+3) + 4(-1-6)$

$= 2\cdot 3 + 3\cdot 5 + 4(-7)$

$= 6 + 15 - 28 = 21 - 28$

$= -7$

$\therefore$ Volume of parallelopiped $= |[\mathbf{abc}]|$

$= |-7| = 7$

99. (c) LHS $= (\mathbf{a}\times\mathbf{b})^2 + (\mathbf{a}\cdot\mathbf{b})^2$

$= (|\mathbf{a}||\mathbf{b}|\sin\theta\,\mathbf{n}) + (|\mathbf{a}||\mathbf{b}|\cos\theta)$

$= (|\mathbf{a}|)^2(|\mathbf{b}|)^2\cdot\sin^2\theta\cdot 1 + |\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta$

$= |\mathbf{a}|^2|\mathbf{b}|^2\cdot(\sin^2\theta + \cos^2\theta)$

$= \mathbf{a}^2\mathbf{b}^2$ $\quad \left\{\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1 \\ \text{and} \quad |\mathbf{a}|^2 = \mathbf{a}^2\end{array}\right\}$

100. (c) If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ be three non-coplanar vectors, then the cross product is distributive over addition of vectors.

i.e., $\quad \mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b} + \mathbf{a}\times\mathbf{c}$

$\Rightarrow \quad (\mathbf{a}+\mathbf{b})\times\mathbf{c} = \mathbf{a}\times\mathbf{c} + \mathbf{b}\times\mathbf{c}$

101. (d) Let θ be the angle between two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$.

Then, $\quad \mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

$\therefore \quad \theta = \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)$

102. Given lines are,

$\mathbf{r} = \mathbf{a} + \mathbf{b} + t\mathbf{c}$

and $\quad \mathbf{r} = \mathbf{a} - \mathbf{b} + t_1(\mathbf{a}+\mathbf{b}-\mathbf{c}) + t_2(\mathbf{a}-\mathbf{b}+\mathbf{c})$

103. (a) Given that,

vertices of triangle are,

$(x_1, y_1, z_1) \to (1, 3, 2)$
$(x_2, y_2, z_2) \to (2, -1, 1)$
$(x_3, y_3, z_3) \to (-1, 2, 3)$

Now, $\Delta x = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}[3(+1-3) - 2(-1-2) + 1(-3-2)]$

$= \frac{1}{2}[-6+6-5] = -\frac{5}{2}$

$\Delta y = \frac{1}{2}\begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ -1 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}[1(1-3) - 2(2+1) + 1(6+1)]$

$= \frac{1}{2}[-2-6+7] = \frac{1}{2}(-1) = -\frac{1}{2}$

and $\Delta z = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ -1 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2}[1(-1-2) - 3(2+1) + 1(4-1)]$

$= \frac{1}{2}[-3-9+3] = = -\frac{9}{2}$

$\therefore$ Required area of triangle $= \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

$= \sqrt{\frac{25}{4} + \frac{1}{4} + \frac{81}{4}} = \frac{1}{2}\sqrt{107}$

104. (c) The system of vectors are closed under addition and multiplication. (by property of vectors)

105. (b) From option (b),

$x^3 \frac{dy}{dx} - x^2 y + y^4 \cos x = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2 y - y^4 \cos x}{x^3}$

106. (c) From option (c), we get

$\frac{d^2y}{dx^2} + x^2 \frac{dy}{dx} + (1+x)\, y = e^x$ is a linear differential equation because it has degree '1' in $y, \frac{dy}{dx}$ and $\left(\frac{d^2y}{dx^2}\right)$

107. (a) $y = Px + f(P)$ is the clairaut's form, if we replace $(P = c)$ then, we get its general solution.

i.e., $y = Cx + f(c)$

108. (c) We know that, the curved surface area of frustum of a sphere is

$= 2\pi ad$

where, a = Radius

and d = Thickness of a frustum of sphere

$\therefore$ Required curved surface area $= 2\pi rK$

109. (d) Volume of a frustum of a cone

$= \frac{K\pi}{3}(r_1^2 + r_2^2 + r_1 r_2)$

where, K = thickness of a frustum of cone.

$\therefore$ Required volume $= \frac{\pi h}{3}(r_1^2 + r_1 r_2 + r_2^2)$

110. (a) Let $I = \int_0^{\pi/2} \log(\tan x)\, dx$ ...(i)

$I = \int_0^{\pi/2} \log\left\{\tan\left(\frac{\pi}{2} - x\right)\right\} dx$

$= \int_0^{\pi/2} \log \cot x\, dx = -\int_0^{\pi/2} \log \tan x\, dx$ ...(ii)

$\{\because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\}$

On adding both equation, we get

$2\pi = 0 \Rightarrow I = \int_0^{\pi/2} \log(\tan x)\, dx = 0$

111. (b) Let $I = \int \frac{x + \sin x}{1 + \cos x} dx$

$I = \int \frac{x}{1+\cos x} dx + \int \frac{\sin x}{1 + \cos x} dx$

$= \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} dn$

$= \frac{1}{2}\int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$

$= \frac{1}{2}\left[x \cdot 2 \tan \frac{x}{2} - 2\int 1 \cdot \tan \frac{x}{2} dx\right] + \int \tan \frac{x}{2} dx$

($\because$ integeration by parts)

$= x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx + c$

$= x \tan \frac{x}{2} + c$

112. (a) Let $I = \int \frac{xe^x}{(1+x)^2} dx$

$= \int \frac{[(1+x)-1]e^x}{(1+x)^2}$

$= \int \frac{e^x}{(1+x)} dx - \int \frac{e^x}{(1+x)^2} dx$

$= \frac{1}{(1+x)} \cdot e^x - \int -\frac{1}{(1+x)^2} \cdot e^x dx - \int \frac{e^x}{(1+x)^2} dx$

($\because$ Integrating by parts)

$= \frac{e^x}{(1+x)} + \int \frac{e^x}{(1+x)^2} dx - \int \frac{e^x}{(1+x)^2} dx$

$= \frac{e^x}{(1+x)} + C$

113. (c) Let $v = x + y$ ...(i)

with given condition;

$$xy = 1 \quad ...(ii)$$

From Eq. (i),

$$v = x + \frac{1}{x}$$

On differentiating w.r.t. x, we get

$$\frac{dv}{dx} = 1 - \frac{1}{x^2}$$

For maximum or minimum value of v, we get

$$\frac{dv}{dx} = 0 \Rightarrow 1 - \frac{1}{x^2} = 0$$

$$\Rightarrow \quad (x = \pm 1)$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2v}{dx^2} = \frac{2}{x^3}$$

$$\left(\frac{d^2v}{dx^2}\right)_{(\text{at } x=1)} = \frac{2}{1} = 2 > 0 \quad (\text{min})$$

$\therefore$ $v = x + y$ is minimum at $x = 1$.

Hence, its minimum value is

$$v = 1 + 1 = 2.$$

114. (d) Given curves,

$$ax^2 + by^2 = 1 \quad ...(i)$$

and $$a'x^2 + b'y^2 = 1 \quad ...(ii)$$

For slope of curve Eq. (i),

On differentiating w.r.t. x, we get

$$2ax + 2by\frac{dy}{dx} = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_1 = -\frac{ax}{by} \quad ...(iii)$$

For slope of curve Eq. (ii),

On differentiating w.r.t. x, we get

$$2a'x + 2b'y\frac{dy}{dx} = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_2 = -\frac{a'x}{b'y} \quad ...(iv)$$

If both curves intersect orthogonally, then

$$\left(\frac{dy}{dx}\right)_1 \times \left(\frac{dy}{dx}\right)_2 = -1$$

$$\Rightarrow -\frac{ax}{by} \times -\frac{a'x}{b'y} = -1$$

$$\Rightarrow aa'x^2 = -bb'y^2 \quad ...(v)$$

On solving Eqs. (i) and (ii), we get

$$ab'x^2 + bb'y^2 = b'$$

$$\underline{a'bx^2 + bb'y^2 = b}$$

$$(ab' - a'b)x^2 = (b' - b)$$

$$\Rightarrow \left(x^2 = \frac{b' - b}{ab' - a'b}\right)$$

and $$by^2 = 1 - \frac{(ab' - ab)}{(ab' - a'b)} = \frac{ab - a'b}{ab' - a'b}$$

$$\Rightarrow \left(y^2 = \frac{a - a'}{ab' - a'b}\right)$$

On putting the value of x^2 and y^2 in Eq. (v), we get

$$\Rightarrow aa'\frac{(b' - b)}{(ab' - a'b)} = -\frac{(a - a')}{(ab' - a'b)} \cdot bb'$$

$$aa'(ab' - a'b)(b' - b) = -bb'(ab' - a'b)(a - a')$$

$$\Rightarrow aa'(b' - b) = -bb'(a - a')$$

$$\Rightarrow aa'b' - aa'b = -abb' + a'b'b$$

$$\Rightarrow aa'b' + b'ab = aba' + ba'b'$$

Dividing on both sides by $aba'b'$

$$\Rightarrow \frac{1}{b} + \frac{1}{a'} = \frac{1}{b'} + \frac{1}{a}$$

$$\therefore \frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}$$

115. (b) Given curve,

$$xy^2 = 4(4 - x)$$

On differentiating both sides w.r.t. x, we get

$$x \cdot 2y\frac{dy}{dx} + y^2 = 4(0 - 1)$$

$$\Rightarrow 2xy\frac{dy}{dx} + y^2 = -4$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(4 + y^2)}{2xy}$$

which is slope of tangent to the curve.

Now, $$\left(\frac{dy}{dx}\right)_{\text{at }(2,2)} = -\frac{(4+4)}{2 \cdot 2 \cdot 2} = -\frac{8}{8} = -1$$

$\therefore$ Equation of tangent to the curve passes through the point $(2, 2)$ is

$$(y - 2) = -1(x - 2)$$

$$\Rightarrow y - 2 = -x + 2$$

$$\Rightarrow x + y = 4$$

116. (a) Let $u = \sin^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$u = \sin^{-1}\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right)$$

$$= \sin^{-1}\cos 2\theta = \sin^{-1}\sin\left(\frac{\pi}{2} - 2\theta\right)$$

$$\Rightarrow \quad u = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1} x$$

Now, differentiating w.r.t. x, we get

$$\frac{du}{dx} = -\frac{2}{1+x^2} \quad \ldots(i)$$

and $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$= 2\tan^{-1} x$$

Again, differentiating w.r.t. x, we get

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \ldots(ii)$$

$$\therefore \quad \frac{du}{dv} = \frac{du}{dx} \cdot \frac{dx}{dv} = \frac{-2}{(1+x^2)} \times \frac{(1+x^2)}{x} = -1$$

117. (a) Given function,

$$f(x) = \begin{cases} x + \frac{x}{3}\sin(\log x^2), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Now, $\text{LHL} = f(0-0) = \lim_{h \to 0} f(0-h)$

$$= \lim_{h \to 0} -h\left[1 + \frac{1}{3}\sin(\log h^2)\right]$$

$$= \lim_{h \to 0} -h\left[1 + \frac{1}{3}\sin(2\log h)\right]$$

$$= -0\left[1 + \frac{1}{3}\cdot\sin(\infty)\right]$$

$$= -0 \times \left[1 + \frac{1}{3}(\text{a finite number persist between } -1 \text{ to } +1)\right]$$

$$= 0 \quad (\because -1 \leq \sin x \leq 1)$$

$\text{RHL} = f(0+0) = \lim_{h \to 0} f(0+h)$

$$= \lim_{h \to 0} h\left[1 + \frac{1}{3}\sin(\log h^2)\right]$$

$$= \lim_{h \to 0} h\left[1 + \frac{1}{3}\sin(2\log h)\right]$$

$$= 0\left[1 + \frac{1}{3}\sin(\infty)\right]$$

$$= 0 \times \left[1 + \frac{1}{3}(\text{adefinite number})\right]$$

$$= 0$$

and $f(0) = 0$

$\because \quad \text{LHL} = \text{RHL} = f(0) = 0$

Hence, $f(x)$ is continuous at $x = 0$.

118. (b) Let $y = \lim_{x \to \infty} (1+x)^{1/x}$

Taking log on both sides, we get

$$\log y = \lim_{x \to \infty} \frac{\log(1+x)}{x} \quad \left(\frac{\infty}{\infty}\text{ form}\right)$$

Now, apply L'hospital rule,

$$\log y = \lim_{x \to \infty} \frac{\frac{1}{(1+x)}\cdot(0+1)}{1}$$

$$= \lim_{x \to \infty} \frac{1}{(1+x)} = 0$$

$$\Rightarrow \quad y = e^0 = 1$$

119. (b)

120. (a) If any line $y = mx + c$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then it satisfy the following condition,

$$c^2 = (a^2m^2 - b^2) \quad \ldots(i)$$

Given line is,

$$lx + my = n$$

$$\Rightarrow \quad y = -\frac{l}{m}x + \frac{n}{m}$$

Here, $m = -\frac{l}{m}$ and $c = \frac{n}{m}$

On putting these value in Eq. (i), we get

$$\left(\frac{n}{m}\right)^2 = \left[a^2\left(-\frac{l}{m}\right)^2 - b^2\right]$$

$$\Rightarrow \quad \frac{n^2}{m^2} = \frac{a^2l^2}{m^2} - b^2$$

$$\Rightarrow \quad n^2 = a^2l^2 - b^2m^2$$

121. (b) If the line $y = mx + c$ is the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then it satisfy the following condition

$$c^2 = (a^2m^2 + b^2) \quad \ldots(i)$$

Give line is,

$$x\cos\alpha + y\sin\alpha = P$$

$$\Rightarrow \quad y = -\cot\alpha\, x + P\operatorname{cosec}\alpha$$

Here, $m = -\cot\alpha$ and $c = P\operatorname{cosec}\alpha$

On putting these values in Eq. (i), we get

$$(P\operatorname{cosec}\alpha)^2 = a^2(-\cot\alpha)^2 + b^2$$

$$\Rightarrow \quad \frac{P^2}{\sin^2\alpha} = \frac{a^2\cos^2\alpha}{\sin^2\alpha} + b^2$$

$$\Rightarrow \quad P^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$$

122. (d) Given equation of parabola

$$y^2 = 8x$$

On comparing with $y^2 = 4ax$, we get

$$4a = 8$$

$$\Rightarrow \quad a = 2$$

Now, focal distance of a point on the parabola $= 2t^2 + a$

where, $(2t^2, 4t)$ is the parametric coordinate of $y^2 = 8x$.

Given, $\quad 2t^2 + a = 4$

$$\Rightarrow \quad 2t^2 + 2 = 4$$

$$\Rightarrow \quad 2t^2 = 2$$

$$\Rightarrow \quad (t^1 = \pm 1)$$

$\therefore$ Its ordinate $= 4t$

$$= 4(\pm 1) = \pm 4$$

123. (c) Given equation of lines,

$$(2x - y + 1)(x - 2y + 3) = 0$$

$$\Rightarrow \quad 2x - y + 1 = 0 \quad \text{and} \quad x - 2y + 3 = 0$$

Now, we will get the intersection points of these lines with the axes

Equation	**Points**
$2x - y + 1 = 0$	$A(0, 1)$ and $B\left(-\frac{1}{2}, 0\right)$
$x - 2y + 3 = 0$	$C\left(0, \frac{3}{2}\right)$ and $D(-3, 0)$

Let the standard equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$$

When circle (i) passes through the point $A(0, 1)$,

$$1 + 2f + c = 0 \quad \ldots(ii)$$

When circle (i) passes through the point $B\left(-\frac{1}{2}, 0\right)$

$$\frac{1}{4} - g + c = 0 \quad \ldots(iii)$$

When circle (i) passes through the point $C(0, 3)$,

$$\frac{9}{4} + 3f + c = 0 \quad \ldots(iv)$$

When circle (i) passes through the point $D(-3, 0)$

$$9 - 6g + c = 0 \quad \ldots(v)$$

On subtracting Eq. (iii) from Eq. (ii), we get

$$1 + 2f - \frac{1}{4} + g = 0$$

$$2f + g = -\frac{3}{4} \quad \ldots(vi)$$

Again, subtracting Eq. (v) from Eq. (iv), we get

$$\frac{9}{4} + 3f - 9 + 6g = 0$$

$$\Rightarrow \quad 3f + 6g = \frac{27}{4}$$

$$\Rightarrow \quad f + 2g = \frac{9}{4} \quad \ldots(vii)$$

On multiplying Eq. (vi) by 2 and then subtracting Eq. (vii) from it, we get

$$4f + 2g = -\frac{3}{2}$$

$$f + 2g = \frac{9}{4}$$

$$- \quad - \quad -$$

$$3f = -\left(\frac{6+9}{4}\right) = -\frac{15}{4} \qquad \left(f = -\frac{5}{4}\right)$$

From Eq. (vii), we get

$$2g = \frac{9}{4} + \frac{5}{4} = \frac{14}{4} = \frac{7}{2}$$

$$\Rightarrow \quad \left(g = \frac{7}{4}\right)$$

From Eq. (v), we get

$$\left(C = 6 \times \frac{7}{4} - 9 = \frac{21}{2} - 9 = \frac{3}{2}\right)$$

$\therefore$ Required equation of circle is

$$x^2 + y^2 + 2\left(\frac{7}{4}\right)x + 2\left(-\frac{5}{4}\right)y + \frac{3}{2} = 0$$

$$\Rightarrow \quad x^2 + y^2 + \frac{7x}{2} - \frac{5y}{2} + \frac{3}{2} = 0$$

$$\Rightarrow \quad 2x^2 + 2y^2 + 7x - 5y + 3 = 0$$

$\therefore$ Required radius $= \sqrt{g^2 + f^2 - c}$

$$= \sqrt{\frac{49}{16} + \frac{25}{16} - \frac{3}{2}}$$

$$= \frac{1}{4}\sqrt{49 + 25 - 24} = \frac{1}{4}\sqrt{50}$$

$$= \frac{5\sqrt{2}}{4} = \frac{5}{2\sqrt{2}}$$

124. (a) Let the equation of required circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$$

which passes through $(-1, 2)$

$$1 + 4 - 2g + 4f + c = 0$$

$$-2g + 4f + c = -5 \quad \ldots(ii)$$

Since, the circle (i) concentric with the circle

$$x^2 + y^2 - 2x - 4y - 4 = 0$$

$$\therefore \quad g = -1 \quad \text{and} \quad f = -2$$

On putting this value in Eq. (ii), we get

$$2 - 8 + c = -5$$

$$\Rightarrow \quad c = 1$$

So, the required equation is

$$x^2 + y^2 - 2x + 4y + 1 = 0$$

125. (d) Given equation of two straight line is

$$6x^2 + 5xy - 4y^2 + 7x + 13y - 3 = 0$$

Compare with,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

we get, $a=6,\ h=\frac{5}{2},\ b=-4$

Let θ be the angle between two straight lines,

then $\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

$\Rightarrow \tan\theta=\left|\frac{2\sqrt{\frac{25}{4}+24}}{6-4}\right|=\left|\sqrt{\frac{25+96}{4}}\right|=\left|\sqrt{\frac{121}{4}}\right|$

$\therefore \theta=\tan^{-1}\left(\frac{11}{2}\right)$

126. (a) Given straight lines

$$4x+3y-8=0 \quad \ldots(i)$$

and $$x+y-1=0 \quad \ldots(ii)$$

On multiplying Eq. (ii) by 3 and then subtracting it from Eq. (i), we get

$$4x+3y-8=0$$
$$3x+3y-3=0$$
$$\underline{-\quad -\quad +}$$
$$x-5=0 \Rightarrow x=5$$

and $y=1-5=-4$

So, the required point is (5, – 4).

Now, the equation of the straight line passing through the points (5, – 4) and (–2, 5) is

$$(y+4)=\frac{5+4}{-2-5}(x-5)$$

$\Rightarrow -7(y+4)=9(x-5)$

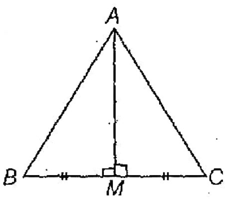
$\Rightarrow -7y-28=9x-45$

$\Rightarrow 9x+7y=17$

127. (b) In ΔABM,

using Pythagoras theorem

$$AB^2=BM^2+AM^2 \quad \ldots(i)$$



and In ΔACM,

Using Pythagoras theorem

$$AC^2=AM^2+MC^2 \quad \ldots(ii)$$

On adding both equations, we get

$AB^2+AC^2=BM^2+MC^2+2AM^2$

$(\because BM=MC)$

$\Rightarrow AB^2+AC^2=2AM^2+BM^2+BM^2$

$\Rightarrow AB^2+AC^2=2AM^2+2BM^2$

128. (c) The equation of straight line which passes through the point $P(2,\sqrt{3})$ and making an angle of 60° with the x-axis is

$$(y-\sqrt{3})=\tan 60°\,(x-2)$$

$\Rightarrow y-\sqrt{3}=\sqrt{3}(x-2)$

$\Rightarrow y-\sqrt{3}=\sqrt{3}x-2\sqrt{3}$

$\Rightarrow y-\sqrt{3}x=-\sqrt{3} \quad \ldots(i)$

the other line is, $x+\sqrt{3}y=12 \quad \ldots(ii)$

On multiplying Eq. (ii) by 3 and then adding in Eq. (i), we get

$$y-\sqrt{3}x=-\sqrt{3}$$
$$\sqrt{3}x+3y=12\sqrt{3}$$

$$4y=11\sqrt{3} \Rightarrow y=\frac{11+\sqrt{3}}{4}$$

and $$x=\frac{15}{4}$$

So, the intersection point of both line is $Q\left(\frac{15}{4},\frac{11\sqrt{3}}{4}\right)$

$\therefore$ Length of the intercept on it (i) between the point P and Q

$$=\sqrt{\left(\frac{15}{4}-2\right)^2+\left(\frac{11\sqrt{3}}{4}-\sqrt{3}\right)^2}$$

$$=\sqrt{\frac{49}{16}+\frac{49}{16}\times 3}=\frac{7}{4}\times 2=\frac{7}{2}=3.5$$

129. (d) **Reflexive** Let abc be any element of G. Then, $aa^{-1}=c\in H$.

Since, H is a subgroup of G. Therefore $a\equiv a \pmod H$ for all $a\in G$. Hence, the relation is reflexive.

Symmetric we have, $a\equiv b \pmod H \Rightarrow ab^{-1}\in H$

$\Rightarrow (ab^{-1})\in H$ $\quad(\because H$ is a subgroup of $G)$

$\Rightarrow ba^{-1}\in H \Rightarrow b\equiv a \pmod H$

Therefore, the relation is symmetric.

Transitive Let $a\equiv b \pmod H$ and $b\equiv c \pmod H$.

Then, $ab^{-1}\in H$ and $bc^{-1}\in H$ but H is a subgroup of G and thus H must be closed with respect to the composition in G. Therefore,

$$(ab^{-1})(bc^{-1})\in H$$

$\Rightarrow a(b^{-1}b)c^{-1}\in H$

$\Rightarrow aec^{-1}\in H$

$\Rightarrow ac^{-1}\in H$

$\Rightarrow a\equiv c \pmod H$

Hence, the relation is transitive.

Therefore, the relation congruence mod H is an equivalence relation in G.

130. (a) Given that, $S=(l_1,l_2,l_3,l_4)$

and $P(l_2)=\frac{1}{3},\ P(l_3)=\frac{1}{6},\ P(l_4)=\frac{1}{9}$

Now, $P(s)=P(l_1,l_2,l_3,l_4)$

$\Rightarrow \quad 1 = P(I_1) + P(I_2) + P(I_3) + P(I_4)$

$\Rightarrow \quad P(I_1) = 1 - \{P(I_2) + P(I_3) + P(I_4)\}$

$$= 1 - \left\{\frac{1}{3} + \frac{1}{6} + \frac{1}{9}\right\} = 1 - \frac{11}{18} = \frac{7}{18}$$

131. (b) Given that,

$U = \{\text{Natural numbers}\}$
$= \{1, 2, 3, \dots\}$
$A = \{\text{Multiples of } 3\}$
$= \{3, 6, 9, 12, 15, \dots\}$
and $B = \{\text{Multiples of } 5\}$
$= \{5, 10, 15, \dots\}$
$\overline{B} = \{\text{Natural numbers except multiple of } 5\}$
$\therefore A - B = \{3, 6, 9, 12, 18, 21, 24, 27, 33, \dots\}$
$= A \cap \overline{B}$
$= \{\text{Multiples of } 3\} \cap \{\text{All natural numbers except multiple of } 5\}$

132. (d) Because same elements in a set can arrange in any order, *i.e.,*

$$\{a, b, c\} = \{c, a, b\} = \{b, a, c\}$$

Option (c), repeatetion is not allowed of the elements in any set.

Option (b), A set is not belongs to the other set.

133. (b) Every subset of a finite set is also finite but every subset of an infinite set is not infinite.

Set $\{x : x + 8 = 8\} = \{x : x = 0\}$ is the singleton set.

134. (b) Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$

and $B = \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 4 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}_{2 \times 3} \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 4 & 1 \end{bmatrix}_{3 \times 2}$$

$$= \begin{bmatrix} -2 + 1 + 4 & 3 - 5 + 1 \\ -6 + 3 + 12 & 9 - 15 + 3 \end{bmatrix}_{2 \times 2}$$

$$= \begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}_{2 \times 2}$$

135. (c) Let $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Given that ω is cube root of unity,

i.e., $\quad 1 + \omega + \omega^2 = 0$

and $\quad \omega^3 = 1$

Using operation, $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} 1 + \omega + \omega^2 & \omega & \omega^2 \\ 1 + \omega + \omega^2 & \omega^2 & 1 \\ 1 + \omega + \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix}$$

$= 0 \quad$ ($\because$ all elements of C_1 is 0)

136. (d) $\dfrac{12}{3 + \sqrt{5} + 2\sqrt{2}} = \dfrac{12}{(3 + 2\sqrt{2}) + \sqrt{5}} \times \dfrac{(3 + 2\sqrt{2}) - \sqrt{5}}{(3 + 2\sqrt{2}) - \sqrt{5}}$

$$= \frac{12\,(3 + 2\sqrt{2} - \sqrt{5})}{(3 + 2\sqrt{2})^2 - 5} = \frac{12\,(3 + 2\sqrt{2} - \sqrt{5})}{9 + 8 + 12\sqrt{2} - 5}$$

$$= \frac{12\,(3 + 2\sqrt{2} - \sqrt{5})}{12\,(1 + \sqrt{2})} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{(3 + 2\sqrt{2} - \sqrt{5})(\sqrt{2} - 1)}{(2 - 1)}$$

$$= \frac{3\sqrt{2} + 4 - \sqrt{10} - 3 - 2\sqrt{2} + \sqrt{5}}{1}$$

$$= 1 + \sqrt{2} + \sqrt{5} - \sqrt{10}$$

137. (c) Given that, ${}^nC_{r-1} = 36$ ...(i)

${}^nC_r = 84$ ...(ii)

and ${}^nC_{r+1} = 126$ ...(iii)

On dividing Eq. (ii) from Eq. (i), we get

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36} \Rightarrow \frac{\dfrac{n!}{r!\,(n-r)!}}{\dfrac{n!}{(r-1)!\,(n-r+1)!}} = \frac{7}{3}$$

$$\Rightarrow \frac{(n-r+1)!}{(n-r)!\,r} = \frac{7}{3} \Rightarrow \frac{(n-r+1)}{r} = \frac{7}{3}$$

$\Rightarrow \quad 3n - 3r + 3 = 7r$

$\Rightarrow \quad 3n - 10r + 3 = 0$ ...(iv)

Again, dividing Eq. (iii) from Eq. (ii), we get

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{126}{84}$$

$$\Rightarrow \frac{\dfrac{n!}{(r+1)!\,(n-r-1)!}}{\dfrac{n!}{r!\,(n-r)!}} = \frac{21}{14}$$

$$\Rightarrow \frac{(n-r)!}{(n-r-1)!\,(r+1)} = \frac{21}{14}$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{21}{14}$$

$\Rightarrow \quad 14n - 14r = 21r + 21$

$\Rightarrow \quad 14n - 35r = 21$ ...(v)

On multiplying Eq. (iv) by 7 and Eq. (v) by 2 and then subtracting, we get

$$21n - 70r = -21$$
$$28n - 70r = 42$$
$$-\quad\ +\quad\ -$$

$-7n = -63$

$\Rightarrow \quad n = 9$

From Eq. (iv),

$3(9) - 10r + 3 = 0$

$\Rightarrow \quad 10r = 27 + 3 = 30$

$\therefore \quad r = 3$

138. (b) Given expansion $(a-b)^n$, $n \geq 5$

General term

$$T_{r+1} = {}^nC_r\,(a)^{n-r}\,(-b)^4 \quad \ldots(i)$$

Now, 5th term,

$$T_{4+1} = {}^nC_4\,(a)^{n-4}\,(-b)^4 = {}^nC_4\,a^{n-4}\cdot b^4 \quad \ldots(ii)$$

and 6th term,

$$T_{5+1} = {}^nC_5\,(a)^{n-5}\,(-b)^5 = -\,{}^nC_5\,a^{n-5}\cdot b^5 \quad \ldots(iii)$$

Given that,

$$T_5 + T_6 = 0$$

$${}^nC_4\cdot a^{n-4}\cdot b^4 - {}^nC_5\,a^{n-5}\cdot b^5 = 0$$

$${}^nC_4\cdot a^{n-4}b^4 = {}^nC_5\cdot a^{n-4}\cdot a^{-1}\cdot b^4\cdot b$$

$$\Rightarrow \quad {}^nC_4\cdot a = {}^nC_5\cdot b$$

$$\Rightarrow \quad \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{\dfrac{n!}{5!\,(n-5)!}}{\dfrac{n!}{4!\,(n-4)!}} = \frac{(n-4)!}{5\cdot(n-5)!}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{n-4}{5}$$

139. (b) Given, series

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots 16$$

Let nth term, $T_n = \dfrac{1^3+2^3+3^3+\ldots}{1+2+3+\ldots}$

$$= \left[\frac{n\,(n+1)}{2}\right]^2 \times \frac{2}{n\,(n+1)}$$

$$= \frac{n\,(n+1)}{2}$$

$$\therefore \quad S_n = \frac{1}{2}\{\Sigma n^2 + \Sigma n\}$$

$$= \frac{1}{2}\left\{\frac{n\,(n+1)\,(2n+1)}{6} + \frac{n\,(n+1)}{2}\right\}$$

$$= \frac{1}{2}\cdot\frac{n\,(n+1)}{2}\left\{\frac{2n+1}{3}+1\right\}$$

$$= \frac{n\,(n+1)}{4}\times\frac{1}{3}(2n+4)$$

$$= \frac{n\,(n+1)}{6}(n+2)$$

$\therefore$ Sum of 16 terms $= S_{16}$

$$= \frac{16\cdot(16+1)\,(16+2)}{6}$$

$$= \frac{16\cdot 17\cdot 18}{6} = 16\cdot 17\cdot 3 = 816$$

140. (b) Given that,

$$x = 1 + a + a^2 + a^3 + \ldots \infty \;(a<1)$$

$$x = \frac{1}{1-a} \text{ (infinite term of GP)} \quad \ldots(i)$$

$$y = 1 + b + b^2 + b^3 + \ldots \infty \quad (b<1)$$

$$y = \frac{1}{1-b} \text{ (infinite term of GP)} \quad \ldots(ii)$$

Then, the series,

$$1 + ab + a^2b^2 + a^3b^3 + \ldots \infty$$

which is infinite term of GP.

$$\therefore S_\infty = \frac{1}{1-ab} = \frac{1}{1-\left(\dfrac{x-1}{x}\right)\left(\dfrac{y-1}{y}\right)} \text{ [from Eqs. (i) and (ii)]}$$

$$= \frac{xy}{xy-(x-1)(y-1)} = \frac{xy}{xy-xy+y+x-1}$$

$$= \frac{xy}{x+y-1}$$

141. (c) Let the first term and common difference of an AP is a and d, respectively.

Given, sum of first p term of an AP $= q$

$$\Rightarrow \quad S_p = q$$

$$\Rightarrow \quad \frac{p}{2}[2a+(p-1)d] = q$$

$$\Rightarrow \quad 2ap + p(p-1)d = 2q \quad \ldots(i)$$

and sum of first q term of an AP $= p$

$$\Rightarrow \quad S_q = p$$

$$\Rightarrow \quad \frac{q}{2}[2a+(q-1)d] = p$$

$$\Rightarrow \quad 2qa + q\,(q-1)\,d = 2p \quad \ldots(ii)$$

On multiplying Eq. (i) by q and Eq. (ii) by p and then subtracting, we get

$$\Rightarrow \quad 2apq + pq\,(p-1)\,d = 2q^2$$

$$\underline{\;2pqa + pq\,(q-1)\,d = 2p^2\;}$$

$$d\,(p-1-q+1)\,pq = (2q^2 - p^2)$$

$$\Rightarrow \quad d\cdot pq\,(p-q) = 2(q-p)\,(q+p)$$

$$\Rightarrow \quad \left\{d = -\frac{2\,(p+q)}{pq}\right\}$$

On putting the value of 'd' in Eq. (i), we get

$$2ap + p(p-1)\left\{\frac{-2\,(p+q)}{pq}\right\} = 2q$$

$\Rightarrow \quad ap - \frac{(p-1)}{q}(p+q) = q$

$\Rightarrow \quad apq - (p-1)(p+q) = q^2$

$\Rightarrow \quad apq = q^2 + p^2 + pq - p - q$

$\therefore \quad a = \frac{p^2 + q^2 + pq - (p+q)}{pq}$

$\therefore S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}\left[\frac{2p^2 + 2q^2 + 2pq - 2p - 2q}{pq} - \frac{2(p+q)(p+q-1)}{pq}\right]$

$= \frac{p+q}{pq}\{p^2 + q^2 + pq - p - q - p^2 - pq - pq - q^2 + p + q\}$

$= \frac{p+q}{pq}\{-pq\} = -(p+q)$

142. (d) Given series,

$$2\frac{1}{2} + 1\frac{7}{13} + 1\frac{1}{9} + \frac{20}{23} + \dots$$

From option (d), we get

$$T_n = \frac{20}{5n+3}$$

Here, $T_1 = \frac{20}{8} = \frac{5}{2} = 2\frac{1}{2}$, $T_2 = \frac{20}{13} = 1\frac{7}{13}$

$T_3 = \frac{20}{18} = \frac{10}{9} = 1\frac{1}{9}$, $T_4 = \frac{20}{23}, \dots$

So, the nth term of given series is $\frac{20}{5n+3}$.

143. (a) $7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80}$

$= 7(\log 16 - \log 15) + 5(\log 25 - \log 24) + 3(\log 81 - \log 80)$

$= 7(4\log 2 - \log 3 - \log 5) + 5(2\log 5 - 3\log 2 - \log 3) + 3(4\log 3 - 4\log 2 - \log 5)$

$= 28\log 2 - 7\log 3 - 7\log 5 + 10\log 5 - 15\log 2 - 5\log 3 + 12\log 3 - 12\log 2 - 3\log 5$

$= \log 2$

144. (a) $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$

$= \frac{1}{2^4}\{(1+i)^8 + (1-i)^8\}$ ...(i)

Let $1 + i = r_1(\cos\theta_1 + i\sin\theta_1)$

$r_1\cos\theta_1 = 1$ and $r_1\sin\theta_1 = 1$

On squaring and adding, we get

$r_1^2 = 2 \Rightarrow (r_1 = \pm\sqrt{2})$

and $\tan\theta_1 = 1 = \tan\frac{\pi}{4} \Rightarrow \theta_1 = \frac{\pi}{4}$

Let $1 - i = r_2(\cos\theta_2 + i\sin\theta_2)$

$r_2\cos\theta_2 = 1$ and $r_2\sin\theta_2 = -1$

On squaring and adding, we get

$r_2^2 = 2 \Rightarrow r_2 = \pm\sqrt{2}$

and $\tan\theta_2 = -1 = \tan\left(-\frac{\pi}{4}\right)$

$\Rightarrow \quad \theta_2 = -\frac{\pi}{4}$

Now, from Eq. (i),

$= \frac{1}{24}\left\{(\pm\sqrt{2})^8\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^8 + (\pm\sqrt{2})^8\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]^8\right\}$

$= \frac{1}{2^4} \times 2^4\left\{\cos\left(\frac{\pi}{4}\cdot 8\right) + i\sin\left(\frac{\pi}{4}\cdot 8\right) + \cos\left(-\frac{\pi}{4}\cdot 8\right) - i\sin\left(+\frac{\pi}{4}\cdot 8\right)\right\}$

(using Demoiver theorem)

$= \{\cos 2\pi + i\sin 2\pi + \cos 2\pi - i\sin 2\pi\}$

$[\because \cos(-\theta) = \cos\theta]$

$= 2\cos 2\pi = 2\cdot\sin 90°$

$= 2\cdot(1) = 2$

145. (b) Let the three conselcutive odd natural numbers are $n, (n+2)$ and $(n+4)$

Given that, $n + (n+2) + (n+4) = 153$

$\Rightarrow \quad 3n = 153 - 6 = 147$

$\Rightarrow \quad n = 49$

$\therefore$ Required numbers,

$n = 49$

$n + 2 = 49 + 2 = 51$

and $n + 4 = 49 + 4 = 53$

146. (a) Given equation is,

$$\frac{x-1}{3} - \frac{4x+1}{4} = \frac{1}{12}$$

$\Rightarrow \quad 4x - 4 - 12x - 3 = 1$

$\Rightarrow \quad -8x - 7 = 1$

$\Rightarrow \quad 8x + 8 = 0$

$\Rightarrow \quad x = -1$

147. (d) Given expression is,

$36x^2 - 12x + 1 - 25y^2$

$= \{(6x)^2 - 2 \cdot 6x + 1\} - (5y)^2$

$= (6x-1)^2 - (5y)^2 \qquad \{\because a^2 - b^2 = (a-b)(a+b)\}$

$= (6x - 1 + 5y)(6x - 1 - 5y)$

$= (6x - 5y - 1)(6x + 5y - 1)$

148. (b) Given that,

$$\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k \qquad \text{(say)}$$

$\Rightarrow \qquad \log x = k(b-c)$

$\Rightarrow \qquad x = e^{k(b-c)}$

Similarly, $\quad y = e^{k(c-a)}$ and $z = e^{k(a-b)}$

$\therefore\ x^a y^b z^c = e^{k(b-c)a}\, e^{K(c-a)b} \cdot e^{k(a-b)c}$

$= e^{k(ab - ca + bc - ab + ac - bc)} = e^{k \cdot 0} = e^0 = 1$

149. (b) The decimal number which is non-terminating but reccurring is represent in the form $\frac{p}{q}$.

i.e., $\quad \frac{68}{9} = 7.555$

150. (a) Let $x = 3.\overline{142857} = 3.142857$ ...(i)

On multiplying both sides by 1000000, we get

$1000000\,x = 3142857.142857$ ...(ii)

On subtracting above equations, we get

$\Rightarrow \qquad 1000000x - x = 3142853$

$\Rightarrow \qquad 999999x = 3142853$

$\Rightarrow \qquad x = \frac{3142853}{999999} = 3.\overline{142857}$

which is equivalent to the decimal representation of $\frac{22}{7}$.