BHU MCA

Solved Paper 2018

1. $Z = \frac{1}{1 - \cos\theta - i\sin\theta}$, then Re($Z$) is equal to

(a) 1 (b) $\cot\frac{\theta}{2}$

(c) $\frac{1}{2}$ (d) $\frac{1}{2}.\cot\theta$

2. If $(x + iy)^{1/3} = a + ib$, $x, y, a, b \in R$, then $x/a + y/b$ is equal to

(a) $a - b$ (b) $4(a^2 - b^2)$ (c) $a + b$ (d) $4(a^2 + b^2)$

3. How many words can be formed from the letters of word 'DAUGHTER' so that the vowels are never together?

(a) $8! - 6!$ (b) 40320 (c) 36000 (d) 4320

4. The equation of the smallest degree with the real coefficient having $(1 + i)$ as one of the roots is

(a) $x^2 + x + 2 = 0$ (b) $x^2 + x + 1 = 0$

(c) $x^2 - 2x + 2 = 0$ (d) $x^2 + 2x + 2 = 0$

5. A rod AB of length 15 cm rests in between two coordinate axis in such a way that the end point A lies on X-axis and end point B lies on Y-axis. A point P is taken on the rod in such a way that $AP = 6$ cm. If the locus of P is an ellipse, then its eccentricity (e) is

(a) $\frac{\sqrt{5}}{3}$ (b) $\sqrt{\frac{5}{7}}$ (c) $\frac{5}{3}$ (d) $\sqrt{\frac{117}{81}}$

6. A line passes through the point (3, − 2). The locus of the middle point of the portion of the line intercepted between the axis is

(a) $3x - 2y = 2xy$ (b) $\frac{2x}{3} + \frac{y}{1} = 1$

(c) $3y - 2x = 2xy$ (d) $3x - 2y = 2$

7. The equation of the parabola, whose vertex is at (2, 1) and directrix is $x = y - 1$, is given by

(a) $x^2 + y^2 - 14x + 2y + 2xy + 17 = 0$

(b) $x^2 - 14x = 4xy$

(c) $3y - 2x = 2xy$

(d) $3x - 2y = 2$

8. The domain of the function $f(x) = \sqrt{x - 3 - 2\sqrt{x - 4}} - \sqrt{x - 3 + 2\sqrt{x - 4}}$ is, where 'f' be a real values of real variable

(a) $[2, \infty)$ (b) $[4, \infty)$ (c) R^+ (d) $(-\infty, 4]$

9. The 4th term from end in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$ is

(a) $\frac{35}{48}x^5$ (b) $\frac{35}{32}x^6$

(c) $\frac{35}{32}x^5$ (d) $\frac{35}{48}x^6$

10. If $\frac{3 + 2i\sin\theta}{1 - 2\sin\theta}$ is a purely real, then the value of θ is (n being integer)

(a) $\theta = (n + 1)\frac{\pi}{2}$ (b) $\theta = n\pi$

(c) $\theta = \frac{n}{2}$ (d) $\theta = \frac{n\pi}{2}$

11. The eccentricity of the conic $9x^2 - 16y^2 = 144$ is

(a) $\frac{5}{4}$ (b) $\frac{\sqrt{7}}{4}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

12. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is

(a) 1.2 (b) 1.6

(c) 0.8 (d) 0.4

13. The coefficient of x^6y^3 in the expansion of $(x + 2y)^9$ is

(a) 670 (b) 1348

(c) 674 (d) 672

14. The value of $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{576} + i^{574}}$ is ($i = \sqrt{-1}$)

(a) −1 (b) 0 (c) −2 (d) 1

15. Two dice are thrown, the probability of getting an odd number on the first die and multiple of 3 on the other, is

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{1}{3}$

16. If $y = \sin^{-1} x + \sin^{-1}\sqrt{1 - x^2}$ and $x \in (-1, 0)$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{1}{\sqrt{1 - x^2}}$ (b) 0

(c) $\frac{2}{\sqrt{1 - x^2}}$ (d) $\frac{1}{\sqrt{1 - x^2}}$

35. If w is a non-real cube root of unity and n is not a multiple of 3, then

$$\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$$ is equal to

(a) $2n$ (b) 1
(c) n^2 (d) 0

36. The point on the curve $9y^2 = x^3$, where normal to the curve makes equal intercepts with the axes is

(a) $\left(1, \pm\frac{1}{3}\right)$ (b) $\left(4, \frac{8}{3}\right)$
(c) $\left(2, \pm\frac{2\sqrt{2}}{3}\right)$ (d) $\left(2, \sqrt{\frac{8}{3}}\right)$

37. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio $1:7:42$, then the value of 'n' is

(a) 30 (b) 55 (c) 52 (d) 50

38. If the third term in the expansion of $\left(\frac{1}{x} + x^{\log_{10} x}\right)^5$ is 1000, then the value of x is

(a) 21 (b) 100 (c) 10 (d) 50

39. If a matrix A is both symmetric and skew-symmetric, then

(a) A is a scalar matrix (b) A is a zero matrix
(c) A is a diagonal matrix (d) A is a square matrix

40. If B is a matrix, such that $B\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then value of B is

(a) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ -1 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -3 \\ 1 & -1 \end{bmatrix}$

41. Probability of solving a problem of A, B, C are 1/3, 2/7 and 3/8, respectively. If all the three to solve the problem simultaneously, find the probability that exactly one of them can solve it.

(a) $\frac{35}{168}$ (b) $\frac{1}{2}$ (c) $\frac{25}{56}$ (d) $\frac{101}{168}$

42. If $(a+b)^2 x^2 + 8(a^2 - b^2)x + 16(a-b)^2 = 0$, then the value of x is

(a) $\frac{a+b}{a-b}$ (b) $\frac{(b-a)}{a+b}$ (c) $\frac{4(b-a)}{a+b}$ (d) $\frac{4(a-b)}{a+b}$

43. The axes of an ellipse are along the coordinate axes, vertices are at $(0, \pm 10)$ and eccentricity $e = 4/5$. The equation of the ellipse is

(a) $\frac{x^2}{164} + \frac{y^2}{36} = 1$ (b) $\frac{x^2}{36} + \frac{y^2}{164} = 1$
(c) $\frac{x^2}{64} + \frac{y^2}{100} = 1$ (d) $\frac{x^2}{36} + \frac{y^2}{100} = 1$

44. The area bounded by the curve $y^2 = 4x$ and $x^2 = 4y$ is

(a) $\frac{14}{3}$ sq units (b) $\frac{16}{3}$ sq units
(c) $\frac{3}{14}$ sq unit (d) $\frac{8}{3}$ sq units

45. A bag contains 4 red and 4 blue balls. Four balls are drawn one-by-one from the bag, then the probability that the drawn are in alternate colour, is

(a) $\frac{6}{35}$ (b) $\frac{7}{31}$ (c) $\frac{9}{31}$ (d) $\frac{1}{35}$

46. The number of terms in the expansion of $(1 - 3x + 3x^2 - x^3)^8$ is

(a) 32 (b) 24 (c) 26 (d) 25

47. If ${}^nC_{15} = {}^nC_8$, then value of ${}^nC_{21}$ is

(a) 251 (b) 554
(c) 250 (d) 253

48. The sum of the series $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \frac{1}{\log_{16} 4} + \dots + \frac{1}{\log_{2^n} 4}$ is

(a) $n \cdot \log_2 4$ (b) $\frac{n(n-1)}{4}$
(c) $\frac{n(n+1)(2n+1)}{12}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$

49. An arc is in the form of a parabola with its axis vertical and one of its end is at the vertex. The arc is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of parabola?

(a) $\frac{\sqrt{5}}{2}$ m (b) $\sqrt{5}$ m (c) 2.5 m (d) $\sqrt{\frac{5}{2}}$ m

50. If a complex number Z is given by $Z = \frac{1+7i}{(2-i)^2}$, then

(a) $\arg(Z) = \frac{5\pi}{4}$ (b) $\arg(Z) = \frac{3\pi}{4}$
(c) $\arg(Z) = \frac{\pi}{4}$ (d) $\arg(Z) = -\frac{\pi}{4}$

51. The equation of the circle passing through (1, 0) and (0, 1) and having the smallest possible radius is

(a) $x^2 + y^2 + x + y + 1 = 0$ (b) $x^2 + y^2 + x - y + 1 = 0$
(c) $x^2 + y^2 + x + y = 0$ (d) $x^2 + y^2 - x - y = 0$

52. Solution of the inequation given below is

$$\left|\frac{2}{x-4}\right| > 1, x \neq 4$$

(a) (2, 6) (b) [2, 4)
(c) $(2, 4) \cup (4, 6)$ (d) [2, 6]

53. The number of terms with integral coefficients in the expansion of $(17^{1/3} + 35^{1/2})^{600}$ is

(a) 200 (b) 301
(c) 100 (d) 101

54. Value of x in the inequation $|x-1|+|x-2|\geq 4$, is

(a) $\left[-\frac{1}{2}, \frac{7}{2}\right]$ (b) $\left(-\infty, +\frac{1}{2}\right] \cup \left[\frac{7}{2}, \infty\right]$

(c) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{7}{2}, \infty\right)$ (d) $\left(-\infty, -\frac{1}{2}\right)$

55. Out of 9 outstanding students in a college, there are 4 boys and 5 girls. A team of 4 students is to be selected for a quiz programme. Find the probability that two are boys and two are girls.

(a) $\frac{11}{21}$ (b) $\frac{10}{21}$ (c) $\frac{20}{41}$ (d) $\frac{2}{63}$

56. The sum of all the natural numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time is

(a) 93324 (b) 90002 (c) 93240 (d) 93004

57. If $z = 2 - 3i$, then value of $4z^3 - 3z^2 + 169$ is

(a) 160 (b) 140 (c) 0 (d) 199

58. The quadratic equation $x^2(a^2+b^2)+2x(ac+bd)+(c^2+d^2)=0$ has no real roots, if

(a) $ab = cd$ (b) $ad \neq bc$

(c) $ab \neq cd$ (d) $ad = bc$

59. It is given that $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2], the value of 'a' is

(a) 100 (b) 119 (c) 120 (d) 140

60. If $f: R \to R$ is a function defined by $f(x) = \frac{x^2}{1+x^2}$, then the range of the function 'f' is

(a) $[0, \infty)$ (b) R (c) $R/\{1\}$ (d) $[0, 1)$

61. If a line perpendicular to the line segment joining the point (1, 0) and (2, 3) divided in ratio $1:n$, then equation of the line is

(a) $(n+1)x + 3(n+1)y = n + 11$

(b) $(n+1)x + 3(n+2)y = n + 10$

(c) $(n+1)x - (n-2)y = n + 11$

(d) $nx + (n+1)y = n + 11$

62. A box contains 5 different red balls and 6 different white balls. In how many ways can 6 balls be selected so that there are at least two balls each colour

(a) 625 (b) 425

(c) 400 (d) 252

63. The radius of the circle $25x^2 + 25y^2 - 20x + 2y - 60 = 0$

(a) $\frac{16}{25}$ (b) $\frac{\sqrt{1601}}{25}$ (c) $\frac{8}{5}$ (d) $\frac{\sqrt{464}}{5}$

64. In a Geometric progression (G.P.), if the $(m+n)$th term is 'p' and $(m-n)$th term of 'q', then its mth term is

(a) $p + q$ (b) $\frac{1}{2}(p+q)$

(c) pq (d) $\sqrt{pq}$

65. The letters of the word 'RANDOM' are written in all possible orders and these words are written in dictionary. The rank of word 'RANDOM' is

(a) 614 (b) 600 (c) 610 (d) 612

66. The value of $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$ is (C being constant of integration):

(a) $e^{2x}\sec^2 x + C$ (b) $\frac{1}{2}e^{2x}\tan x + C$

(c) $e^{2x}\tan x + C$ (d) $\frac{1}{2}e^{2x}\sec^2 x + C$

67. The value of $\int \sin^{-1}(\cos x)\,dx$, $0 < x < 11/2$, is (C being constant of integration)

(a) $\frac{\pi}{2} - x + C$ (b) $\frac{\pi}{2}x - \frac{x^2}{2} + C$

(c) $\frac{\pi}{2}x + \frac{x^2}{2} + C$ (d) $-\frac{\sin x}{\sqrt{1-x^2}} + C$

68. If a arithmetic means are inserted between 20 and 60 such that the ratio of the first mean to the last mean is 1 : 3, then the value of 'n' is

(a) 15 (b) 10 (c) 11 (d) 12

69. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $3f(x)$ (b) $[f(x)]^3$ (c) $2f(x)$ (d) $[f(x)]^2$

70. In how many ways 5 boys and 3 girls can be seated in a row so that no two girls are together?

(a) 2400 (b) 7200 (c) 14400 (d) 1440

71. A particle moves along a curve, $6y = x^3 + 2$ such that at some instant the y-coordinate is changing 8 times as fast as the x-coordinate. The position of the particle at the instant is

(a) (4, 11) (b) (4, 6)

(c) (1, 4) (d) (2, 11)

72. If $y = \log_x 2$, then $\frac{dy}{dx}$ is equal to

(a) $-\frac{1}{(\log_2 x)}\cdot\frac{1}{(x\cdot\log_e 2)}$ (b) $-\frac{1}{(\log_2 x)^2}\cdot\frac{1}{(x\cdot\log_e 2)}$

(c) $(\log_x 2)\cdot\log_e 2$ (d) $2\log_x 2.\left(\frac{\log_e 2}{x}\right)$

73. Set of solution of the equation $z^2 + |z| = 0$, where z is a complex number $(z = x + iy)$, is

(a) $\{0, 1, 2i\}$ (b) $\{0, i, 1\}$

(c) $\{0, 1\}$ (d) $\{0, i, -i\}$

74. For a post three persons A, B and C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is thrice that of C. The probability of A being selected is

(a) $\frac{1}{10}$ (b) $\frac{3}{5}$ (c) $\frac{3}{10}$ (d) $\frac{2}{5}$

75. If $R = \{(x, y) : x, y, \in Z, x^2 + y^2 \le 4\}$ is a relation defined on the set Z of integers, then the domain of R is

(a) $\{0, 1, 2\}$
(b) $\{-2, -1, 0, 1, 2\}$
(c) $\{0, 1, 2, 3, 4\}$
(d) $\{0, 1, 2\}$

76. In a flight of 600 km an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/h and the time increased by 30 min. Duration of the flights is

(a) 1 h (b) 2 h
(c) 1 h 20 min (d) 30 min

77. Find the value of x, if

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

(a) 5 (b) 3 (c) 2 (d) 4

78. If $S = 1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \frac{1+2+3+4}{4} + \ldots + n$ terms. Then the value of 'S' is

(a) $\frac{n(n+1)(2n+1)}{12}$ (b) $\frac{n^2}{2}$
(c) $\frac{n(n+2)}{4}$ (d) $\frac{n(n+3)}{4}$

79. If $x = 1 + a + a^2 + a^3 + \ldots \infty\ |a| < 1$ and $y = 1 + b + b^2 + b^3 + \ldots, \infty\ |b| < 1$ then value of $1 + ab + a^2b^2 + a^3b^3 + \ldots \infty$, is

(a) $\frac{1+xy}{x+y+1}$ (b) $1 + xy$
(c) xy (d) $\frac{xy}{x+y-1}$

80. The length of the latus rectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is

(a) 2 (b) 8 (c) 4 (d) 10

Directions (Q. Nos. 81 to 85) *In each of the following questions one number-series is given in which one term is wrong. Find out the wrong term.*

81. 4, 11, 21, 34, 49, 69, 91

(a) 49 (b) 34 (c) 69 (d) 21

82. 8, 36, 149, 596, 2388, 9556

(a) 2388 (b) 9556 (c) 149 (d) 596

83. 5, 7, 11, 20, 35, 67

(a) 35 (b) 11 (c) 20 (d) 67

84. 5, 12, 19, 33, 47, 75, 104

(a) 47 (b) 75 (c) 33 (d) 104

85. 0, 3, 8, 15, 24, 36, 48

(a) 15 (b) 48 (c) 36 (d) 24

86. In the following question one word is different from the rest. Find out the word which does not belong to the group.

(a) Sun (b) Star
(c) Moon (d) Sky

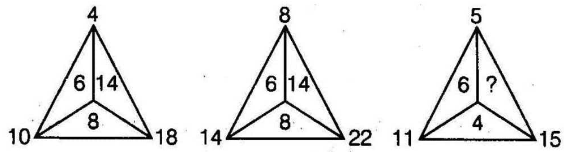
Directions (Q. Nos. 87 - 96) *Which number should come in place of question mark (?) in the following?*

87.

1	7	9
2	14	?
3	105	117

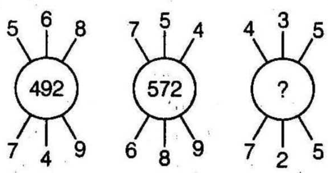
(a) 16 (b) 26
(c) 20 (d) 12

88.



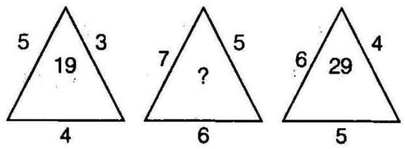
(a) 14 (b) 10
(c) 8 (d) 6

89.



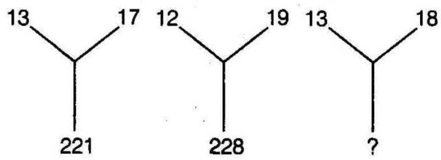
(a) 130 (b) 115 (c) 140 (d) 135

90.



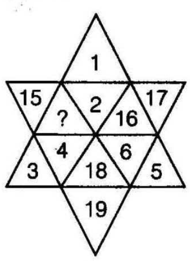
(a) 37 (b) 41 (c) 25 (d) 47

91.

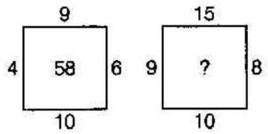


(a) 234 (b) 31
(c) 312 (d) 229

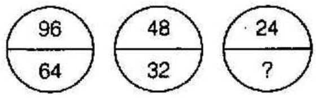
92.



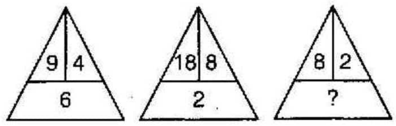
(a) 20 (b) 13 (c) 21 (d) 14

93. 

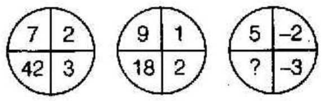
(a) 63 (b) 117 (c) 78 (d) 100

94. 

(a) 21 (b) 16 (c) 10 (d) 8

95. 

(a) 7 (b) 6 (c) 5 (d) 4

96. 

(a) 13 (b) –30 (c) 8 (d) 30

97. If the following words are arranged in the dictionary order, then which will be the last word?

(a) Dream (b) Drench (c) Dread (d) Dredge

Directions (Q. Nos. 98-101) *Read the following information carefully and answer the questions given below:*

Ravi and Kunal are good in Hockey and Volleyball, Sachin and Ravi are good in Hockey and baseball. Gaurav and Kunal are good in Cricket and Volleyball. Sachin, Gaurav and Michael are good in Football and Baseball.

98. Who is good in Hockey, Cricket and Volleyball?

(a) Gaurav (b) Sachin (c) Kunal (d) Ravi

99. Who is good in Baseball, Cricket, Volleyball and Football?

(a) Ravi (b) Sachin
(c) Gaurav (d) Kunal

100. Who is good in Baseball, Volleyball and Hockey?

(a) Sachin (b) Kunal
(c) Gaurav (d) Ravi

101. Who is good in Hockey, Baseball and Football?

(a) Ravi (b) Kunal
(c) Gaurav (d) Sachin

102. As a "Shirt is related to "Cloth" in the same way "Chair" is related to what?

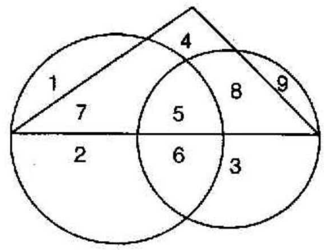
(a) Weaving (b) Repairing
(c) Wood (d) Sit

103. As 'Author' is related to 'Writing'. Similarly "Theif is related to what?

(a) To steal (b) To night
(c) To feel (d) To wonder

Directions (Q. Nos. 104-108)

These questions are based on the following diagram in which the triangle represents female graduates, small circle represents self-employed females having a car and the big circle represents self-employed females with bank loan facility. Number are shown in the different sections of the diagram. On the basis of these numbers answer the following questions:



104. How many non-graduate self- employed females are with bank loan facility?

(a) 9 (b) 8 (c) 3 (d) 12

105. How many female graduates are not self-employed?

(a) 15 (b) 10 (c) 12 (d) 4

106. How many female graduates are self-employed and having a car?

(a) 15 (b) 12 (c) 20 (d) 13

107. How many non-graduate females are self-employed?

(a) 12 (b) 11 (c) 9 (d) 21

108. How many self- employed female graduates are with bank loan facility?

(a) 12 (b) 20
(c) 7 (d) 5

Directions (Q. Nos. 109-112) *In each of the following questions, statement/group of statements is given followed by some conclusions, choose the conclusion which logically follows from the given statements.*

109. Statements

1. Processed meat is a perishable food.
2. All perishable foods are packed in sealed tins.
3. Sealed tins some times do not contain processed meat.

Conclusion

(a) Processed meat is always packed in sealed tins.
(b) Processed meat is sometimes not packed in sealed tins.
(c) Non-perishable foods are never packed in sealed tins.
(d) Sealed tins always contain perishable food.

110. Statements

1. Only students can participate in the race.
2. Some participants in the race are females.
3. All females participants in the race are invited for coaching.

Conclusion

(a) All participants in the race are invited for coaching.

(b) All participants in the race are males.

(c) All participants in the race are students.

(d) All students are invited for coaching.

111. Statements

1. I watch T.V. only if I am bored.
2. I am never bored when I have my brother's company.
3. Whenever I go to the theatre I take my brother along.

Conclusion

(a) If I am bored, I seek my brother's company.

(b) If I am bored, I watch T.V.

(c) If I am not with my brother then I watch T.V.

(d) If I am not bored, I do not watch T.V.

112. Statements

1. All members of Mohan's family are honest.
2. Some members of Mohan's family are not employed.
3. Some employed persons are not honest.
4. Some honest persons are not employed.

Conclusion:

(a) The employed members of Mohan's family are not honest.

(b) All members of Mohan's family are employed.

(c) The employed members of Mohan's family are honest.

(d) The honest members of Mohan's family are not employed.

113. Letters of which of the alternative answers when placed at the blank places one after another will complete the given letter – series?

a __ b b c __ a a b __ c c a __ b b c c

(a) c a b a (b) a c b a (c) a b b a (d) b a c b

114. As "Earthquake' is related to "Earth", similarly, Thundering' is related to what?

(a) Fair (b) Sea

(c) Earth (d) Sky

Directions (Q. Nos. 115-117)

In each of the following problems, there is one questions and three statements I, II and III given below the question. You have to decide whether the data given in the statements is sufficient to answer the question. Read all the question. Any one such alternative which contains the statements I and II together should not be accepted as anwers to the questions. Remember out of the three statements, each of them alone can also be sufficient to answer the question. In such cases for example, your answer should be taken as Only I or Only II or Only III and not Only I.

115. What does 'comes' represent in a code language?

I. 'pit na tac' means 'come and go' in that code language.

II. 'ja ta da' means 'you are good' in that code language.

III. 'na da rac' means 'You can come' in that code language.

(a) I and II together

(b) I and III together

(c) I, II and III all together

(d) II and III together

116. Five persons - A, B, C, D and E are sitting in a row. Who is sitting in the middle?

I. B is between E and C.

II. B is to the right of E.

III. D is between A and E.

(a) I and III together

(b) I, II and III together

(c) II and III together

(d) I and II together

117. Pankaj is younger than Sunita and Rupali is older than Tom. Who among them is the oldest?

I. Rupali is older than Pankaj.

II. Sunita is older than Rupali.

III. Tom is youngest among all.

(a) Only II

(b) I and II together

(c) Only III

(d) I, II and III together

118. If in a class of 37 students the places of Anuradha and Saroj are 10th and 16th respectively, what are their places from the last?

(a) 22th and 21st (b) 28th and 22nd

(c) 28th and 20th (d) 27th and 22nd

119. In the following series, find the term in place of question mark (?)

3, 8, 27, 112, 565, ?

(a) 1596 (b) 3400

(c) 2266 (d) 3396

120. Statement "Some kings are not beggars." If this fact is false then which of the following conclusion is false:

Conclusion

I. All kings are beggars.

II. Some beggars are king.

III. Some kings are beggars.

IV. All beggars are king.

V. No king is beggars.

(a) II and III (b) All

(c) I and IV (d) Only V

Answer *with* Explanations

1. ***(c)*** We have,

$$Z = \frac{1}{1-\cos\theta - i\sin\theta}$$

$$= \frac{1}{2\sin^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{1}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right)}$$

$$= \frac{\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right)}$$

$$= \frac{\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}} = \frac{1}{2} + \frac{1}{2}i\cot\frac{\theta}{2}$$

$\therefore$ $\quad \text{Re}(Z) = \frac{1}{2}$

2. ***(b)*** We have,

$$(x+iy)^{1/3} = a + ib$$

$\Rightarrow \quad x + iy = (a+ib)^3$

$\Rightarrow \quad x + iy = a^3 + 3a^2ib + 3ai^2b^2 + i^3b^3$

$\Rightarrow \quad x + iy = (a^3 - 3ab^2) + (3a^2b - b^3)i$

$\therefore \quad x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

$\Rightarrow \quad \frac{x}{a} = a^2 - 3b^2$

and $\quad \frac{y}{b} = 3a^2 - b^2$

$\therefore \quad \frac{x}{a} + \frac{y}{b} = (a^2 - 3b^2) + (3a^2 - b^2)$

$= 4a^2 - 4b^2$

$= 4(a^2 - b^2)$

3. ***(c)*** 'DAUGHTER' has 5 consonants and 3 vowels. These 3 vowels treated as a single word.

So, $8 - 3 + 1 = 6$

Number of ways vowels always together is $6! \times 3!$

$= 720 \times 6 = 4320$

$\because$ Vowels are never together.

$\therefore$ Number of ways vowels never together $= 8! - 4320$

$= 40320 - 4320 = 36000$

4. ***(c)*** Let $\alpha = 1 + i$

Then, $\beta = 1 - i$

[since, complex are always in conjugate pair]

Here, $\quad \alpha + \beta = 1 + i + 1 - i = 2$

and $\quad \alpha\beta = (1+i)(1-i)$

$= 1^2 - i^2 = 1 + 1 = 2$

So, required equation is

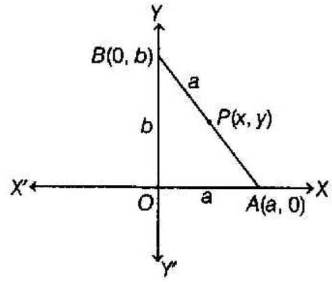
$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

[by putting the values of $(\alpha + \beta)$ and $\alpha\beta$]

$\Rightarrow \quad x^2 - 2x + 2 = 0$

5. ***(a)*** Let AB be the rod and $P(x, y)$ be any point on the rod such that $AP = 6$ cm.

$$AB = 15\text{ cm} \Rightarrow BP = 9\text{ cm}$$



Let $A(a, 0)$ and $B(0, b)$ be the coordinates of points A and B respectively.

Clearly, point $P(x, y)$ divides AB in the ratio 6 : 9 i.e. 2 : 3.

By section formula,

$$x = \frac{2(0) + 3(a)}{2+3} \text{ and } y = \frac{2(b) + 3(0)}{2+3}$$

$\Rightarrow \quad x = \frac{3a}{5}$ and $y = \frac{2b}{5}$

$\Rightarrow \quad a = \frac{5x}{3}$ and $b = \frac{5y}{2} \quad$ ...(i)

In ΔOAB,

$$(OA)^2 + (OB)^2 = (AB)^2$$

$\Rightarrow \quad a^2 + b^2 = (15)^2$

$\Rightarrow \quad \left(\frac{5x}{3}\right)^2 + \left(\frac{5y}{2}\right)^2 = 225$

$\Rightarrow \quad \frac{25x^2}{9} + \frac{25y^2}{4} = 225$

$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 9$

$\Rightarrow \quad \frac{x^2}{81} + \frac{y^2}{36} = 1$

$\therefore$ Eccentricity $\Rightarrow \quad b^2 = a^2(1 - e^2)$

$\Rightarrow \quad 36 = 81(1 - e^2)$

$\Rightarrow \quad 1 - e^2 = \frac{36}{81}$

$\Rightarrow \quad e^2 = 1 - \frac{36}{81} = \frac{45}{81}$

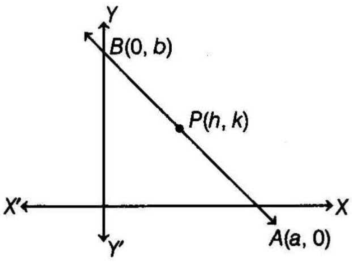
$\Rightarrow \quad e = \sqrt{\frac{45}{81}}$

$= \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

6. ***(c)*** Let $P(h, k)$ be the middle point of the given line. Let a and b be the intercept of the line on coordinate axes.

$\therefore \quad \frac{a+0}{2} = h$ and $\frac{0+b}{2} = k$

$\Rightarrow \quad a = 2h$ and $b = 2k$.



∴ Equation of line is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\Rightarrow \quad \frac{x}{2h}+\frac{y}{2k}=1$$

Since, line passes through (3, – 2), therefore

$$\frac{3}{2h}-\frac{2}{2k}=1$$

$$\Rightarrow \quad \frac{3k-2h}{2hk}=1$$

$$\Rightarrow \quad 3k-2h=2hk$$

∴ Locus of point P is $3y - 2x = 2xy$.

7. ***(a)*** Given that,

Equation of directrix = $x - y + 1 = 0$ ...(i)

Let the coordinate of focus = (a, b)

∴ The axis of parabola is perpendicular to directrix.

So, the equation of axis of parabola may be taken as

$$x+y+k=0$$

∵ It passes through (2, 1).

$$\therefore \quad 2+1+k=0$$

$$\Rightarrow \quad k=-3$$

So, the equation of axis of parabola is $x + y - 3 = 0$ ...(ii)

Now, for the point of intersection of directrix and axis of parabola.

On solving Eqs. (i) and (ii), we get

$$y=2 \text{ and } x=1$$

Thus, the point of intersection of directrix and axis of parabola $(x, y) = (1, 2)$

∵ Vertex $A(2, 1)$ is the mid-point of focus and $Z = (1, 2)$

$$\Rightarrow \quad 2=\frac{a+1}{2} \text{ and } 1=\frac{b+2}{2}$$

$$\Rightarrow \quad 4=a+1 \text{ and } 2=b+2$$

$$\Rightarrow \quad a=3 \text{ and } b=0$$

∴ Coordinate of focus = (3, 0)

Hence, required equation of parabola is

$$\sqrt{(x-3)^2+(y-0)^2}=\left|\frac{x-y+1}{\sqrt{1^2+1^2}}\right|$$

$$\Rightarrow \quad \sqrt{(x-3)^2+y^2}=\left|\frac{x-y+1}{\sqrt{2}}\right|$$

On squaring both sides, we get

$$\Rightarrow \quad (x-3)^2+y^2=\frac{(x-y+1)^2}{2}$$

$$\Rightarrow \quad x^2+9-6x+y^2=\frac{x^2+y^2+1-2xy-2y+2x}{2}$$

$$\Rightarrow \quad 2x^2+18-12x+2y^2=x^2+y^2+1-2xy-2y+2x$$

$$\Rightarrow \quad 2x^2+18-12x+2y^2-x^2-y^2-1+2xy+2y-2x=0$$

$$\Rightarrow \quad x^2+y^2-14x+2y+2xy+17=0$$

8. ***(b)*** We have,

$$f(x)=\sqrt{x-3-2\sqrt{x-4}}-\sqrt{x-3+2\sqrt{x-4}}$$

$$\Rightarrow \quad x-3-2\sqrt{x-4}\ge 0,\ x-4\ge 0,\ x-3+2\sqrt{x-4}\ge 0$$

$$\Rightarrow \quad x-3\ge 2\sqrt{x-4},\ x\ge 4,\ x-3\ge -2\sqrt{x-4}$$

$$\Rightarrow \quad (x-3)^2\ge 4(x-4),\ x\ge 4,\ (x-3)^2\ge 4(x-4)$$

$$\Rightarrow \quad (x-3)^2\ge 4(x-4),\ x\ge 4$$

$$\Rightarrow \quad x^2-6x+9\ge 4x-16,\ x\ge 4$$

$$\Rightarrow \quad x^2-10x+25\ge 0,\ x\ge 4$$

$$\Rightarrow \quad (x-5)^2\ge 0,\ x\ge 4$$

$$\therefore \quad x\in[4,\infty)$$

9. ***(d)*** Given, $\left(\frac{3}{x^2}-\frac{x^3}{6}\right)^7$

Here, it is clear that given expansion contains 8 terms.

So, 4th term from the end.

= (8 – 4 + 1)th = 5th term from the beginning

∴ Required term = $T_5 = T_{4+1}$

$$={}^7C_4\left(\frac{3}{x^2}\right)^{7-4}\left(-\frac{x^3}{6}\right)^4$$

$$={}^7C_3\left(\frac{3}{x^2}\right)^3\left(\frac{x^3}{6}\right)^4 \quad [\because {}^7C_4={}^7C_3]$$

$$=\frac{7\times 6\times 5}{3\times 2\times 1}\left(\frac{3^3}{x^6}\right)\left(\frac{x^{12}}{6^4}\right)=\frac{35}{48}x^6$$

10. ***(b)*** Let $Z=\frac{3+2i\sin\theta}{1-2\sin\theta}$

$$\therefore \quad I_m(Z)=\frac{2\sin\theta}{1-2\sin\theta}$$

For Z to be pure real, $I_m(Z)=0$

$$\therefore \quad \frac{2\sin\theta}{1-2\sin\theta}=0$$

$$\Rightarrow \quad \sin\theta=0$$

$$\Rightarrow \quad \theta=n\pi$$

11. ***(a)*** We have,

$$9x^2-16y^2=144$$

$$\Rightarrow \quad \frac{x^2}{16}-\frac{y^2}{9}=1$$

which is a hyperbola.

Here, $a = 4$ and $b = 3$

∴ Eccentricity, $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{9}{16}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$

12. ***(a)*** Given that,

$$P(A\cup B)=0.6,\ P(A\cap B)=0.2$$

$$\because \quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$0.6=P(A)+P(B)-0.2$$

$$\Rightarrow \quad P(A)+P(B)=0.8$$

Since, $P(\bar{A})+P(\bar{B})=[1-P(A)]+[1-P(B)]$

$$=2-[P(A)+P(B)]$$

$$=2-0.8$$

$$=1.2$$

13. ***(d)*** We know that,

General term of expansion $(a+b)^n$ is

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

For $(x+2y)^9$, putting $n=9, a=x, b=2y$

$$\therefore \quad T_{r+1} = {}^9C_r (x)^{9-r}.(2y)^r$$
$$= {}^9C_r (x)^{9-r} (y^r)(2)^r \quad \text{...(i)}$$

We need to find coefficient of x^6y^3

Comparing $\quad y^r = y^3$

$\therefore \quad r = 3$

Putting $r=3$ in Eq. (i), we get

$$T_{3+1} = {}^9C_3 x^{9-3} y^3.(2)^3$$
$$= \frac{9!}{3!\cdot 6!}\cdot x^6\cdot y^3\cdot(2)^3$$
$$= \frac{9\times 8\times 7}{3\times 2\times 1}\times 8\cdot x^6y^3$$
$$= \frac{9\times 8\times 7\times 8}{3\times 2}.x^6y^3$$
$$T_4 = 672.x^6y^3$$

Hence, coefficient of x^6y^3 is 672.

14. ***(*)*** $\dfrac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{576}+i^{574}}$

$$= \frac{i^{584}(i^8+i^6+i^4+i^2+1)}{i^{574}(i^8+i^6+i^2+1)}$$
$$= \frac{i^{10}(1-1+1-1+1)}{1-1-1+1}$$
$$= i^{10}\times\frac{1}{0}$$
$$= \infty$$

There is no option match.

15. ***(b)*** Let the event A is getting an odd number on the first die and event B is getting an multiples of 3 on the other die.

$\therefore P(A) = \dfrac{3}{6}$ $\quad$ [$\because$ odd numbers are 1, 3, 5]

$= \dfrac{1}{2}$

and $P(B) = \dfrac{2}{6}$ $\quad$ [$\because$ multiple of 3 are 3 and 6]

$= \dfrac{1}{3}$

$\therefore$ Required probability $P(A\cap B) = P(A)\cdot P(B)$

$= \dfrac{1}{2}\times\dfrac{1}{3}$ $\quad$ [$\because$ events are independent]

$= \dfrac{1}{6}$

16. ***(b)*** We have,

$$y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$$
$$= \sin^{-1}x + \cos^{-1}x \quad [\because \sin^{-1}\sqrt{1-x^2} = \cos^{-1}x]$$
$$= \frac{\pi}{2}$$
$$\therefore \quad \frac{dy}{dx} = 0$$

17. ***(*)*** Let

$$I = \int\frac{x^2+1}{(x+1)^2}\,dx$$
$$= \int\frac{(x^2+1+2x-2x)\,dx}{x^2+1+2x}$$
$$= \int\frac{x^2+2x+1}{x^2+1+2x}dx - \int\frac{2x}{x^2+2x+1}dx$$
$$= \int dx - \int\frac{2x}{x^2+2x+1}dx$$
$$= x - \int\frac{2(t-1)}{t^2}dt$$

[put $x^2+2x+1=t^2 \Rightarrow (x+1)^2=t^2, \Rightarrow x+1=t \Rightarrow dx=dt$]

$$= x - \int\frac{2t}{t^2}dt + \int\frac{1}{t^2}dt$$
$$= x - \int\frac{2}{t}dt + \int\frac{dt}{t^2}$$
$$= x - 2\log t - \frac{1}{t}$$
$$= x - 2\log|x+1| - \frac{1}{x+1}$$

18. ***(*)*** Let Rishi get x marks on fifth proper, then according to question

$$75 \le \frac{95+72+83+\text{marks in IVth paper}+x}{5} < 80$$
$$\Rightarrow \quad 75 \le \frac{250+\text{marks in IVth paper}+x}{5} < 80$$

Here, marks of IVth paper is not given. So, we cannot determine range of marks of fifth paper.

None option is correct.

19. ***(b)*** It is given that

$f(x+y) = f(x)\,f(y)\ \forall\ x, y \in N$ $\quad$ ...(i)

$f(1) = 3$

Taking $x=y=1$ in Eq. (i), we obtain

$f(1+1) = f(2) = f(1).f(1) = 3\times 3 = 9$

Similarly,

$f(1+1+1) = f(3) = f(1+2) = f(1)\,f(2)$

$= 3\times 9 = 27$

$f(4) = f(1+3) = f(1)\,f(3) = 3\times 27 = 81$

$\therefore f(1), f(2), f(3), \ldots$ that is 3, 9, 27, ... terms an G.P. with both the first term and common ratio equal to 3.

It is known that, $S_n = \dfrac{a(r^n-1)}{r-1}$

$$\sum_{n=1}^{n} f(x) = 120$$
$$\therefore \quad 120 = \frac{3(3^n-1)}{3-1}$$
$$\Rightarrow \quad 120 = \frac{3}{2}(3^n-1)$$
$$\Rightarrow \quad 3^n - 1 = 80$$
$$\Rightarrow \quad 3^n = 81 \Rightarrow 3^4$$
$$\Rightarrow \quad n = 4$$

20. ***(d)*** We have,

$$A=\begin{bmatrix}1 & na\\0 & 1\end{bmatrix}$$

Now, $A^2=A\cdot A=\begin{bmatrix}1 & na\\0 & 1\end{bmatrix}\begin{bmatrix}1 & na\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2na\\0 & 1\end{bmatrix}$

$$A^3=A^2\cdot A=\begin{bmatrix}1 & 2na\\0 & 1\end{bmatrix}\begin{bmatrix}1 & na\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 3na\\0 & 1\end{bmatrix}$$

$\therefore \quad A^n=\begin{bmatrix}1 & n(na)\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & n^2a\\0 & 1\end{bmatrix}$

21. ***(a)*** Let $P(x, y)$ be the point which moves such that sum of its distances from $F(3, 0)$ and $F'(-3, 0)$ is 9. The curve traced by P is an ellipse length of whose major axes $(2a = 9)$ and whose centre is at the mid-point of FF' at $(0, 0)$, where F, F' are the foci.

The major axis is the X-axis, since the foci lie an it.

Now, $\quad CF = ae = 3$

$$a=\frac{9}{2}$$

Therefore, $\quad e=\frac{3}{a}=\frac{3\times 2}{9}=\frac{2}{3}$

$$b=a\sqrt{1-e^2}$$

$\Rightarrow \quad b=\frac{9}{2}\sqrt{1-\frac{4}{9}}=\frac{3\sqrt{5}}{2}$

$\therefore \quad b^2=\frac{45}{4}$

Hence, the locus of the point is

$$\frac{x^2}{\frac{81}{4}}+\frac{y^2}{\frac{45}{4}}<1 \Rightarrow \frac{4x^2}{81}+\frac{4y^2}{45}<1$$

$\Rightarrow \quad 20x^2+36y^2<405$

22. ***(d)*** Let $I=\int_0^{\pi/4}\sqrt{1+\sin 2x}\,dx.$

Then, $\quad I=\int_0^{\pi/4}\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}\,dx$

$$=\int_0^{\pi/4}\sqrt{(\sin x+\cos x)^2}\,dx$$

$$=\int_0^{\pi/4}1(\cos x+\sin x)\,dx$$

$$=\int_0^{\pi/4}(\cos x+\sin x)\,dx=[\sin x-\cos x]_0^{\pi/4}$$

$$=\left(\sin\frac{\pi}{4}-\cos\frac{\pi}{4}\right)-(\sin 0-\cos 0)$$

$$=\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-(0-1)=1$$

23. ***(a)*** We have,

$$S_\infty=\frac{1}{1+x}-\frac{1-x}{(1+x)^2}+\frac{(1-x)^2}{(1+x)^3}-\frac{(1-x)^3}{(1+x)^4}+\dots\infty$$

$$=\frac{\frac{1}{1+x}}{1-\left(-\left(\frac{1-x}{1+x}\right)\right)}$$

$$\left[\because \text{sum of infinite GP } =\frac{a}{1-r}, r<1\right]$$

$$=\frac{\frac{1}{1+x}}{1+\frac{1-x}{1+x}}$$

$$=\frac{1}{1+x+1-x}=\frac{1}{2}$$

24. ***(b)*** The lowest number is 101 and the highest number is 998.

So, the A.P. is 101, 104, 107, ..., 998.

Here, $a = 101$, $d = 3$ and $a_n = 998$

$\because \quad a_n = a+(n-1)d$

$\Rightarrow \quad 998=101+(n-1)3$

$\Rightarrow \quad 998-101=(n-1)3$

$\Rightarrow \quad n-1=\frac{897}{3}$

$\Rightarrow \quad n-1=299$

$\Rightarrow \quad n=300$

$\therefore$ Sum of all 3-digit numbers is

$$S=\frac{n}{2}(\text{Ist term + last term})$$

$$=\frac{300}{2}(101+998)$$

$$=150\,(1099)$$

$$=164850$$

25. ***(d)*** Given equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$

Here, $\quad a=5, b=3$

$\because \quad a>5$

$\therefore$ Eccentricity $\quad e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}$

$$=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

Hence, foci are $(\pm 4, 0)$.

Given, eccentricity of hyperbola = 2

$\therefore \quad a=\frac{ae}{e}=\frac{4}{2}=2$

and $\quad b=2\sqrt{4-1}=2\sqrt{3}$

$\therefore$ Required equation of hyperbola is

$$\frac{x^2}{(2)^2}-\frac{y^2}{(2\sqrt{3})^2}=1$$

$\Rightarrow \quad \frac{x^2}{4}-\frac{y^2}{12}=1$

26. ***(d)*** The number of diagonals,

$$={}^nC_2-n$$

$$=\frac{n(n-1)-n}{2}$$

$$=\frac{n(n-1)-2n}{2}$$

$$=\frac{n^2-n-2n}{2}$$

$$=\frac{n^2-3n}{2}=\frac{n(n-3)}{2}$$

27. *(c)* Let $A=\begin{bmatrix}\cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7}\\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7}\end{bmatrix}$

$$\therefore \quad A^2 = A\cdot A = \begin{bmatrix}\cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7}\\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7}\end{bmatrix}\begin{bmatrix}\cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7}\\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7}\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\frac{2\pi}{7}-\sin^2\frac{2\pi}{7} & -2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7}\\ 2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7} & \cos^2\frac{2\pi}{7}-\sin^2\frac{2\pi}{7}\end{bmatrix}$$

$$= \begin{bmatrix}\cos 2\left(\frac{2\pi}{7}\right) & -\sin 2\left(\frac{2\pi}{7}\right)\\ \sin 2\left(\frac{2\pi}{7}\right) & \cos 2\left(\frac{2\pi}{7}\right)\end{bmatrix}$$

$$\therefore \quad A^k = \begin{bmatrix}\cos\frac{2k\pi}{7} & -\sin\frac{2k\pi}{7}\\ \sin\frac{2k\pi}{7} & \cos\frac{2k\pi}{7}\end{bmatrix}$$

Now, according to the question

$$\begin{bmatrix}\cos\frac{2k\pi}{7} & -\sin\frac{2k\pi}{7}\\ \sin\frac{2k\pi}{7} & \cos\frac{2k\pi}{7}\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\Rightarrow \quad \cos\frac{2k\pi}{7} = 1$ and $\sin\frac{2k\pi}{7} = 0$

$\therefore \quad \frac{2k\pi}{7} = 2\pi$

$\Rightarrow \quad k = 7$

$\therefore$ Least value of k is 7.

28. *(d)* Since, Salim occupies the second position Sita and Rita are always adjacent. It means that both girls are not occupying the first position. Thus, we can fill the first position by other two students in two different methods. Second position is filled by Salim in only one method.

	I	II	III	IV	V
1	X	Salim	Sita	Rita	X
2	X	Salim	Rita	Sita	X
3	X	Salim	X	Sita	Rita
4	X	Salim	X	Sita	Sita

Hence, only one position is vacant and this position is filled by 5 students in only one method.

So, required numbers of arrangements

$= 2 \times 4 \times 1 = 8$

29. *(c)* Given word is 'MATHEMATICS'

Here, $M = 2, T = 2, E = 1, C = 1$

$A = 2, H = 1, I = 1, S = 1$

There are some cases arrises which are following

${}^8C_4 \cdot 4!$ .... 4 different words

${}^3C_1 \cdot {}^7C_2 \cdot \frac{4!}{2!}$ .... 2 different and one pair

${}^3C_2 \cdot \frac{4!}{2!2!}$ .... 2 pairs

$\therefore$ Total numbers of words

$= {}^8C_4 \cdot 4! + {}^3C_1 \cdot {}^7C_2 \cdot \frac{4!}{2!} + {}^3C_2 \cdot \frac{4!}{2!2!}$

$= 8 \times 7 \times 6 \times 5 + 3 \times 21 \times 12 + 3 \times 6$

$= 1680 + 756 + 18 = 2454$

30. *(b)* We have,

$$f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$$

$\therefore \quad 1 - x > 0,\ 1 - x \neq 1$ and $x + 2 \geq 0$

$\Rightarrow \quad x < 1, x \neq 0$ and $x \geq -2$

$\therefore \quad x \in [-2, 0) \cup (0, 1)$

31. *(a)* Given,

$$I = \int_0^{\pi/2} \sin 2x \log \tan x \, dx \qquad \text{...(i)}$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right) \log \tan\left(\frac{\pi}{2} - x\right) dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \sin 2x \log \cot x \, dx \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii),

$$2I = \int_0^{\pi/2} \sin 2x [\log \tan x + \log \cot x] \, dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \sin 2x \log (\tan x \cdot \cot x) \, dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \sin 2x \cdot \log 1 \, dx$$

$\Rightarrow \quad 2I = 0$

$\Rightarrow \quad I = 0$

32. *(c)* Given, $f(2) = 4$ and $f'(2) = 1$

$$\therefore \quad \lim_{x \to 2} \frac{x f(2) - 2f(x)}{x - 2}$$

At $x = 2$, it is in $\frac{0}{0}$ form.

Using L' Hospital rule,

$$= \lim_{x \to 2} \frac{1 \times f(2) - 2f'(x)}{1}$$

$= 4 - 2 \times 1 = 4 - 2 = 2$

33. *(c)* Given,

$S_n = 3n^2 + 5n$

$S_1 = a_1 = 3(1)^2 + 5(1) = 8$

$S_2 = 3(2)^2 + 5(2) = 22$

$S_2 = 22 = a_1 + a_2$

$a_2 = 22 - 8 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

nth term value is 164, then

nth term $= a + (n-1)d$

$\Rightarrow \quad 8 + (n-1)6 = 164$

$\Rightarrow \quad (n-1)6 = 164 - 8$

$\Rightarrow \quad (n-1)6 = 156$

$\Rightarrow \quad n - 1 = \frac{156}{6} = 26$

$\Rightarrow \quad n = 26 + 1 = 27$th term

34. *(d)* Let

$$S_n = 3+7+13+21+31+\ldots+a_{n-1}+a_n \quad \ldots(i)$$

and $S_n = 0+3+7+13+21+\ldots\ldots+a_{n-2}+a_{n-1}+a_n \quad \ldots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get

$$S_n - S_n = 3-0+[(7-3)+(13-7)+(21-13)+\ldots+(a_{n-1}-a_{n-2})+(a_n-a_{n-1})]-a_n$$

$$0 = 3+[4+6+8+\ldots+a_{n-1}]-a_n$$

$$a_n = 3+[4+6+8+\ldots+a_{n-1}] \quad \ldots(iii)$$

Now, $4+6+8+\ldots\ldots+a_{n-1}$ is an A.P.

Here, $a=4$ and $d=6-4=2$

$\because$ Sum of n terms of A.P. $=\frac{n}{2}[2a+(n-1)d]$

On putting $n=n-1$, $a=4$, $d=2$

$$\therefore \quad [4+6+8+\ldots\ldots+(n-1)\text{ terms}]$$

$$=\left(\frac{n-1}{2}\right)[2a+(n-1-1)d]$$

$$=\left(\frac{n-1}{2}\right)[2(4)+(n-2)2]$$

$$=\left(\frac{n-1}{2}\right)[8+2n-4]=\left(\frac{n-1}{2}\right)[2n+4]$$

$$=(n-1)(n+2)$$

Thus, $[4+6+8+\ldots\ldots$ upto $(n-1)$ terms$]=(n-1)(n+2)$

From Eq. (iii),

$$a_n = 3+[4+6+8+\ldots\ldots+a_{n-1}]$$

On putting values

$$=3+(n-1)(n+2)$$

$$=3+n^2+2n-n-2=n^2+n+1$$

Now, $S_n=\sum_{n-1}^{n} a_n = \sum_{n=1}^{n} n^2+n+1$

$$=\sum_{n=1}^{n} n^2+\sum_{n=1}^{n} n+\sum_{n=1}^{n} 1$$

$$=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}+n$$

$$=\frac{n(n+1)(2n+1)+3n(n+1)+6n}{6}$$

$$=n\left[\frac{2n^2+n+2n+1+3n+3+6}{6}\right]$$

$$=n\left(\frac{2n^2+6n+10}{6}\right)$$

$$=\frac{n}{6}\times 2(n^2+3n+5)$$

$$=\frac{n}{3}(n^2+3n+5)$$

35. *(d)* Let $\Delta=\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$

On applying $C_1 \to C_1+C_2+C_3$, we get

$$\Delta=\begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \end{vmatrix}=\begin{vmatrix} 0 & \omega^n & \omega^{2n} \\ 0 & 1 & \omega^n \\ 0 & \omega^{2n} & 1 \end{vmatrix}$$

[$\because 1+\omega^n+\omega^{2n}=0$, if n is not multiple of 3]

$=0$

36. *(b)* The equation of the given curve is $9y^2=x^3$

Differentiating w.r.t. x,

$$9(2y)\frac{dy}{dx}=3x^2$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{x^2}{6y}$$

The slope of the normal to the given curve at point (x_1, y_1) is

$$\frac{-1}{\left.\frac{dy}{dx}\right]_{(x_1,y_1)}}=\frac{-6y_1}{x_1^2}$$

$\therefore$ Equation of the normal to the curve at (x_1, y_1) is

$$y-y_1=\frac{-6y_1}{x_1^2}(x-x_1)$$

$$\Rightarrow \quad x_1^2y-x_1^2y_1=-6xy_1+6x_1y_1$$

$$\Rightarrow \quad 6xy_1+x_1^2y=6x_1y_1+x_1^2y_1$$

$$\Rightarrow \quad \frac{6xy_1}{6x_1y_1+x_1^2y_1}+\frac{x_1^2y}{6x_1y_1+x_1^2y_1}=1$$

$$\Rightarrow \quad \frac{x}{\frac{x_1(6+x_1)}{6}}+\frac{y}{\frac{y_1(6+x_1)}{x_1}}=1$$

It is given that the normal makes equal intercepts with the axes.

Therefore,

$$\frac{x_1(6+x_1)}{6}=\frac{y_1(6+x_1)}{x_1}$$

$$\Rightarrow \quad \frac{x_1}{6}=\frac{y_1}{x_1}$$

$$\Rightarrow \quad x_1^2=6y_1 \quad \ldots(i)$$

Also, the point (x_1, y_1) lies on the curve, so we have

$$9y_1^2=x_1^3 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$9\left(\frac{x_1^2}{6}\right)^2=x_1^3$$

$$\Rightarrow \quad \frac{x_1^4}{4}=x_1^3 \Rightarrow x_1=4$$

From Eq. (ii),

$$9y_1^2=(4)^3=64$$

$$\Rightarrow \quad y_1^2=\frac{64}{9}\Rightarrow y_1=\pm\frac{8}{3}$$

Hence, required point is $\left(4, \frac{8}{3}\right)$ or $\left(4, -\frac{8}{3}\right)$.

37. *(b)* Let $(r+1)$th, $(r+2)$th and $(r+3)$th be three consecutive terms, it is given that

$${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 1:7:42$$

Now,

$$\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{1}{7}\Rightarrow\frac{r+1}{n-r}=\frac{1}{7}$$

$$\Rightarrow \quad n-8r=7 \quad \ldots(i)$$

and $\frac{{}^nC_{r+1}}{{}^nC_{r+2}}=\frac{7}{42}\Rightarrow\frac{r+2}{n-r-1}=\frac{1}{6}$

$$\Rightarrow \quad n-7r=13 \quad \ldots(ii)$$

By solving Eqs. (i) and (ii), we get

$$n=55$$

38. *(b)* Given,

$$\left(\frac{1}{x}+x^{\log_{10} x}\right)^5$$

$$T_3 = {}^5C_2\left(\frac{1}{x}\right)^{5-2}(x^{\log_{10} x})^2$$

$\because$ $T_3 = 1000$ [given]

$\therefore$ $10(x^{-3})(x^{\log_{10} x})^2 = 1000$

$\Rightarrow$ $x^{2\log x - 3} = 100$

Taking log on both sides, we get

$\Rightarrow$ $\log_x \{x^{2\log_{10} x - 3}\} = \log_2 . 100$

$\Rightarrow$ $2\log_{10} x - 3 = 2\log_x 10$

$\Rightarrow$ $2\log_{10} x - 3 = \dfrac{2}{\log_{10} x}$

$\Rightarrow$ $2(\log_{10} x)^2 - 3\log_{10} x - 2 = 0$

$\Rightarrow$ $2(\log_{10} x)^2 - 4\log_{10} x + \log_{10} x - 2 = 0$

$\Rightarrow$ $2\log_{10} x(\log_{10} x - 2) + 1(\log_{10} x - 2) = 0$

$\Rightarrow$ $(2\log_{10} x + 1)(\log_{10} x - 2) = 0$

When ,

$2\log_{10} x + 1 = 0$

$\Rightarrow$ $\log_{10} x = -\dfrac{1}{2}$

$\Rightarrow$ $x = 10^{\frac{-1}{2}}$

$\Rightarrow$ $x = \dfrac{1}{\sqrt{10}}$

or $\log_{10} x - 2 = 0$

$\Rightarrow$ $\log_{10} x = 2$

$\Rightarrow$ $x = 10^2$

$\Rightarrow$ $x = 100$

39. *(b)* If a matrix is both symmetric and skew-symmetric matrix, then A is symmetric matrix.

$\Rightarrow$ $a_{ij} = a_{ji}$

A is a skew-symmetric matrix.

$\Rightarrow$ $a_{ij} = -a_{ji}$

If $a_{ij} = a_{ji} = -a_{ji}$

$\Rightarrow$ $a_{ij} = 0$

Hence, A is a zero matrix.

40. *(c)* Given that,

$$B\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$$

$\therefore$
$$B = \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}^{-1}$$

$$= \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}\left\{\frac{1}{4+2}\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}\right\}$$

$$= 6\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\cdot\frac{1}{6}\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}4+0 & 2+0\\ 0-1 & 0+1\end{bmatrix}$$

$$= \begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$$

41. *(c)* $P(A) = \dfrac{1}{3} \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

$$P(B) = \frac{2}{7} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{2}{7} = \frac{5}{7}$$

$$P(C) = \frac{3}{8} \Rightarrow P(\overline{C}) = 1 - P(C) = 1 - \frac{3}{8} = \frac{5}{8}$$

Probability that exactly one of them can solve

$= P(A)\,P(\overline{B})\,P(\overline{C}) + P(\overline{A})\,P(B)\,P(\overline{C}) + P(\overline{A})\,P(\overline{B})\,P(C)$

$= \dfrac{1}{3}\times\dfrac{5}{7}\times\dfrac{5}{8} + \dfrac{2}{3}\times\dfrac{2}{7}\times\dfrac{5}{8} + \dfrac{2}{3}\times\dfrac{5}{7}\times\dfrac{3}{8}$

$= \dfrac{25}{168} + \dfrac{20}{168} + \dfrac{30}{168} = \dfrac{75}{168} = \dfrac{25}{56}$

42. *(c)* We have,

$(a+b)^2x^2 + 8(a^2 - b^2)x + 16(a-b)^2 = 0$

$\Rightarrow (a+b)^2x^2 + 4(a^2-b^2)x + 4(a^2-b^2)x + 16(a-b)^2 = 0$

$\Rightarrow (a+b)x[(a+b)x + 4(a-b)] + 4(a-b)[(a+b)x + 4(a-b)] = 0$

$\Rightarrow [(a+b)x + 4(a-b)][(a+b)x + 4(a-b)] = 0$

$\Rightarrow [(a+b)x + 4(a-b)]^2 = 0$

$\Rightarrow x = -\dfrac{4(a-b)}{a+b} = \dfrac{4(b-a)}{a+b}$

43. *(d)* We have,

Vertices $= (0, \pm 10)$ and $e = \dfrac{4}{5}$

$\therefore$ $b = 10$ and $e = \dfrac{4}{5}$

Now, $e = \dfrac{4}{5}$

$\Rightarrow$ $e^2 = \dfrac{16}{25}$

$\Rightarrow$ $1 - \dfrac{a^2}{b^2} = \dfrac{16}{25}$

$\Rightarrow$ $\dfrac{a^2}{b^2} = \dfrac{9}{25}$ $\Rightarrow a^2 = \dfrac{9}{25}\times(10)^2$

$\Rightarrow$ $a^2 = 36$

$\therefore$ Equation of ellipse is

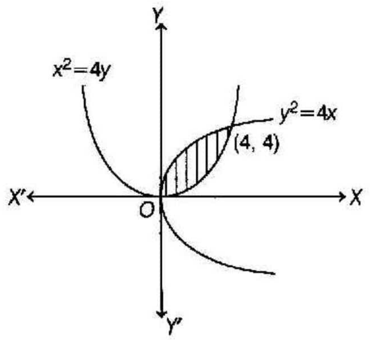
$$\frac{x^2}{36} + \frac{y^2}{100} = 1$$

44. *(b)* Given equation of curve is

$y^2 = 4x$ ...(i)

and $x^2 = 4y$ ...(ii)

Intersection point of curve (i) and (ii) is (4, 4).



$\therefore$ Required area

$$= \int_0^4 \sqrt{4x}\,dx - \int_0^4 \frac{x^2}{4}\,dx$$

$$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - 0\right] - \frac{1}{12}[(4)^3 - 0]$$

$$= \frac{4}{3} \times 8 - \frac{1}{12} \times 64$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq unit}$$

45. ***(a)*** Required probability = Probability of drawing 2 red and 2 blue balls $= \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}$

$$= \frac{\left(\frac{4 \times 3}{2}\right) \times \left(\frac{4 \times 3}{2}\right)}{\left(\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}\right)} = \frac{6}{35}$$

46. ***(d)*** Given, $(1 - 3x + 3x^2 - x^3)^8$

$$= \{(1 - x)^3\}^8 \quad [\because (a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2]$$

$$= (1 - x)^{24}$$

Hence, the number of terms are 25.

47. ***(d)*** Given,

$${}^nC_{15} = {}^nC_8$$

As we know that,

$${}^nC_x = {}^nC_y \Rightarrow n = x + y$$

$\therefore \quad n = 15 + 8 = 23$

So, $\quad {}^nC_{21} = {}^{23}C_{21} = {}^{23}C_2 = \frac{23 \cdot 22}{2}$

$$= 23 \cdot 11 = 253$$

48. ***(*)*** $\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \frac{1}{\log_{16} 4} + \ldots + \frac{1}{\log 2^n \, 4}$

$$= \log_4 2 + \log_4 2^2 + \log_4 2^3 + \log_4 2^4 + \ldots + \log_4 2^n \quad \left[\because \frac{1}{\log_a b} = \log_b a\right]$$

$$= \log_4 2\,[1 + 2 + 3 + \ldots + n]$$

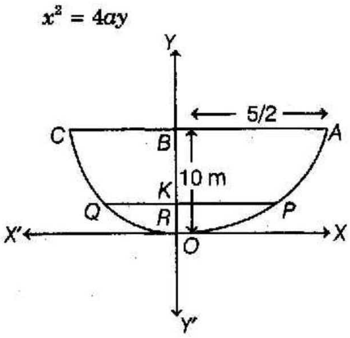
$$= \log_4 2 \cdot \frac{n(n+1)}{2} = \frac{n(n+1)}{2 \cdot \log_2 4}$$

$$= \frac{n(n+1)}{2\log_2 2^2} = \frac{n(n+1)}{4\log_2 2} \quad [\because \log_e e = 1]$$

$$= \frac{n(n+1)}{4}$$

49. ***(*)*** Here, axis is vertical, so let arch of parabola is in the form

$$x^2 = 4ay \quad \ldots(i)$$



Given, $\quad OB = 10\,\text{m}$

and $\quad AC = 5\,\text{m} \Rightarrow AB = \frac{5}{2}\,\text{m} \quad \left[\because AB = BC = \frac{AC}{2}\right]$

Hence, coordinates of $A = \left(\frac{5}{2}, 10\right)$ will satisfy Eq. (i), i.e.

$$\left(\frac{5}{2}\right)^2 = 4a \times 10 \Rightarrow \frac{25}{4} = 40a \Rightarrow a = \frac{5}{32}$$

From Eq. (i), $x^2 = 4 \times \frac{5}{32}y \Rightarrow x^2 = \frac{5}{8}y$

Now, let $OR = 2$ and $PQ = k \Rightarrow RP = \frac{k}{2}$

Therefore, $P = \left(\frac{k}{2}, 2\right)$ will lie on parabola.

$$\therefore \left(\frac{k}{2}\right)^2 = \frac{5}{8} \times 2 \Rightarrow \frac{k^2}{4} = \frac{5}{4} \Rightarrow k^2 = 5 \Rightarrow k = \sqrt{5}\,\text{m}$$

None option is correct.

50. ***(b)*** We have, $z = \frac{1 + 7i}{(2 - i)^2}$

$$z = \frac{1 + 7i}{(2 - i)^2} \times \frac{(2 + i)^2}{(2 + i)^2} = \frac{(1 + 7i)(2 + i)^2}{[(2)^2 - (i)^2]^2}$$

$$= \frac{(1 + 7i)(4 + i^2 + 4i)}{(4 - i^2)^2}$$

$$= \frac{(4 + i^2 + 4i + 28i + 7i^3 + 28i^2)}{(4 + 1)^2}$$

$$z = \frac{25i - 25}{25} = i - 1$$

Here, $\quad z = i - 1$

$\because \quad z = x + iy$

$\therefore \quad x = -1, y = 1$

$$x < 0, y > 0$$

$\arg(z) = \theta = \pi - \alpha,\ x < 0,\ y > 0$

$$\theta = \pi - \tan^{-1}\left|\frac{y}{x}\right| \Rightarrow \theta = \pi - \frac{\pi}{4}$$

$\Rightarrow \quad \theta = \frac{3\pi}{4}$

Hence, $\quad \arg(z) = \frac{3\pi}{4}$.

51. ***(d)*** Let the equation of a circle passing through the given points be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$$

Since, the circle passes through the points (1, 0).

On putting $x = 1$, $y = 0$ in Eq. (i), we get

$$1 + 2g + c = 0 \Rightarrow g = -\left(\frac{c+1}{2}\right) \quad \ldots(ii)$$

Also, circle passes through the points (0, 1).

On putting $x = 0$, $y = 1$ in Eq. (i), we get

$$1 + 2f + c = 0 \Rightarrow f = -\left(\frac{c+1}{2}\right) \quad \ldots(iii)$$

Put the value of g and f in Eq. (i), we get

$$x^2 + y^2 - (c + 1)x - (c + 1)y + c = 0$$

Radius $\quad = \sqrt{\frac{(c+1)^2}{4} + \frac{(c+1)^2}{4} - c}$

$$= \sqrt{\frac{(c+1)^2 - 2c}{2}}$$

$$=\sqrt{\frac{c^2+1}{2}} \ge \frac{1}{\sqrt{2}}, \text{ when } c=0$$

[∵ radius is smallest]

Put the value of c in Eqs. (ii) and (iii), we get

$$g=-\frac{1}{2}, f=-\frac{1}{2}$$

Now, put the value of g, f and c in Eq. (i), we get

$$x^2+y^2+2\left(-\frac{1}{2}\right)x+2\left(-\frac{1}{2}\right)y+0=0$$

$\Rightarrow \quad x^2+y^2-x-y=0$

52. *(c)* $\left|\frac{2}{x-4}\right|>1, x\ne 4$

$\Rightarrow \quad \left|\frac{x-4}{2}\right|<1, x\ne 4$

$\Rightarrow \quad |x-4|<2, x\ne 4$

$\Rightarrow \quad -2<x-4<2, x\ne 4$

$\Rightarrow \quad -2+4<x<2+4$ [adding 4 from both sides]

$\Rightarrow \quad 2<x<6, x\ne 4$

i.e. $\quad x\in(2,6), x\ne 4$

$\therefore \quad x\in(2,4)\cup(4,6)$

53. *(d)* The general term T_{r+1} in the expansion of $(17^{1/3}+35^{1/2})^{600}$ is given by

$$T_{r+1}={}^{600}C_r\left(17^{\frac{1}{3}}\right)^{600-r}(35)^{\frac{r}{2}}$$

$$={}^{600}C_r\left(17^{200-\frac{r}{3}}\right)\left(35^{\frac{r}{2}}\right)$$

$$={}^{600}C_r 17^{200-r}\left(17^{\frac{2r}{3}}\cdot 35^{\frac{r}{2}}\right)$$

$$={}^{600}C_r 17^{200-r}(17^{4r}\cdot 35^{3})^{\frac{r}{6}}$$

Cl`early, T_{r+1} will be integer, iff $\frac{r}{6}$ is an integer such that

$0\le r\le 600$

$\Rightarrow r$ is a multiple of 6 lying satisfying $0\le r\le 600$

$\Rightarrow r=0, 6, 12, \ldots, 600$

$\therefore$ There are total 101 integral terms.

54. *(c)* We have, $\quad |x-1|+|x-2|\ge 4$

Put $\quad x-1=0\Rightarrow x=1$

And $\quad x-2=0\Rightarrow x=2$

Thus, $x=1$ and 2 are critical points, so we will consider three intervals $(-\infty,1)$, $[1,2]$ and $[2,\infty)$.

Case I When $-\infty<x<(1)$

Then, $|x-1|=-(x-1)$ and $|x-2|=-(x-2)$

$$\left[\because |x-a|=\begin{cases} x-a, & \text{if } x\ge a\\ -(x-a), & \text{if } x<a\end{cases}\right]$$

$\therefore \quad |x-1|+|x-2|\ge 4$

$\Rightarrow \quad -(x-1)-(x-2)\ge 4$

$\Rightarrow \quad -x+1-x+2\ge 4\Rightarrow -2x+3\ge 4$

$\Rightarrow \quad -2x\ge 1$ [subtracting 3 from both sides]

$\Rightarrow \quad x\le -\frac{1}{2}$ [dividing both sides by -2]

$\therefore$ Solution set of given inequality in this case is

$$-\infty<x\le -\frac{1}{2} \qquad \text{...(i)}$$

Case II When $1\le x<2$

Then, $\quad |x-1|=x-1$

and $\quad |x-2|=-(x-2)$

$\therefore \quad |x-1|+|x-2|\ge 4\Rightarrow x-1-x+2\ge 4$

$\Rightarrow 1\ge 4$, which is absurd.

So, in this case, given inequality has no solution.

Case III When $2\le x<\infty$

Then, $|x-1|=x-1$ and $|x-2|=x-2$

$\therefore \quad |x-1|+|x-2|\ge 4$

$\Rightarrow \quad x-1+x-2\ge 4\Rightarrow 2x-3\ge 4$

$\Rightarrow \quad 2x\ge 7$ [adding 3 on both sides]

$\Rightarrow \quad x\ge\frac{7}{2}$ [dividing both sides by 2]

Here, $2\le x<\infty$

$\therefore$ Solution set of given inequality in this case is

$$\frac{7}{2}\le x<\infty \qquad \text{...(ii)}$$

On combining inequalities (i) and (ii), we get

$$-\infty<x\le -\frac{1}{2} \text{ and } \frac{7}{2}\le x<\infty$$

i.e. $\quad x\in\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{7}{2},\infty\right)$

55. *(b)* Total outstanding student = 9

There are 4 boys and 5 girls.

Number of ways of selecting 2 boys out of 4 boys = ${}^4C_2$

Number of ways of selecting 2 girls out of 5 girls = ${}^5C_2$

Required probability = $\frac{{}^4C_2\times{}^5C_2}{{}^9C_4}$

$$=\frac{\frac{4!}{2!\,2!}\times\frac{5!}{2!\,3!}}{\frac{9!}{5!\,4!}}=\frac{60}{126}=\frac{10}{21}$$

56. *(a)* The total number of numbers formed with the digits 2, 3, 4, 5 taken all at a times = Number of arrangement of 4 digits, taken all at a time = ${}^4P_4=4!=24$

To find the sum of these 24 members, we will find the sum of digits at unit's, ten's, hundred's and thousand's places in all these numbers.

Consider the digits in the unit' places in all these numbers. Each of the digits 2, 3, 4, 5 occurs in $3!=(6)$ times in the unit's place.

So, total for the digits in the units place in all the numbers $=(2+3+4+5)\times 3!=84$

Since, each of the digits 2, 3, 4, 5 occurs 3! times in any one of the remaining places.

So, the sum of digits in the ten's, hundred's and thousand's place in all the numbers $=(2+3+4+5)\times 3!=84$

Hence, the sum of all the numbers

$$=84\,(10^0+10^1+10^2+10^3)=93324$$

57. ***(c)*** Given, $z = 2 - 3i$

Then, $4z^3 - 3z^2 + 169$

$= 4(2-3i)^3 - 3(2-3i)^2 + 169$

$= 4[8 + 27i - 54 - 36i] - 3[4 - 9 - 12i] + 169$

$[\because (a-b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b,$
$(a-b)^2 = a^2 + b^2 - 2ab]$

$= 4[-46 - 9i] - 3[-5 - 12i] + 169$

$= -184 - 36i + 15 + 36i + 169$

$= -184 + 184 = 0$

58. ***(*)*** The quadratic equation

$x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real root.

For no real roots, discriminant must be negative.

$\therefore \quad 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) < 0$

$\Rightarrow \quad 4a^2c^2 + 4b^2d^2 + 8abcd - (4a^2 + 4b^2)(c^2 + d^2) < 0$

$\Rightarrow \quad 4a^2c^2 + 4b^2d^2 + 8abcd - 4a^2c^2 - 4a^2d^2 - 4b^2c^2 - 4b^2d^2 < 0$

$\Rightarrow \quad 8abcd < 4a^2d^2 + 4b^2c^2$

$\Rightarrow \quad 4a^2d^2 + 4b^2c^2 - 8abcd > 0$

$\Rightarrow \quad 4(a^2d^2 + b^2c^2 - 2abcd) > 0$

$\Rightarrow \quad 4(ad - bc)^2 > 0$

$\Rightarrow \quad (ad - bc)^2 > 0$

$\Rightarrow \quad ad > bc$

None option is correct.

59. ***(c)*** $f(x) = x^4 - 62x^2 + ax + 9$

$f'(x) = 4x^3 - 124x + a$

Put $x = 1$ and $f'(1) = 0$

$f'(1) = 0 = 4(1)^3 - 124(1) + a$

$\Rightarrow \quad 4 - 124 + a = 0$

$\Rightarrow \quad a = 120$

60. ***(d)*** Given, $f : R \to R$

$x \in R$ as $1 + x^2 \neq 0$ for any $x \in R$.

Now, let $y = f(x) = \dfrac{x^2}{1+x^2}$

$\Rightarrow \quad y(1 + x^2) = x^2$

$\Rightarrow \quad y + yx^2 = x^2$

$\Rightarrow \quad yx^2 + y - x^2 = 0$

$\Rightarrow \quad x^2(y - 1) = -y$

$\Rightarrow \quad x^2 = \dfrac{y}{1-y}$

$\Rightarrow \quad x = \pm\sqrt{\dfrac{y}{1-y}}$

Clearly, x will be real, if

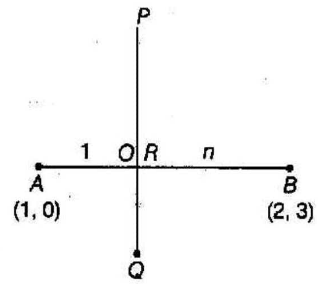
$\dfrac{y}{1-y} \geq 0$ and $y \neq 1$

$\Rightarrow \quad \dfrac{y}{y-1} \leq 0$ and $y \neq 1$

$\Rightarrow \quad 0 \leq y < 1$

$\therefore$ Range $(f) = [0, 1)$

61. ***(a)*** Let the given points be $A(1, 0)$ and $B(2, 3)$. Let the line PQ divide AB in the ratio $1:n$ at R internally.



Then coordinates of R

$= \left[\dfrac{1 \times x_2 + n \times x_1}{1+n}, \dfrac{1 \times y_2 + n \times y_1}{1+n}\right]$

[by internal division formula]

$= \left(\dfrac{1 \times 2 + n \times 1}{1+n}, \dfrac{1 \times 3 + n \times 0}{1+n}\right)$

$[\because x_1 = 1, y_1 = 0, x_2 = 2, y_2 = 3]$

$= \left(\dfrac{n+2}{n+1}, \dfrac{3}{1+n}\right)$

Let slope of line PQ be m.

$\because \quad PQ \perp AB$

$\therefore$ Slope of line $PQ \times$ Slope of line $AB = -1 \quad [\because m_1 m_2 = -1]$

$\Rightarrow \quad m \times \dfrac{y_2 - y_1}{x_2 - x_1} = -1 \Rightarrow m \times \dfrac{3-0}{2-1} = -1$

$[\because x_1 = 1, y_1 = 0, x_2 = 2, y_2 = 3]$

$\Rightarrow \quad m \times 3 = -1 \Rightarrow m = -\dfrac{1}{3}$

Now, equation of line PQ by using $y - y_0 = m(x - x_0)$ is

$y - \dfrac{3}{1+n} = -\dfrac{1}{3}\left(x - \dfrac{n+2}{n+1}\right)$

$\left[\because R\left(\dfrac{n+2}{n+1}, \dfrac{3}{1+n}\right) = (x_0, y_0)\right]$

$\Rightarrow \quad \dfrac{3(n+1)y - 9}{1+n} = \dfrac{-x(n+1) + (n+2)}{n+1}$

$\Rightarrow \quad 3(n+1)y - 9 = -x(n+1) + (n+2)$

$\Rightarrow \quad x(n+1) + 3(n+1)y = n + 2 + 9$

$\Rightarrow \quad x(n+1) + 3(n+1)y = n + 11$

Which is the required equation of line.

62. ***(b)*** A box contains 5 different red balls and 6 different white balls.

Number of ways of selecting 6 balls in which atleast two balls each colours must be these, is given by

Red	White
2	4
3	3
4	2

$\therefore$ Required ways $= {}^5C_2 \times {}^6C_4 + {}^5C_3 \times {}^6C_3 + {}^5C_4 \times {}^6C_2$

$= \dfrac{5!}{2!\,3!} \times \dfrac{6!}{4!\,2!} + \dfrac{5!}{3!\,2!} \times \dfrac{6!}{3!\,3!} + \dfrac{5!}{4!\,1!} \times \dfrac{6!}{2!\,4!}$

$= 10 \times 15 + 10 \times 20 + 5 \times 15$

$= 150 + 200 + 75 = 425$

63. ***(b)*** Given equation of circle is

$$25x^2 + 25y^2 - 20x + 2y - 60 = 0$$

On dividing both sides by 25, we get

$$\Rightarrow \quad x^2 + y^2 - \frac{20}{25}x + \frac{2}{25}y - \frac{60}{25} = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{4}{5}x + \frac{2}{25}y - \frac{12}{5} = 0$$

Radius $= \sqrt{\alpha^2 + \beta^2 - \text{(Constant term)}}$

Where $\alpha = -\frac{1}{2}$ (Coefficient of x) $= -\frac{1}{2} \times -\frac{4}{5} = \frac{2}{5}$

$\beta = -\frac{1}{2}$ (coefficient of y) $= -\frac{1}{2} \times \frac{2}{25} = -\frac{1}{25}$

Constant term $= -\frac{12}{5}$

$$\therefore \text{Radius} = \sqrt{\frac{4}{25} + \frac{1}{625} + \frac{12}{5}} = \sqrt{\frac{100 + 1 + 1500}{625}}$$

$$= \sqrt{\frac{1601}{625}} = \frac{\sqrt{1601}}{25}$$

64. ***(d)*** Let a be the first term and r be the common ratio.
Then, $a_{m+n} = p$ and $a_{m-n} = q$

$$\Rightarrow \quad ar^{m+n-1} = p \text{ and } ar^{m-n-1} = q$$

$$\Rightarrow \quad \frac{ar^{m+n-1}}{ar^{m-n-1}} = \frac{p}{q} \Rightarrow r^{2n} = \frac{p}{q}$$

$$\Rightarrow \quad r = \left(\frac{p}{q}\right)^{\frac{1}{2n}} \Rightarrow \frac{1}{r} = \left(\frac{q}{p}\right)^{\frac{1}{2n}}$$

Now, $\quad a_m = ar^{m-1}$

$$\Rightarrow \quad a_m = ar^{(m+n-1)}\left(\frac{1}{r}\right)^n$$

$$\Rightarrow \quad a_m = a_{m+n}\left(\frac{1}{r}\right)^n \quad [\because a_{m+n} = ar^{m+n-1}]$$

$$\Rightarrow \quad a_m = p\left(\frac{q}{p}\right)^{\frac{n}{2n}} \left[\because a_{m+n} = p \text{ and } \frac{1}{r} = \left(\frac{q}{p}\right)^{\frac{1}{2n}}\right]$$

$$\Rightarrow \quad a_m = p\left(\frac{q}{p}\right)^{\frac{1}{2}} = \sqrt{pq}$$

Hence, mth term is $\sqrt{pq}$.

65. ***(a)*** In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with A, D, M, N, O, R in order. A will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time i.e. A will occur 5! times. Similarly D, M, N, O will occur in the first place the same number of times.

$\therefore$ Number of words starting with A = 5! = 120

Number of words starting with D = 5! = 120

Number of words starting with M = 5! = 120

Number of words starting with N = 5! = 120

Number of words starting with O = 5! = 120

Number of words beginning with R is 5!, but one of these word is the word RANDOM. So, we first find the number of words beginning with RAD and RAM.

Number of words starting with RAD = 3! = 6

Number of words starting with RAM = 3! = 6

Now, the words beginning with 'RAN' must follow. There are 3! words beginning with RAN. One of these word is the word RANDOM itself. The first word beginning with RAN is the word RANDMO and the next word is RANDOM.

$\therefore$ Rank of RANDOM $= 5 \times 120 + 2 \times 6 + 2 = 614$

66. ***(b)*** $\int e^{2x}\left(\frac{1 + \sin 2x}{1 + \cos 2x}\right) dx$

Let, $\quad 2x = t \Rightarrow dx = \frac{1}{2} dt$

$$= \frac{1}{2}\int e^t\left(\frac{1 + \sin t}{1 + \cos t}\right) dt$$

$$= \frac{1}{2}\int e^t\left(\frac{1 + 2\sin\frac{t}{2}\cos\frac{t}{2}}{1 + 2\cos^2\frac{t}{2} - 1}\right) dt$$

$$= \frac{1}{2}\int e^t\left(\frac{1 + 2\sin\frac{t}{2}\cos\frac{t}{2}}{2\cos^2\frac{t}{2}}\right) dt$$

$$= \frac{1}{2}\int e^t\left(\frac{1}{2}\sec^2\frac{t}{2} + \tan\frac{t}{2}\right) dt$$

$$= \frac{1}{2} e^t \tan\frac{t}{2} + C$$

$$[\because \int e^x (f(x) + f'(x))\, dx = e^x f(x) + C]$$

$\because \quad t = 2x$

$\therefore \quad = \frac{1}{2} e^{2x} \tan x + C$

67. ***(b)*** Let $I = \int \sin^{-1}(\cos x)\, dx$

$$= \int \sin^{-1} \sin\left(\frac{\pi}{2} - x\right) dx$$

$$= \int\left(\frac{\pi}{2} - x\right) dx = \frac{\pi}{2}x - \frac{x^2}{2} + C$$

68. ***(*)*** Let $A_1, A_2, A_3, \ldots, A_n$ be n arithmetic means between 20 and 60 and let d be the common difference between the terms of A.P.

Then, $\quad d = \frac{(b - a)}{n + 1} = \frac{(60 - 20)}{n + 1} = \frac{40}{n + 1}$

$$A_1 = 20 + d = 20 + \frac{40}{n + 1}$$

$$= 20\left[\frac{n + 1 + 2}{n + 1}\right] = 20\left[\frac{(n + 3)}{(n + 1)}\right]$$

$$A_n = 20 + \frac{40n}{n + 1}$$

$$= 20\left[1 + \frac{2n}{n + 1}\right] = 20\left[\frac{3n + 1}{n + 1}\right]$$

$$\because \quad \frac{A_1}{A_n} = \frac{1}{3} \Rightarrow \frac{\frac{n + 3}{n + 1}}{\frac{3n + 1}{n + 1}} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{n + 3}{3n + 1} = \frac{1}{3}$$

$$\Rightarrow \quad 3n + 9 = 3n + 1$$

$$\Rightarrow \quad n = 0$$

None option is correct.

69. *(c)* Given, $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$$\therefore \quad f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right) \qquad [\because \log a^n = n \log a]$$

$$= 2f(x)$$

70. *(c)* The seating arrangement would be as follows

_G_G_G_G_G_

$\because$ 5 girls can sit in any of 5 places

$\therefore$ Number of ways they can sit = 5!

Similarly, 3 boys can sit in any of 6 places marked.

So, number of ways they can sit = ${}^6P_3$

Hence, total number of ways they can sit

$$= 5! \times {}^6P_3$$

$$= 120 \times \frac{6!}{3!}$$

$$= 120 \times \frac{720}{6}$$

$$= 720 \times 20 = 14400 \text{ ways}$$

71. *(a)* The equation of the curve is given as

$$6y = x^3 + 2$$

The rate of change of the position of the particle w.r.t time is given by

$$6\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt} + 0$$

$$\Rightarrow \quad 2\frac{dy}{dt} = x^2 \frac{dx}{dt}$$

When the y-coordinate of the particle changes 8 times as fast as the x - coordinate

i.e. $\left(\frac{dy}{dt} = 8\frac{dx}{dt}\right)$

We have, $2\left(8\frac{dx}{dt}\right) = x^2 \frac{dx}{dt}$

$$\Rightarrow \quad 16 \cdot \frac{dx}{dt} = x^2 \cdot \frac{dx}{dt}$$

$$\Rightarrow \quad (x^2 - 16) \cdot \frac{dx}{dt} = 0$$

$$\Rightarrow \quad x^2 = 16$$

$$\Rightarrow \quad x = \pm 4$$

When, $x = 4$, $y = \frac{66}{6} = 11$

When, $x = -4$, $y = -\frac{62}{6} = -\frac{31}{3}$

Hence, the required points on the curves are (4, 11) and $\left(-4, -\frac{31}{3}\right)$

72. *(*)* We have

$$y = \log_x 2$$

$$\Rightarrow \quad y = \frac{\log 2}{\log x}$$

$$\therefore \quad \frac{dy}{dx} = \log 2 \cdot \frac{d}{dx}\left(\frac{1}{\log x}\right) = \frac{-\log 2}{(\log x)^2} \cdot \frac{1}{x}$$

None option is correct.

73. *(d)* Let $z = x + iy$. Then,

$$z^2 + |z| = 0$$

$$\Rightarrow \quad (x+iy)^2 + \sqrt{x^2+y^2} = 0$$

$$\Rightarrow \quad x^2 - y^2 + \sqrt{x^2+y^2} + 2ixy = 0$$

$$\Rightarrow \quad x^2 - y^2 + \sqrt{x^2+y^2} = 0 \qquad \text{...(i)}$$

and $\quad 2xy = 0 \qquad$ ...(ii)

Now, $\quad 2xy = 0 \Rightarrow xy = 0 \Rightarrow x = 0$ or $y = 0$

Case I When $y = 0$

Putting $y = 0$ in Eq. (i),

$$x^2 + \sqrt{x^2} = 0 \Rightarrow x^2 + |x| = 0$$

Clearly, $x^2 + |x| > 0$ for all $x > 0$. So let $x < 0$.

In this case, we have

$$x^2 + |x| = 0$$

$$\Rightarrow \quad x^2 - x = 0 \qquad [\because x < 0, \therefore |x| = -x]$$

$$\Rightarrow \quad x(x-1) = 0$$

$$\Rightarrow \quad x = 0, x = 1$$

But $x < 0$, so, the equation $x^2 + |x| = 0$ has no solutions for $x < 0$.

Clearly, $x = 0$ satisfies the equation $x^2 + |x| = 0$

Thus, we have $x = 0$, $y = 0$

$$\therefore \quad z = 0$$

Case II When $x = 0$

Putting $x = 0$ in Eq. (i), we get

$$-y^2 + \sqrt{y^2} = 0 \Rightarrow -y^2 + (y) = 0$$

$$\Rightarrow \quad y = 1$$

If $y > 0$, then $|y| = y$

$$\therefore \quad -y^2 + |y| = 0$$

$$\Rightarrow \quad y = 0, y = 1 \qquad [\because y > 0]$$

$$\Rightarrow \quad y = 1$$

If $y < 0$, then $|y| = -y$

$$-y^2 + |y| = 0$$

$$\Rightarrow \quad -y^2 - y = 0$$

$$\Rightarrow \quad y = 0, -1 \qquad [\because y < 0]$$

$$\Rightarrow \quad y = -1$$

Thus, we obtain $x = 0$, $y = 1$ or $x = 0$, $y = -1$

$$\therefore \quad z = 0, +i \text{ or } z = 0, -i$$

Hence, $z = 0$, i and $-i$ are solutions of $z^2 + |z| = 0$

74. *(b)* Let E_1, E_2, E_3 be the events of selection of A, B and C respectively.

Let $\quad P(E_3) = x$

Then $\quad P(E_2) = 3P(E_3) = 3x$

And $\quad P(E_1) = 2P(E_2)$

$$= 2 \times 3x = 6x$$

Since E_1, E_2, E_3 are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) = 1$

$\therefore \quad P(E_1) + P(E_2) + P(E_3) = 1$

$\Rightarrow \quad 6x + 3x + x = 1$

$\Rightarrow \quad x = \dfrac{1}{10}$

Hence, probability of A being selected is $P(E_1)$

$\Rightarrow \quad \dfrac{6}{10} = \dfrac{3}{5}$

75. ***(b)*** $\because R = \{(x, y) \mid x, y \in z, x^2 + y^2 \le 4\}$

$\therefore R = \{(-2, 0), (-1, 0), (-1, 1), (0, -1), (0, 1), (0, 2), (0, -2), (1, -1), (0, 0), (1, 0), (1, 1), (2, 0)\}$

Hence, domain of R is $= \{-2, -1, 0, 1, 2\}$

76. ***(a)*** Let the duration of the flight is x h. Then according to question,

$$\frac{600}{x} - \frac{600}{x + \frac{1}{2}} = 200$$

$\Rightarrow \quad 600\left[\dfrac{1}{x} - \dfrac{1}{x + \frac{1}{2}}\right] = 200$

$\Rightarrow \quad 3\left[\dfrac{1}{x} - \dfrac{2}{2x + 1}\right] = 1$

$\Rightarrow \quad \dfrac{3}{x} - \dfrac{6}{2x + 1} = 1$

$\Rightarrow \quad 3(2x + 1) - 6x = 2x^2 + x$

$\Rightarrow \quad 6x + 3 - 6x = 2x^2 + x$

$\Rightarrow \quad 2x^2 + x - 3 = 0$

$\Rightarrow \quad 2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow \quad x(2x + 3) - 1(2x + 3) = 0$

$\Rightarrow \quad (2x + 3)(x - 1) = 0$

$\Rightarrow \quad x = -\dfrac{3}{2}$ or 1

$\Rightarrow \quad x = 1 \qquad \left[\because x \ne -\dfrac{3}{2}\right]$

Hence, duration of flight is 1 h.

77. ***(d)*** Let the given determinant be Δ. Then,

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}$$

$$= \begin{vmatrix} x-2 & 1 & 2 \\ x-4 & -1 & -4 \\ x-8 & -11 & -40 \end{vmatrix} \quad [C_2 \to (C_2 - 2C_1) \text{ and } C_3 \to (C_3 - 3C_1)]$$

$$= \begin{vmatrix} x-2 & 1 & 2 \\ -2 & -2 & -6 \\ -6 & -12 & -42 \end{vmatrix} \quad [R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1]$$

$$= (-2)(-6)\begin{vmatrix} x-2 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & 2 & 7 \end{vmatrix}$$

$$= 12\begin{vmatrix} x-3 & 1 & 2 \\ 0 & 1 & 3 \\ -1 & 2 & 7 \end{vmatrix} \quad [C_1 \to (C_1 - C_2)]$$

$= 12[(x - 3)(7 - 6) - 1 \cdot (3 - 2)] = 12(x - 4)$

$\therefore \Delta = 0 \Rightarrow 12(x - 4) = 0 \Rightarrow x = 4$

78. ***(d)*** Given,

$$S = 1 + \frac{1 + 2}{2} + \frac{1 + 2 + 3}{3} + \frac{1 + 2 + 3 + 4}{4} + \ldots$$

$\therefore \quad T_n = \dfrac{1 + 2 + 3 \ldots + n}{n} = \dfrac{n(n + 1)}{2n}$

$= \dfrac{n + 1}{2}$

Hence, $S = \dfrac{1}{2}(\Sigma n + n)$

$= \dfrac{1}{2}\left\{\dfrac{n(n + 1)}{2} + n\right\}$

$= \dfrac{1}{2}\left[\dfrac{n(n + 1) + 2n}{2}\right]$

$= \dfrac{n(n + 3)}{4}$

79. ***(d)*** Given,

$x = 1 + a + a^2 + a^3 + \ldots \infty$

and $\quad y = 1 + b + b^2 + b^3 + \ldots \infty$

$\therefore \quad x = \dfrac{1}{1 - a} \Rightarrow 1 - a = \dfrac{1}{x} \Rightarrow a = \dfrac{x - 1}{x}$

and $\quad y = \dfrac{1}{1 - b} \Rightarrow b = \dfrac{y - 1}{y}$

Let, $\quad z = 1 + ab + a^2b^2 + a^3b^3 + \ldots \infty$

$= \dfrac{1}{1 - ab}$

$= \dfrac{1}{1 - \left(\dfrac{x - 1}{x}\right)\left(\dfrac{y - 1}{y}\right)}$

$= \dfrac{xy}{xy - (x - 1)(y - 1)}$

$= \dfrac{xy}{xy - [xy - x - y + 1]}$

$= \dfrac{xy}{xy - xy + x + y - 1} = \dfrac{xy}{x + y - 1}$

80. ***(b)*** Given equation of parabola is

$x^2 - 4x - 8y + 12 = 0$

$x^2 - 4x = 8y - 12$

$x^2 - 4x + 4 - 4 = 8y - 12$

$\Rightarrow \quad (x - 2)^2 = 8y - 8$

$\Rightarrow \quad (x - 2)^2 = 8(y - 1)$

Given equation is in the form of $x^2 = 4ay$

$\therefore$ Length of latusrectum $\Rightarrow 4a = 8$

81. ***(a)*** The pattern of the series is as follows

50 ↑

4, 11, 21, 34, [49] 69, 91

+7 +10 +13 +16 +19 +22

$\therefore$ 49 is the wrong term.

82. *(c)* The pattern of the series is as follows

148 ↑

8, 36, [149], 596, 2388, 9556

×4+4, ×4+4, ×4+4, ×4+4, ×4+4

∴ 149 is the wrong term.

83. *(c)* The pattern of the series is as follows

19 ↑

5, 7, 11, [20], 35, 67

+2, +4, +8, +16, +32

×2, ×2, ×2, ×2

∴ 20 is the wrong term.

84. *(d)* The pattern of the series is as follows

103 ↑

5, 12, 19, 33, 47, 75, [104]

+7, +7, +14, +14, +28, +28

∴ 104 is the wrong term.

85. *(c)* The pattern of the series is as follows

35 ↑

0, 3, 8, 15, 24, [36], 48

+3, +5, +7, +9, +11, +13

+2, +2, +2, +2, +2

86. *(d)* In the given group Sky is different from rest of the words, because Sun, Star and Moon all are the part of Solar system.

87. *(d)* As, $1 \times 2 + 1 = 2 + 1 = 3$

and $7 \times 14 + 7 = 98 + 7 = 105$

Similarly, $9 \times ? + 9 = 117$

$\therefore \quad ? = \frac{117 - 9}{9} = 12$

So, the missing number is 12.

88. *(b)* As, $18 - 10 = 8$

$10 - 4 = 6$

$18 - 4 = 14$

and $22 - 14 = 8$

$14 - 8 = 6$

$22 - 8 = 14$

Similarly, $15 - 11 = 4$

$11 - 5 = 6$

$15 - 5 = ? = 10$

So, the missing number will be 10.

89. *(a)* As, $5 \times 6 \times 8 = 240$, $7 \times 4 \times 9 = 252$] $240 + 252 = 492$

and, $7 \times 5 \times 4 = 140$, $6 \times 8 \times 9 = 432$] $140 + 432 = 572$

Similarly, $4 \times 3 \times 5 = 60$, $7 \times 2 \times 5 = 70$] $60 + 70 = ? = 130$

So, the missing number will be 130.

90. *(b)* As, $5 \times 3 + 4 = 19$

and $6 \times 4 + 5 = 29$

Similarly, $7 \times 5 + 6 = ? = 41$

So, the missing number will be 41.

91. *(a)* As, $13 \times 17 = 221$

and $12 \times 19 = 228$

Similarly, $13 \times 18 = ? = 234$

So, the missing number will be 234.

92. *(d)* As, $3 + 17 = 4 + 16 = 20$

$19 + 1 = 18 + 2 = 20$

Similarly, $15 + 5 = 6 + ? = 20$

$\therefore \quad ? = 20 - 6 = 14$

93. *(c)* As, $9 \times 10 = 90$, $4 \times 8 = 32$] $90 - 32 = 58$

Similarly, $15 \times 10 = 150$, $9 \times 8 = 72$] $150 - 72 = ? = 78$

94. *(b)* As, $16 \times 6 = 96$

$16 \times 4 = 64$

and $8 \times 6 = 48$

$8 \times 4 = 32$

Similarly, $4 \times 6 = 24$

$4 \times 4 = 16$

95. *(d)* As, $9 \times 4 = 36, \sqrt{36} = 6$

and $18 \times 8 = 144, \sqrt{144} = 12$

Similarly, $8 \times 2 = 16, \sqrt{16} = 4$

96. *(d)* As, $7 \times 2 \times 3 = 42$

and $9 \times 1 \times 2 = 18$

Similarly, $5 \times (-2) \times (-3) = ? = 30$

97. *(b)* The given words can be arranged as per dictionary as,

Dread → Dream → Dredge → [Drench]

Sol. (Q. Nos. 98-101)

Person\Sport	Hockey	Volleyball	Football	Baseball	Cricket
Ravi	✓	✓	×	✓	×
Kunal	✓	✓	×	×	✓
Sachin	✓	×	✓	✓	×
Gaurav	×	✓	✓	✓	✓
Micheal	×	×	✓	✓	×

98. *(c)* Kunal is good in Hockey, Cricket and Volleyball.

99. *(c)* Gaurav is good in Baseball, Cricket, Volleyball and Football.

100. *(d)* Ravi is good in Baseball, Volleyball and Hockey.

101. *(d)* Sachin is good in Hockey, Baseball and Football.

102. *(c)* As a shirt is made up of cloth, in the same way chair is made up of wood.

103. ***(a)*** As, by the act of writing any person is classified as an **Author.**

In the same way, by the act of stealing any person is classified as a **Thief.**

104. ***(b)*** The non-graduate self-employed females with bank loan facility are $2 + 6 = 8$.

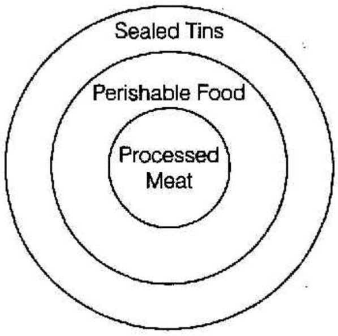
105. ***(d)*** Female graduates who are not self-employed are (4).

106. ***(d)*** Female graduates who are self-employed and having a car are $8 + 5 = 13$.

107. ***(b)*** Non-graduate females who are self-employed are $6 + 3 + 2 = 11$

108. ***(a)*** Female graduates who are self-employed and having bank loan facility are $7 + 5 = 12$.

109. ***(a)***

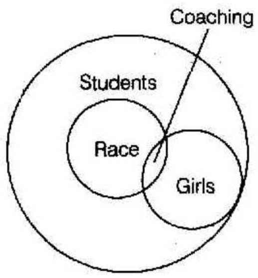


Conclusions

a. (✓) b. (×)
c. (×) d. (×)

Only, Conclusion (a) follows.

110. ***(c)***



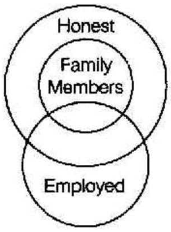
Conclusions

a. (×) b. (×)
c. (✓) d. (×)

Only, Conclusion (a) follows.

111. ***(d)*** According to statement (I), I watch T.V. only if I am bored it means that if I am not bored, I do not watch T.V.

112. ***(c)***



Conclusions

a. (×) c. (×)
b. (✓) d. (×)

Only, Conclusion (c) follows.

113. ***(b)*** The pattern of the series is,
a a b b c c/a a b b c c/a a b b c c
So, **acba** will be the answer.

114. ***(d)*** As, Earthquake is occurred on Earth's surface. In the similar way Thundering is occurred in the Sky.

115. ***(b)*** (I) 'pit na tac' → come and go

(II) 'ja ta da' → you are good

(III) 'na da rac' → you can come

Here, 'come' is in (I) and (III) and it is notified by 'na' because 'na' is common in both sentences.

So, 'come' can be represent by statement (I) and (III) together.

116. ***(b)*** From I, the order is → E, B, C or C, B, E
From II, the order is → E, B
From III, the order is → A, D, E or E, D, A
By (I), (II) and (III), we get the order A, D, E, B, C
So, clearly E is sitting in the middle.

117. ***(a)*** From the given information,

Sunita > Pankaj
Rupali > Tom

From Statement I,

Rupali > Pankaj

But we don't know whether Rupali is older than Sunita or not.

From Statement II,

Sunita > Rupali

∴ Sunita > Pankaj (given)

and Sunita > Rupali > Tom

Hence, from Statement II, Sunita is oldest.

118. ***(b)*** Total number of students = 37

From Left

Anuradha's position = 10 th
Saroj's position = 16th

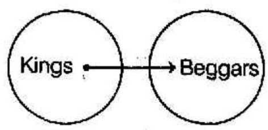
From Right

Their position will be
Anuradha's position = $37 - 10 + 1 = 27 + 1 = 28$ th
Saroj's position = $37 - 16 + 1 = 21 + 1 = 22$ nd

119. ***(d)***

$3 \times 2 + 2 = 8$
$8 \times 3 + 3 = 27$
$27 \times 4 + 4 = 112$
$112 \times 5 + 5 = 565$
$565 \times 6 + 6 = 3396$

120. ***(d)***



If the fact is false then according to above diagram, only Conclusion V is false.