KIITEE MCA
Solved Paper 2008

Mathematics

1. The least period of the function $f(x) = [x] + [x + 1/3] + [x + 2/3] - 3x + 10$, where $[x]$ denotes the greatest integer $\leq n$ is
(a) 2/3 (b) 1
(c) 1/3 (d) 1/2

2. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is
(a) {1, 2, 3, 4} (b) {1, 2, 3}
(c) {1, 2, 3, 4, 5} (d) {3, 4, 5, 6}

3. Let $A = \{x \mid -1 < x < 1\} = B$. If $f : A \to B$ be bijective, then $f(x)$ could be defined as
(a) $|x|$ (b) $\sin \pi x$
(c) $x|x|$ (d) None of these

4. Let $f : R \to R$ be a mapping such that $f(x) = \frac{x^2}{1+x^2}$, then the property of the function f is
(a) one-one (b) one-many
(c) many-one (d) onto

5. "Congruence modulo m" is a relation with property
(a) symmetric relation only
(b) an equivalence relation
(c) reflexive relation only
(d) transitive relation only

6. For a given set $A = \{1, 2, 3\}$ the total number of distinct relations defined over A is
(a) 2^3 (b) 3^3
(c) 6 (d) 2^9

7. The radius of a circle given by the equation $z\bar{z} + (4-3i)z + (4+3i)\bar{z} + 5 = 0$ is
(a) 5/2 (b) $2\sqrt{5}$
(c) 5 (d) None of these

8. The equation $|z + i| - |z - i| = k$ represents a hyperbola, if
(a) $0 < k < 2$ (b) $-2 < k < 2$
(c) $k > 2$ (d) None of these

9. $\frac{(1+ix)(1+2ix)}{1-ix}$ is purely real, then the non-zero real value of x is
(a) $\sqrt{2}$ (b) 2
(c) 1 (d) -1

10. The fourth roots of unity are given as z_1, z_2, z_3 and z_4. The value of $z_1^2 + z_2^2 + z_3^2 + z_4^2$ is
(a) i (b) 1
(c) $-i$ (d) 0

11. If $|z - i| \leq 2$ and $z_0 = 5 + 3i$, then the maximum value of $|iz + z_0|$
(a) $\sqrt{32} - 2$ (b) $2 + \sqrt{32}$
(c) 7 (d) 4

12. If z is different from $\pm i$ and $|z| = 1$, then $\frac{z+i}{z-i}$ is
(a) purely imaginary
(b) purely real
(c) non-real with equal real and imaginary parts
(d) None of the above

13. Find the total number of ways a child can be given atleast one rupee from four 25 paise coins, three 50 paise coins and two rupee coins.
(a) 53 (b) 51
(c) 54 (d) None of these

14. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in arithmetic progression, then n is
(a) 9 (b) 8
(c) 17 (d) 14

15. Twenty apples are to be given among three boys so that each gets atleast four apples. How many ways it can be distributed
(a) ${}^{22}C_{20}$ (b) 90
(c) ${}^{10}C_8$ (d) None of these

16. The number of arrangements of the letters of the women SWAGAT taking three at a time is
(a) 72 (b) 120
(c) 14 (d) None of these

17. The number of points (x, y, z) in space, whose each coordinate is a negative integer such that $x + y + z + 12 = 0$
(a) 110 (b) 385
(c) 55 (d) None of these

18. The number of even proper divisors of 1008 is
(a) 24 (b) 23
(c) 22 (d) None of these

19. The value of $\begin{vmatrix} x & x^2-yz & 1 \\ y & y^2-zx & 1 \\ z & z^2-xy & 1 \end{vmatrix}$ is

(a) -1 (b) 1
(c) 0 (d) xyz

20. If $i=\sqrt{-1}$ and $\sqrt[4]{1}=\alpha, \beta, \gamma$ and δ, then

$\begin{vmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \alpha \\ \gamma & \delta & \alpha & \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ is equal to

(a) 1 (b) i (c) $-i$ (d) 0

21. $ax+4y+z=0$, $bx+3y+z=0$, $cx+2y+z=0$ can be a system of equation with non-trivial solutions, if a, b and c are in

(a) HP (b) AP
(c) GP (d) None of these

22. The system of equations $2x+3y=8$, $7x-5y=-3$ and $4x-6y+\lambda=0$ is solvable when λ is

(a) -6 (b) -8 (c) 6 (d) 8

23. The value of $\sum_{r=1}^{10} r \frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to

(a) $9(n-4)$ (b) $5(2n-9)$
(c) $10n$ (d) None of these

24. The sum of the series $\sum_{r=1}^{n} (-1)^{r-1}\, {}^nC_r(a-r)$ is equal to

(a) $n\cdot 2^{n-1}+a$ (b) 0
(c) a (d) None of these

25. The sum of the numerical coefficients in the expansion of $\left(1+\frac{x}{3}+\frac{2y}{3}\right)^{12}$ is

(a) 2^{12} (b) 1
(c) 2 (d) None of these

26. The coefficients of x^3 in the expansion of $(1-x+x^2)^5$ is

(a) 10 (b) -20 (c) -30 (d) -50

27. For $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 5 & 4 & 6 \\ 4 & 1 & 2 \\ -5 & -1 & 1 \end{bmatrix}$

(a) AB exists (b) $A+B$ exists
(c) BA exists (d) None of these

28. If $A=\begin{bmatrix} 0 & -4 & 1 \\ 2 & \lambda & -3 \\ 1 & 2 & -1 \end{bmatrix}$, then A^{-1} exists of

(a) $\lambda=4$ (b) $\lambda\neq 8$
(c) $\lambda\neq 4$ (d) None of these

29. If $A=\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, then A^2 is equal to

(a) A (b) A^T
(c) I (d) None of these

30. If $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, then $x\cdot y$ is equal to

(a) -5 (b) 5 (c) 4 (d) 6

31. The value of $\sin\frac{\pi}{n}+\sin\frac{3\pi}{n}+\sin\frac{5\pi}{n}+\ldots+n$ terms is equal to

(a) 0 (b) 1
(c) $\frac{n}{2}$ (d) None of these

32. If $e^{\sin x}-e^{-\sin x}-4=0$, then the number of real values of x is

(a) 0 (b) 1
(c) infinite (d) None of these

33. In a ΔABC, $a=5, b=4$ and $\tan\frac{C}{2}=\sqrt{\frac{7}{9}}$, then the side c is

(a) 3 (b) 6
(c) 2 (d) None of these

34. In a ΔABC, $A=90°$. Then, $\tan^{-1}\frac{b}{a+c}+\tan^{-1}\frac{c}{a+b}$ is equal to

(a) $\frac{\pi}{4}$ (b) $\tan^{-1}\frac{a}{b+c}$
(c) $\frac{\pi}{2}$ (d) None of these

35. If in a ΔABC, $3a=b+c$, then $\tan\frac{B}{2}\cdot\tan\frac{C}{2}$ is equal to

(a) 1 (b) $\tan\frac{A}{2}$
(c) 2 (d) None of these

36. $\cos^{-1}(\cos x)=x$ is satisfied by

(a) $x\in R$ (b) $x\in[-1, 1]$
(c) $x\in[0, \pi]$ (d) None of these

37. The value of $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$ is

(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) None of these

38. The coordinates of a point on the line $x+y=3$ such that the points is at equal distances from the lines $|x|=|y|$ are

(a) $(3, 0)$ (b) $(-3, 0)$
(c) $(0, -3)$ (d) None of these

39. Lines are drawn through the points $P(-2, -3)$ to meet the circle $x^2+y^2-2x-10y+1=0$. The length of the line segment PA, A being the point on the circle where the line meets the circle is

(a) $4\sqrt{3}$ (b) 16
(c) 48 (d) None of these

40. If the common chord of the circles $x^2+(y-\lambda)^2=16$ and $x^2+y^2=16$ subtend a right angle at the origin, then λ is equal to

(a) $\pm 4\sqrt{2}$ (b) $4\sqrt{2}$ (c) 4 (d) 8

41. The equation of any tangent to the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ is
(a) $y = mx + 3\sqrt{1 + m^2}$
(b) $y = mx - 2 + 3\sqrt{1 + m^2}$
(c) $y = mx - m - 2 + 3\sqrt{1 + m^2}$
(d) None of the above

42. The tangents of the circle $x^2 + y^2 = 4$ at the points A and B meet at $P(-4, 0)$. The area of the quadrilateral $PAOB$ where O is the origin is
(a) 4 (b) $6\sqrt{2}$
(c) $4\sqrt{3}$ (d) None of these

43. The circle $x^2 + y^2 + 2\lambda x = 0$, $\lambda \in R$ touches the parabola $y^2 = 4x$ externally. Then,
(a) $\lambda > 1$ (b) $\lambda < 0$
(c) $\lambda > 0$ (d) None of these

44. A point P on the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ has the eccentric angle $\frac{\pi}{8}$. The sum of the distances of P from the two foci is
(a) 10 (b) 6 (c) 5 (d) 3

45. For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$ which of the following remains constant when α varies?
(a) directrix (b) eccentricity
(c) abscissae of foci (d) abscissae of vertices

46. The projection of a line segment on the axes of reference are 3, 4 and 12 respectively. The length of the line segment is
(a) 13 (b) 5 (c) $\frac{19}{3}$ (d) 19

47. The points $(0, 0, 0)$, $(0, 2, 0)$, $(1, 0, 0)$, $(0, 0, 4)$ are
(a) vertices of a rectangle
(b) on a sphere
(c) vertices of a parallelogram
(d) coplanar

48. If $y = (x^2 + 1)^{\sin x}$, then $y'(0)$ is equal to
(a) 1/2 (b) e^2
(c) 0 (d) 3/2

49. The vertices of a ΔABC are $A(-1, 0, 2)$, $B(1, 2, 0)$ and $C(2, 3, 4)$. The moment of a force of magnitude 10 acting a A along AB about C is
(a) $20\sqrt{6}$ (b) $\frac{50\sqrt{6}}{3}$
(c) $\frac{50}{\sqrt{3}}$ (d) None of these

50. The coplanar points A, B, C, D are $(2 - x, 2, 2)$, $(2, 2 - y, 2)$, $(2, 2, 2 - z)$ and $(1, 1, 1)$ respectively. Then, one the following is true, find it.
(a) $\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z} = 1$ (b) $x + y + z = 1$
(c) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ (d) None of these

51. If **a**, **b** and **c** be three vectors such that $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = 4$, then $[\mathbf{a} \times \mathbf{b}\ \mathbf{b} \times \mathbf{c}\ \mathbf{c} \times \mathbf{a}]$ is equal to
(a) 64 (b) 16
(c) 8 (d) None of these

52. Let **a**, **b** and **c** be three unit vectors of which **b** and **c** are non-parallel. Let the angle between **a** and **b** be α and then between **a** and **c** be β. If $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{1}{2}\mathbf{b}$, then
(a) $\alpha = \frac{\pi}{2}, \beta = \frac{\pi}{3}$ (b) $\alpha = \frac{\pi}{3}, \beta = \frac{\pi}{2}$
(c) $\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$ (d) None of these

53. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$ when [.] denotes the greatest integer, function then $\lim_{x \to 0} f(x)$ is equal to
(a) 0 (b) 1
(c) -1 (d) None of these

54. If $f(x) = e^{-\frac{1}{x^2}}$, $x \neq 0$ and $f(0) = 0$, then $f'(0)$ is
(a) 1 (b) 0
(c) e (d) None of these

55. The sum of the intercepts made on the axes of coordinates by any tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ is equal to
(a) 4 (b) 8
(c) 2 (d) None of these

56. If $f(x) = a \log e^{|x| + bx^2 + x}$ has the extremums at $x = 1$ and $x = 3$, then
(a) $a = \frac{3}{4}, b = -\frac{1}{8}$ (b) $a = -\frac{3}{4}, b = \frac{1}{8}$
(c) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (d) None of these

57. Let $f(x) = \cos \pi x + 10x + 3x^2 + x^3$, if $-2 \le x \le 3$, the absolute minimum value of $f(x)$ is
(a) 0 (b) -15
(c) $3 - 2\pi$ (d) None of these

58. A function $y = f(x)$ has a second derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is
(a) $(x - 1)^3$ (b) $(x + 1)^3$ (c) $(x - 1)^2$ (d) $(x + 1)^2$

59. A card is drawn from a pack. The card is replaced and the pack is reshuffled. If this is done six times, the probability that 2 hearts, 2 diamonds and 2 club cards are drawn is
(a) $\frac{45}{2}\left(\frac{3}{4}\right)^2$ (b) $90\left(\frac{1}{4}\right)^6$
(c) $\frac{90}{2^{10}}$ (d) None of these

60. There are three piles of identical yellow, black and green balls and each pile contains atleast 20 balls. The number of ways of selecting 20 balls, if the number of black balls to be selected is twice the number of yellow balls is
(a) 6 (b) 7
(c) 8 (d) 9

Computer Awareness

61. The earliest calculating devices are
(a) Abacus (b) Clock
(c) Difference Engine (d) None of these

62. Computers built before the First Generation of computers were
(a) Mechanical (b) Electro-mechanical
(c) Electrical (d) None of these

63. Programs stored in ROM are called
(a) Hardware (b) Firmware
(c) Software (d) None of these

64. A Compiler (Choose the most appropriate one)
(a) is a computer program
(b) translates a high level language into machine language
(c) is a part of software
(d) None of the above

65. Assembly language
(a) uses alphabetic codes in place of binary numbers used in machine language
(b) is the easiest language to write programs
(c) need not be translated into machine language
(d) None of the above

66. In a computer ...... is capable to store single binary bit.
(a) Capacitor (b) Flip-flop
(c) Register (d) Inductor

67. A set of flip-flops integrated together is called
(a) Counter (b) Adder
(c) Register (d) None of these

68. Which of the following is the best units of data on an external storage device?
(a) Bits (b) Bytes
(c) Hertz (d) Clock cycles

69. A register that allows left or right move operations is called a ......
(a) Counter (b) Loader
(c) Adder. (d) Shift register

70. ...... is a semi conductor memory.
(a) Dynamic (b) Static
(c) Bubble (d) Both (a) and (b)

71. Which of the following is not a valid capacity of a floppy disk?
(a) 360 KB (b) 720 KB
(c) 1.24 MB (d) 1.44 MB

72. Data (information) is stored in computers as
(a) Files (b) Directories
(c) Floppies (d) Matter

73. Which of the memories below is used for fast read & write operation in computer?
(a) RAM (b) ROM
(c) FDD (d) HDD

74. Time taken to move from one cylinder of a hard disk to another is called
(a) Transfer rate (b) Average seek time
(c) Latency (d) Roundtrip time

75. Which of the following RAM have to be refreshed often in order to retain its contents?
(a) SIMM (b) DIMM
(c) SDMM (d) DSMM

76. Which of the following is not a logic gate?
(a) AND (b) OR (c) NOT (d) NAT

77. The Analytical Engine developed during First Generation of computers and used ...... as a memory unit.
(a) RAM (b) Floppies
(c) Cards (d) Counter Wheels

78. MOS stands for ......
(a) Metal Oxide Semiconductor
(b) Most Often Store
(c) Method Organised Stack
(d) None of the above

79. When a key is pressed on the keyboard, which standard is used for converting the keystroke into the corresponding bits
(a) ANSI (b) ASCII
(c) EBCDIC (d) ISO

80. Which storage device is mounted on 'reels'?
(a) Floppy Disk (b) Hard Disk
(c) Magnetic Tapes (d) CDROM

81. Primary memory stores.
(a) Data alone
(b) Programs alone
(c) Results alone
(d) All of the above

82. EPROM can be used for
(a) Erasing the contents of ROM
(b) Reconstructing the contents of ROM
(c) Erasing and reconstructing the contents of ROM
(d) Duplicating ROM

83. 2's complement (0110101)
(a) 1001010 (b) 1001100
(c) 1001011 (d) 1100110

84. Gray Code of (1010)
(a) 1010 (b) 0101
(c) 1111 (d) 0000

85. Boolean algebra is different from ordinary algebra in which way?
(a) Boolean algebra can represent more than 1 discrete level between 0 and 1.
(b) Boolean algebra have only 2 discrete levels : 0 and 1.
(c) Boolean algebra can describe upto 3 levels of logic.
(d) They are actually the same.

86. What is the difference between digital and analog techniques?
(a) Digital quantities can take on any value over a continuous range.
(b) Digital quantities can take on discrete value over a range.
(c) Actually, they are not different, only digital is a new technology invented in 1980s.
(d) None of the above

87. What is the largest decimal number that can be represented using 8 bits?
(a) 128
(b) 255
(c) 256
(d) 1024

88. Which of the following is not an advantage of digital system?
(a) Digital system is easier to design.
(b) Accuracy and precision are greater.
(c) Digital circuits are less affected by noise.
(d) Digital quantities are equivalent to real-world physical quantities.

89. If $(28)_r = (18)_{16}$, then
(a) $r = 2$ (b) $r = 8$
(c) $r = 10$ (d) $r = 16$

90. Find the value of (1010) 2 * (100) 2
(a) 101010 (b) 1010100
(c) 101000 (d) 100100

Analytical Ability & Logical Reasoning

91. The least number which must be added to 893304 to obtain a perfect square is
(a) 1612 (b) 6121 (c) 279 (d) 729

92. The cube root of $\frac{9261}{42875}$ is
(a) $\frac{21}{55}$ (b) $\frac{31}{55}$ (c) $\frac{3}{5}$ (d) $\frac{21}{45}$

93. The smallest number by which 120393 should be multiplied so that the product was a cube root, is
(a) 168 (b) 961
(c) 962 (d) 169

94. The value of $(0.000729)^{-3/4} \times (0.09)^{-3/4}$ is
(a) $\frac{729}{1000000}$ (b) $\frac{5861}{1000000}$
(c) $\frac{1000000}{5861}$ (d) $\frac{1000000}{729}$

95. Some chocolates are bought at the rate of 11 of ₹ 10 and the same number at the rate of 9 for ₹ 10. If the whole lot is sold at one rupee per chocolate, the transaction is at.
(a) 1%, profit (b) 1%, loss
(c) No loss-No gain (d) 2%, loss

96. A man takes 5 h 45 min for walking to a certain place and riding back. He could have gained 2 h by riding both ways. The time he would take to walk both ways is
(a) 3 h 45 min (b) 7 h 30 min
(c) 7 h 45 min (d) 11 h 45 min

Directions (Q. Nos. 97-100) *Answer the questions on the basis of information given below.*

Four rooms are numbered as 1, 2, 3 and 4 and have different colours as yellow, blue, green and pink. These rooms are shared by Anshu, Dushmanta, Gaurang, Krishna, Jahnavi, Shankutala, Sharmistha and Sandhya. Each room is shared by two and the following facts are found to be true.

(i) Odd number rooms are neither green nor pink in colour.
(ii) Rooms of Krishna and Dushmanta have even numbers.
(iii) Shankutala and Sharmistha have rooms with colours yellow and blue respectively and their room numbers are in increasing order.
(iv) Room numbers of Anshu and Gaurang are in decreasing order.
(v) Dushmanta shares pink colour room with Jahnavi.
(vi) No other's room number is larger than that of Krishna.

97. Sandhya's room mate, room number and the colour of the room are
(a) Gaurang, 2, green (b) Krishna, 4, green
(c) Jahnavi, 3, pink (d) Anshu, 3, blue

98. Find Gaurang's room mate, room number and its colour.
(a) Shankutala, 1, yellow
(b) Sharmistha, 3, blue
(c) Sandhya, 4, green
(d) Krishna, 4, pink

99. Find room mates, room colour for the room number 2
(a) Gaurang, Shankutala, blue
(b) Anshu, Dushmanta, green
(c) Jahnavi, Dushmanta, pink
(d) Krishna, Sandhya, pink

100. Find the room mates and room number for the blue colour room.
(a) Gaurang, Shankutala, 3
(b) Anshu, Sharmistha, 3
(c) Krishna, Sandhya, 4
(d) Dushmanta, Krishna, 4

101. "Take admission in *xyz* coaching centre to succeed" : an advertisement reads, from the assumptions
1. People like to attend coaching centre to succeed.
2. People sometimes respond to advertisement.
3. Advertisement fools people.
Find the most appropriate from
(a) only 1 is implicit (b) only 2 is implicit
(c) only 3 is implicit (d) only 1 and 2 are implicit

102. Considering the following statements to be true.
1. Some computers are cell phones.
2. All cell phones are radios.
Choose the one from the following conclusions that logically follows from the statement.
(i) All radios are cell phones.
(ii) All computers are radios.
(iii) Some computers which are not cell phones are radios.
(iv) Some radios are computers.
(a) All follow (b) Only (iv) follows
(c) Only (iii) and (iv) follow (d) Only (ii) and (iii) follow

103. Two trains, one from Bhubaneswar to Delhi and the other from Delhi to Bhubaneswar, started simultaneously. After they meet at the mid-point, the trains reached their destinations after 12 h and 9 h respectively. The ratio of their speeds is
(a) 2 : 3 (b) 3 : 4
(c) 4 : 3 (d) 3 : 2

104. A watch that gains uniformly, was 5 min slow at 8 am on a Sunday, then it was 5 min 48 s fast at 8 pm on the following Sunday. When was it correct?
(a) The following Wednesday 7:20 pm.
(b) The following Thursday 8 pm.
(c) The following Friday 8:20 pm.
(d) The following Saturday 7:20 pm.

105. At the end of a board meeting the ten board members shake hands with each other once. How many hand shakes were there altogether?
(a) 55 (b) 90
(c) 45 (d) 81

106. A driver knows three different routes from Delhi to Varanasi and four different routes from Varanasi to Patna and two different routes from Patna to Balasore and one route from Balasore to Bhubaneswar. How many different routes the driver knows to reach Bhubaneswar from Delhi?
(a) 10 (b) 9
(c) 24 (d) 14

107. Saroja was returning after buying some fruits. Supriya asked her to tell how many fruits she bought. Saroja replied that she bought all oranges but eight, all mangoes but eight and all guava but eight. How many fruits Saroja bought?
(a) 16 (b) 12
(c) 24 (d) 20

108. Sneha moved a distance 50 m towards North. She then turned to the left and walked for 40 m, turned left and again walked 90 m. Finally she turned to the right at angle of 45°. In which direction she is moving finally?
(a) North-West (b) North-East
(c) South-East (d) South-West

109. Five girls took part in a race. Renu finished before Malli but behind Garima. Ani finished before Sangeeta but behind Malli. Who won the race?
(a) Renu (b) Malli
(c) Garima (d) Ani

110. A watch reads 4:30. If the minute hand points west, in what direction with the hour hand point?
(a) North-East (b) South-East
(c) North-West (d) South-West

Directions (Q. Nos. 111-115) *Answer the questions on the basis of information given below.*

"Priti, Kranti, Rudra, Sarat, Tarun, Uma and Vakul take a series of tests and the score of each is always unique. Vakul always scores more than Priti and Priti scores always more than Kranti. Each time either Rudra scores the highest and Tarun gets the least or alternatively. Sarat scores the highest and Uma or Kranti scores the least."

111. If Sarat is ranked sixth and Kranti is ranked fifth, which of the following can be true?
(a) Rudra is ranked second or third.
(b) Vakul is ranked first or fourth.
(c) Priti is ranked second or fifth.
(d) Uma is ranked third or fourth.

112. If Rudra gets most, Vakul should be ranked not lower than
(a) third (b) second
(c) fourth (d) fifth

113. If Rudra is ranked second and Kranti is ranked fifth, which of the following must be true?
(a) Tarun is ranked sixth.
(b) Priti is ranked sixth.
(c) Uma is ranked sixth.
(d) Sarat is ranked third.

114. If Sarat is ranked second, which of the following can be true?
(a) Uma gets more than Vakul.
(b) Vakul gets more than Sarat.
(c) Priti gets more than Rudra.
(d) Tarun gets more than Kranti.

115. If Vakul is ranked fifth, which of the following must be true?
(a) Rudra is ranked second.
(b) Tarun is ranked third.
(c) Sarat scores the highest.
(d) Uma scores the least.

116. Choose one word out of the given alternatives, which cannot be formed from the letters of the word "CONSULTATION"?
(a) CONSTANT (b) STATION
(c) SALUTE (d) NATION

117. Anwesha was counting down from 32 and Neha was count upwards the numbers starting from 1 and she was calling to the odd numbers. What common numbers will they call out the same time, if they were calling out at the same speed?
(a) 23
(b) They will not call out the same number
(c) 19
(d) 27

118. In a queue of girls, Deepika is eighth from the right and Neha is twelfth from the left. When Deepika and Neha interchange positions, Neha becomes twenty first from the left which of the following will be Deepika's position from the right?
(a) 21st (b) 17th
(c) Can't be told (d) 8th

119. In a group of 15 Indians, 7 read Oriya, 8 read English while 3 of them read none of these two. How many of them read Oriya and English both?
(a) 5 (b) 0
(c) 3 (d) 4

120. Choose the term which will continue the following series
P 3 C R 5 F T 8 I V 12 L?
(a) X 17 O (b) Y 17 O
(c) X 17 M (d) X 16 O

Answers with Solutions

1. (c) $f(x+T)=f(x)$

$\Rightarrow [x+T]+[x+T+1/3]+[x+T+2/3]-3(x+T)+10$
$= [x]+[x+1/3]+[x+2/3]-3x+10$

$\Rightarrow ([x+T]-[x])+([x+T+1/3]-[x+1/3])$
$+([x+2/3+T]-[x+2/3])=3T$ ...(i)

$\Rightarrow$ Least value of $T=1/3$, for which LHS and RHS of Eq. (i) becomes 1.

2. (b) Given, $f(x)={}^{7-x}P_{x-3}$

which is defined when

$x-3\geq 0$ and $7-x\geq x-3$

$\Rightarrow$ $x\geq 3$ and $x\leq 5$

$\Rightarrow$ $x=3,4,5$

$\therefore$ Range of $f(x)=\{f(3),f(4),f(5)\}$
$=\{{}^4P_0,{}^3P_1,{}^2P_2\}$
$=\{1,3,2\}=\{1,2,3\}$

3. (c) Given, $f:(-1,1)\to(-1,1)$

$f(x)=x|x|=x^2,\ x\geq 0$
$=-x^2,\ x<0$

$\Rightarrow$ $f'(x)=2x,\ x\geq 0=-2x,\ x<0$

$\Rightarrow f(x)$ is one-one and onto both.

4. (c) $f(x)=\dfrac{x^2}{1+x^2}$ is a many-one function, as $f(-1)=f(1)=\dfrac{1}{2}$

5. (b) $a\equiv b \pmod m$

$\Rightarrow$ $a-b$ is divisible by m.

(i) $a\equiv a \pmod m$;
as $a-a=0$ is $M(m)\Rightarrow$ Reflexive

(ii) $a\equiv b \pmod m\Rightarrow b\equiv a \pmod m$;
as $m\mid a-b$

$\Rightarrow$ $m\mid -(a-b)$

$\Rightarrow$ $m\mid (b-a)$

$\Rightarrow$ Symmetric

(iii) $a\equiv b \pmod m$ and
$b\equiv c \pmod m$

$\Rightarrow$ $m\mid (a-b)$ and $m\mid (b-c)$

$\Rightarrow$ $m\mid (a-b)+(b-c)$

$\Rightarrow$ $m\mid (a-c)$

$\Rightarrow$ $a\equiv c \pmod m\Rightarrow$ Transitive

So, an equivalence relation.

6. (d) $A=\{1,2,3\}$

Total number of distinct relations defined over $A=2^{[n(A)]^2}=2^9$

7. (b) Radius of circle
$z\bar{z}+\bar{a}z+a\bar{z}+b=0$ is $\sqrt{a\bar{a}-b}$.

Given equation of circle is

$z\bar{z}+(4-3i)z+(4+3i)\bar{z}+5=0$

$\Rightarrow$ $\bar{a}=4-3i,\ a=4+3i,\ b=5$

$\therefore$ Radius $=\sqrt{(4+3i)(4-3i)-5}$
$=\sqrt{25-5}=\sqrt{20}=2\sqrt{5}$

8. (b) $|z+i|-|z-i|=k$ represents a hyperbola, if $|k|<|-i-i|$

$\Rightarrow$ $|k|<2$

$\Rightarrow$ $-2<k<2$

9. (a) $\dfrac{(1+ix)(1+2ix)}{(1-ix)}=\dfrac{(1+ix)^2(1+2ix)}{(1+ix)(1-ix)}$

$=\dfrac{(1-x^2+2ix)(1+2ix)}{(1+x^2)}$

$=\dfrac{(1-5x^2)+2ix(2-x^2)}{(1+x^2)}$

It is purely real, if imaginary part is equal to zero.

$\Rightarrow$ $2x(2-x^2)=0$

$\Rightarrow$ $x=\sqrt{2}$ (as $x\neq 0$)

10. (d) Fourth roots of unity are

$z_1=1,\ z_2=-1,\ z_3=i,\ z_4=-i$

$z_1^2+z_2^2+z_3^2+z_4^2=1^2+(-1)^2+(i)^2+(-i)^2$
$=1+1-1-1=0$

11. (c) $|iz+z_0|=|i|\left|z+\dfrac{1}{i}z_0\right|$ $[\because |i|=1]$

$=|z-i(5+3i)|$
$=|z+3-5i|$
$=|z-i+3-4i|\leq|z-i|+|3-4i|\leq 2+5$

$\Rightarrow$ $|iz+z_0|\leq 7$

12. (a) Let $z=x+iy$, then $|z|=1$

$\Rightarrow$ $x^2+y^2=1$ ...(i)

$\therefore$ $\dfrac{z+i}{z-i}=\dfrac{x+iy+i}{x+iy-i}=\dfrac{x+i(y+1)}{x+i(y-1)}$

$=\dfrac{[x+i(y+1)][x-i(y-1)]}{x^2+(y-1)^2}$

$=\dfrac{(x^2+y^2-1)+2ix}{x^2+(y-1)^2}$ [from Eq. (i)]

$=\dfrac{2ix}{2-2y}=i\dfrac{x}{1-y}$

which is purely imaginary.

13. (c) Number of ways of giving child the coins from 4,25 paise 3,50 paise and 2, ₹ 1 coin $=(4+1)(3+1)(2+1)=60$

In the following ways amount will be less than one rupee

25 Paise	50 Paise	₹ 1
0	0	0
0	1	0
1	0	0
1	1	0
2	0	0
3	0	0

6 cases are invalid, so required number of ways
$=60-6=54$

14. (d) ${}^nC_4,{}^nC_5,{}^nC_6$ are in AP

$\Rightarrow$ $2({}^nC_5)={}^nC_4+{}^nC_6$

$\Rightarrow$ $2\cdot\dfrac{n!}{5!(n-5)!}=\dfrac{n!}{4!(n-4)!}+\dfrac{n!}{6!(n-6)!}$

$\Rightarrow$ $\dfrac{2}{5\times 4!(n-5)(n-6)!}=\dfrac{1}{4!(n-4)(n-5)(n-6)!}+\dfrac{1}{6\times 5\times 4!(n-6)!}$

$\Rightarrow$ $\dfrac{2}{5(n-5)}=\dfrac{1}{(n-4)(n-5)}+\dfrac{1}{30}$

$\Rightarrow$ $\dfrac{1}{(n-5)}\left(\dfrac{2}{5}-\dfrac{1}{n-4}\right)=\dfrac{1}{30}$

$\Rightarrow$ $\dfrac{1}{(n-5)}\times\dfrac{2(n-4)-5}{5(n-4)}=\dfrac{1}{30}$

$\Rightarrow$ $\dfrac{2n-13}{(n-4)(n-5)}=\dfrac{1}{6}$

$\Rightarrow$ $12n-78=n^2-9n+20$

$\Rightarrow$ $n^2-21n+98=0$

$\Rightarrow$ $(n-7)(n-14)=0$

$\Rightarrow$ $n=7$ or $n=14$

15. (c) Number of ways is equal to total number of integral solutions of

$$x_1 + x_2 + x_3 = 20 \quad \ldots(i)$$

where $x_1 \geq 4,\ x_2 \geq 4,\ x_3 \geq 4$

Eq. (i) can also be written as

$$(x_1 - 4) + (x_2 - 4) + (x_3 - 4) = 20 - 12$$

$\Rightarrow \quad y_1 + y_2 + y_3 = 8 \quad \ldots(ii)$

where $y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0$

which solution is ${}^{8+3-1}C_{3-1} = {}^{10}C_2 = {}^{10}C_8$

16. (a) In SWAGAT, we have two A and S, W, G, T each appearing once.

Selection of three letters and then their arrangement will be as follows.

Variety	Selection	Permutation	Total
2 identical, 1 different	$1 \times {}^4C_1 = 4$	$\frac{3!}{2!} = 3$	12
3 different	${}^5C_3 = 10$	$3! = 6$	60
		Total	72

17. (c) To have negative integral solution of $x + y + z + 12 = 0$, after giving -1 to each of the three variables -9 is distributed freely among those 3 variable in ${}^{9+3-1}C_{3-1}$ ways $= {}^{11}C_2 = 55$ ways

18. (a) As, $1008 = 2^4 \times 3^2 \times 7$, so

number of even divisors $= 4 \times (2 + 1) \times (1 + 1) = 24$

19. (c) $\Delta = \begin{vmatrix} x & x^2 - yz & 1 \\ y & y^2 - zx & 1 \\ z & z^2 - xy & 1 \end{vmatrix}$

By $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\Rightarrow \quad \Delta = \begin{vmatrix} x & x^2 - yz & 1 \\ y - x & (y - x)(x + y + z) & 0 \\ z - x & (z - x)(x + y + z) & 0 \end{vmatrix} = 0$$

20. (d) $(1)^{\frac{1}{4}} = \alpha, \beta, \gamma, \delta = 1, -1, i, -i$

$\Rightarrow \quad \alpha + \beta + \gamma + \delta = 0$

$$\Delta = \begin{vmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \alpha \\ \gamma & \delta & \alpha & \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$$

By $R_1 \to R_1 + R_2 + R_3 + R_4$

$$\Rightarrow \quad \Delta = \begin{vmatrix} 0 & 0 & 0 & 0 \\ \beta & \gamma & \delta & \alpha \\ \gamma & \delta & \alpha & \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix} = 0$$

21. (b) For non-trivial solution of homogeneous equation determinant of coefficient matrix equals to zero.

$$\Rightarrow \quad \begin{vmatrix} a & 4 & 1 \\ b & 3 & 1 \\ c & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad a - 2b + c = 0$

$\Rightarrow \quad a + c = 2b$

$\Rightarrow$ a, b, c are in AP.

22. (d) Since, the system of equations is solvable, so each pair of equation have solution.

$2x + 3y = 8$ and $7x - 5y = -3$

$\Rightarrow \quad x = 1$ and $y = 2$

So, $\quad 4x - 6y + \lambda = 0$

$\Rightarrow \quad 4 - 12 + \lambda = 0 \Rightarrow \lambda = 8$

23. (b) $\displaystyle\sum_{r=1}^{10} r \frac{{}^nC_r}{{}^nC_{r-1}} = \sum_{r=1}^{10} r \frac{n!\, r-1!\, n-r+1!}{r!\, n-r!\, n!}$

$$= \sum_{x=1}^{10} r. \frac{r-1!\,(n-r+1)\ n-r!}{r \cdot r-1!\,(n-r)!}$$

$$= \sum_{r=1}^{10} n - r + 1$$

$$= n + (n-1) + (n-2) + \ldots\ldots + (n-9)$$

$$= 10n - 45 = 5(2n - 9)$$

24. (c) As, $(1 - x)^n = \displaystyle\sum_{r=0}^{n} (-1)^r\ {}^nC_r \cdot x^r \quad \ldots(i)$

Differentiating both sides, we get

$$\Rightarrow \quad -n(1 - x)^{n-1} = \sum_{r=1}^{n} (-1)^r\ {}^nC_r \cdot rx^{r-1}$$

$$\Rightarrow \quad n(1 - x)^{n-1} = \sum_{r=1}^{n} (-1)^{r-1} r\,{}^nC_r x^{r-1}$$

By putting $x = 1$, we get

$$\sum_{r=1}^{n} (-1)^{r-1} r\ {}^nC_r = 0 \quad \ldots(ii)$$

By putting $x = 1$ in Eq. (i), we get

$$0 = \sum_{r=0}^{n} (-1)^r\ {}^nC_r$$

$$\Rightarrow \quad \sum_{r=0}^{n} (-1)^{r-1}\ {}^nC_r = 0$$

$$\Rightarrow \quad \sum_{r=1}^{n} (-1)^{r-1}\ {}^nC_r = 1 \quad \ldots(iii)$$

Now, $\displaystyle\sum_{r=1}^{n} (-1)^{r-1}\ {}^nC_r (a - r)$

$$= a \sum_{r=1}^{n} (-1)^{r-1}\ {}^nC_r - \sum_{r=1}^{n} r\,(-1)^{r-1}\ {}^nC_r$$

$= a$; [by Eqs. (ii) and (iii)]

25. (a) To get the sum of the numerical coefficients in the expansion of $\left(1 + \frac{x}{3} + \frac{2y}{3}\right)^{12}$

Put $x = y = 1$ in it $= \left(1 + \frac{1}{3} + \frac{2}{3}\right)^{12} = 2^{12}$

26. (c) $(1 - x + x^2)^5 = \dfrac{(1 + x^3)^5}{(1 + x)^5}$

$$= (1 + x^3)^5 (1 + x)^{-5}$$

$$= (1 + {}^5C_1 x^3 + {}^5C_2 x^6 + \ldots)$$

$$\left(1 - 5x + \frac{5 \cdot 6}{2!} x^2 - \frac{5 \cdot 6 \cdot 7}{3!} x^3 + \ldots\right)$$

$\Rightarrow$ Coefficient of x^3 in it

$$= -\frac{5 \cdot 6 \cdot 7}{3!} + {}^5C_1 = -35 + 5 = -30$$

27. (c) A is 3×2 and B is 3×3 matrix so, BA exists. But AB is not exist.

28. (b) A^{-1} exist, if $|A| \neq 0$

$$\Rightarrow \quad \begin{vmatrix} 0 & -4 & 1 \\ 2 & \lambda & -3 \\ 1 & 2 & -1 \end{vmatrix} \neq 0$$

$\Rightarrow \quad -4 + 12 - \lambda \neq 0$

$\Rightarrow \quad \lambda \neq 8$

29. (c) $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

So, A is also called involutory matrix.

30. (a) $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\Rightarrow 2(x+y) - y = 3$

and $2(2x) - (x-y) = 2$

$\Rightarrow 2x + y = 3$ and $3x + y = 2$

$\Rightarrow x = -1;\ y = 5$

$\Rightarrow x \cdot y = -5$

31. (a) $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \ldots\ldots + n$ terms

$= \sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \ldots\ldots + \sin\frac{(2n-1)\pi}{n}$

$$= \frac{\left[2\sin\frac{\pi}{n}\sin\frac{\pi}{n} + 2\sin\frac{\pi}{n}\sin\frac{3\pi}{n} + \ldots\ldots + 2\sin\frac{(2n-1)\pi}{n}\sin\frac{\pi}{n}\right]}{2\sin\frac{\pi}{n}}$$

$$= \frac{\left(\cos 0 - \cos\frac{2\pi}{n}\right) + \left(\cos\frac{2\pi}{n} - \cos\frac{4\pi}{n}\right) + \ldots + \left(\cos\frac{(2n-2)\pi}{n} - \cos\frac{2\pi n}{n}\right)}{2\sin\frac{\pi}{n}}$$

$$= \frac{1-1}{2\sin\frac{\pi}{n}} = 0$$

32. (a) $e^{\sin x} - e^{-\sin x} - 4 = 0$

$\Rightarrow y - \frac{1}{y} - 4 = 0$; by putting $e^{\sin x} = y$

$\Rightarrow y^2 - 4y - 1 = 0$

$\Rightarrow y = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$ $\quad (\because -1 \le \sin x \le 1, e \ge 0)$

$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5}$; which is not possible.

Hence, we have no solution.

33. (b) $\tan\frac{C}{2} = \sqrt{\frac{7}{9}}$ (given)

$\Rightarrow \sin\frac{C}{2} = \frac{\sqrt{7}}{4};\ \cos\frac{C}{2} = \frac{3}{4}$

$\Rightarrow \cos C = 2\cos^2\frac{C}{2} - 1 = \frac{1}{8}$

As, $c^2 = a^2 + b^2 - 2ab\cos C$ (by cosine rule)

$= 25 + 16 - \frac{40}{8} = 36$

$\Rightarrow c = 6$

34. (a) $\because A = 90°,\ 2bc\cos A = b^2 + c^2 - a^2$

$\Rightarrow a^2 = b^2 + c^2$ ...(i)

$$\tan^{-1}\frac{b}{a+c} + \tan^{-1}\frac{c}{a+b} = \tan^{-1}\frac{\frac{b}{a+c} + \frac{c}{a+b}}{1 - \frac{bc}{(a+c)(a+b)}}$$

$$= \tan^{-1}\frac{\left(\frac{ab + b^2 + ac + c^2}{a^2 + bc + ac + bc}\right)}{\frac{a^2 + ac + ab}{a^2 + bc + ac + bc}} = \tan^{-1}\left(\frac{a^2 + ab + ac}{ac + ab + a^2}\right)$$

$= \tan^{-1} 1 = \frac{\pi}{4}$ [by using Eq. (i)]

35. (d) $3a = b + c$ (given)

$\Rightarrow 3\sin A = \sin B + \sin C$

$\Rightarrow \frac{\sin B + \sin C}{\sin(B+C)} = 3;$

$[\because A + B + C = \pi, \text{so } A = \pi - (C+B)]$

$\Rightarrow \frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B+C}{2}\right)} = 3$

$\Rightarrow \frac{\cos\frac{B}{2}\cos\frac{C}{2} + \sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}} = 3$

$\Rightarrow \frac{1 + \tan\frac{B}{2}\tan\frac{C}{2}}{1 - \tan\frac{B}{2}\tan\frac{C}{2}} = 3$

$\Rightarrow$ By Dividendo-componendo, we get

$\tan\frac{B}{2}\tan\frac{C}{2} = \frac{3-1}{3+1} = \frac{1}{2}$

$\Rightarrow \tan\frac{B}{2}\tan\frac{C}{2} = \frac{1}{2}$

36. (c) $\cos^{-1}(\cos x) = x$ will be valid for $x \in [0, \pi]$

37. (c) $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$ $\quad \left[\because 2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$= \tan^{-1}\frac{\frac{2}{3}}{1 - \frac{1}{9}} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}}$

$= \tan^{-1} 1 = \frac{\pi}{4}$

38. (a) $|x| = |y|$ implies the straight lines

$y - x = 0$ and $y + x = 0$ ...(i)

Any arbitrary point on the line $x + y = 3$ will be $(\alpha, 3 - \alpha)$. It's distance from the straight lines will be equal, if

$\left|\frac{3 - \alpha - \alpha}{\sqrt{2}}\right| = \left|\frac{3 - \alpha + \alpha}{\sqrt{2}}\right|$

$\Rightarrow |3 - 2\alpha| = 3$

$\Rightarrow 3 - 2\alpha = 3$ or $3 - 2\alpha = -3$

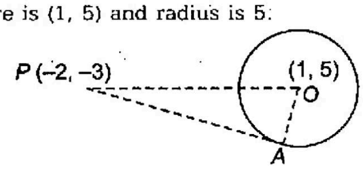
$\Rightarrow \alpha = 0, 3$

So, the required points are (0, 3) and (3, 0).

39. (a) Given circle is $x^2 + y^2 - 2x - 10y + 1 = 0$

$\Rightarrow (x-1)^2 + (y-5)^2 = (5)^2$

Centre is (1, 5) and radius is 5.



$$OP = \sqrt{3^2 + 8^2} = \sqrt{73}, \quad OA = 5$$

$\Rightarrow \quad PA = \sqrt{OP^2 - OA^2} = \sqrt{73 - 25}$

$= \sqrt{48} = 4\sqrt{3}$

40. (a) The point of intersection of $x^2 + (y - \lambda)^2 = 16$ and $x^2 + y^2 = 16$ is given by

$$16 - y^2 + (y - \lambda)^2 = 16$$

$\Rightarrow \quad (y - \lambda)^2 - y^2 = 0$

$\Rightarrow \quad (2y - \lambda)(-\lambda) = 0$

$\Rightarrow \quad y = \frac{\lambda}{2}$

$\Rightarrow \quad x = \pm\sqrt{16 - \frac{\lambda^2}{4}} = \pm\frac{\sqrt{64 - \lambda^2}}{2}$

So, the points of intersection *i.e.*, end points of the chord are

$$A\left(\frac{\sqrt{64 - \lambda^2}}{2}, \frac{\lambda}{2}\right) \text{ and } B\left(\frac{-\sqrt{64 - \lambda^2}}{2}, \frac{\lambda}{2}\right)$$

The chord will subtend right angle at origin, if product of slopes of AO and BO is $= -1$.

$\Rightarrow \quad \left[\frac{\frac{\lambda}{2}}{\frac{\sqrt{64 - \lambda^2}}{2}}\right]\left[\frac{\frac{\lambda}{2}}{\frac{-\sqrt{64 - \lambda^2}}{2}}\right] = -1$

$\Rightarrow \quad \frac{\lambda^2}{4} = \frac{64 - \lambda^2}{4}$

$\Rightarrow \quad \lambda^2 = 32$

$\Rightarrow \quad \lambda = \pm 4\sqrt{2}$

41. (c) Given circle is $x^2 + y^2 - 2x + 4y - 4 = 0$

$\Rightarrow \quad (x - 1)^2 + (y + 2)^2 = (3)^2 \quad \ldots(i)$

$y = mx + c$ will be tangent to the circle (i), if perpendicular from centre $(1, -2)$ is equal to the radius 3.

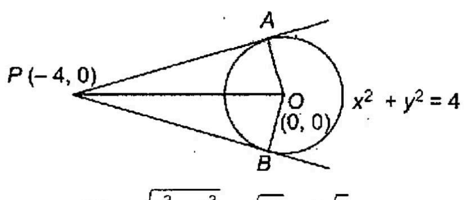
$\Rightarrow \quad \left|\frac{m + c + 2}{\sqrt{1 + m^2}}\right| = 3$

$\Rightarrow \quad c = -m - 2 + 3\sqrt{1 + m^2}$

So, the equation of tangent is

$$y = mx - m - 2 + 3\sqrt{1 + m^2}$$

42. (c) $OP = 4$ and $OA = 2$



$\Rightarrow \quad PA = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$

Also, $\quad PB = 2\sqrt{3}$

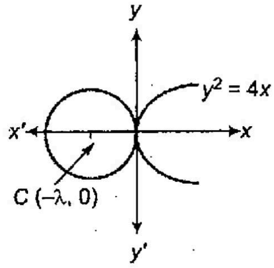
Area of $\Delta OPA = \frac{1}{2} \times OA \times PA = 2\sqrt{3}$

Area of $\Delta OPB = \frac{1}{2} \times OB \times PB = 2\sqrt{3}$

$\Rightarrow$ Area of quadrilateral $OAPB = 2\sqrt{3} + 2\sqrt{3} = 4\sqrt{3}$

43. (c) $x^2 + y^2 + 2\lambda x = 0$

$\Rightarrow \quad (x + \lambda)^2 + y^2 = \lambda^2 \quad \ldots(i)$



Circle in Eq. (i) touches the parabola $y^2 = 4x$ externally, if the centre $(-\lambda, 0)$ is on the left side of the origin on x-axis.

$\Rightarrow \quad -\lambda < 0$

$\Rightarrow \quad \lambda > 0$

44. (a) The sum of the focal distances of any point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to $2a$.

So, for ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$; $a = 5$

$\Rightarrow \quad 2a = 10$

45. (c) For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we have eccentricity '$e$' given by $\quad b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{1}{\cos\alpha} = \sec\alpha$

Directrix is $x = \pm a/e = \pm a^2/\sqrt{a^2 + b^2} = \pm \cos^2\alpha$

Abscissae of foci $= \pm ae = \pm\sqrt{a^2 + b^2}$

Abscissae of vertices $= \pm a = \pm \cos\alpha$

Here, given hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

$a = \cos\alpha$ and $b = \sin\alpha$

$\Rightarrow \quad a^2 + b^2 = 1$

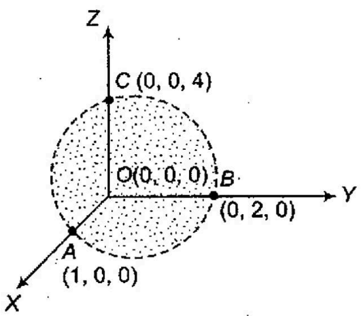
So, abscissae of foci $= \pm 1$, which is independent of α.

46. (a) $A_x = 3$; $A_y = 4$; $A_z = 12$

$\Rightarrow$ Length of line segment $= \sqrt{A_x^2 + A_y^2 + A_z^2}$

$= \sqrt{3^2 + 4^2 + (12)^2} = 13$

47. (b)



The above four point from a sphere passing through origin.

48. (c) $y = (x^2 + 1)^{\sin x}$

$\Rightarrow \quad \ln y = \sin x \ln(x^2 + 1)$, $y(0) = 1$.

$\Rightarrow \quad \frac{1}{y}y' = \cos x \ln(x^2 + 1) + \frac{[\sin x][2x]}{x^2 + 1}$

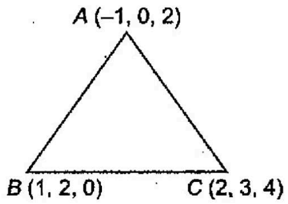
$\Rightarrow \quad y' = y\left[\frac{2x\sin x}{1 + x^2} + (\cos x)\ln(x^2 + 1)\right]$

$\Rightarrow \quad y'(0) = 1[0 + 0] = 0$

49. (d) For $A(-1,0,2)$ and $B(1,2,0)$, we have

$$\mathbf{AB} = 2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$F = 10 \cdot \frac{\mathbf{OC}}{|\mathbf{OC}|} = \frac{10(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})}{\sqrt{29}}$$



$\Rightarrow$ Moment $= |\mathbf{AB} \times \mathbf{F}|$

$$\mathbf{AB} \times \mathbf{F} = \frac{1}{\sqrt{29}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & -2 \\ 20 & 30 & 40 \end{vmatrix} = \frac{160\mathbf{i} - 120\mathbf{j} + 20\mathbf{k}}{\sqrt{29}}$$

$$\Rightarrow |\mathbf{AB} \times \mathbf{F}| = \frac{\sqrt{(160)^2 + (120)^2 + (20)^2}}{\sqrt{29}}$$

$$= 20\sqrt{\left(\frac{101}{29}\right)}$$

50. (c) Points $(2-x, 2, 2)$, $(2, 2-y, 2)$, $(2, 2, 2-z)$ and $(1, 1, 1)$ will be coplanar, if

$$\begin{vmatrix} 2-x & 2 & 2 & 1 \\ 2 & 2-y & 2 & 1 \\ 2 & 2 & 2-z & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

Using $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ and $C_4 \to C_4 - C_1$

$$\Rightarrow \begin{vmatrix} 2-x & x & x & x-1 \\ 2 & -y & 0 & -1 \\ 2 & 0 & -z & -1 \\ 1 & 0 & 0 & 0 \end{vmatrix} = 0$$

Expanding along R_4

$$\Rightarrow \begin{vmatrix} x & x & x-1 \\ -y & 0 & -1 \\ 0 & -z & -1 \end{vmatrix} = 0$$

Explanding along C_1

$\Rightarrow$ $x(-z) + y(-x + xz - z) = 0$

$\Rightarrow$ $xyz = xy + yz + zx$

$\Rightarrow$ $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

51. (b) $[\mathbf{abc}] = 4$ $\quad (\because [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2 = [\mathbf{a} \times \mathbf{b}\ \ \mathbf{b} \times \mathbf{c}\ \ \mathbf{c} \times \mathbf{a}])$

$[\mathbf{a} \times \mathbf{b}\ \ \mathbf{b} \times \mathbf{c}\ \ \mathbf{c} \times \mathbf{a}] = [\mathbf{abc}]^2 = 4^2 = 16$

52. (a) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{1}{2}\mathbf{b}$

$\Rightarrow$ $(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = \frac{1}{2}\mathbf{b}$

$(|\mathbf{a}||\mathbf{c}|\cos\beta)\mathbf{b} - (|\mathbf{a}||\mathbf{b}|\cos\alpha)\ \mathbf{c} = |\mathbf{c}| = \frac{1}{2}\mathbf{b}$

$\Rightarrow$ $(\cos\beta)\mathbf{b} - (\cos\alpha)\mathbf{c} = \frac{1}{2}\mathbf{b}$ $\quad (\because |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1)$

$\Rightarrow$ $\left(\cos\beta - \frac{1}{2}\right)\mathbf{b} = (\cos\alpha)\mathbf{c}$

$\Rightarrow$ $\cos\beta - \frac{1}{2} = 0$ and $\cos\alpha = 0$

as $\mathbf{b}$ and $\mathbf{c}$ are non-parallel.

$\Rightarrow$ $\beta = \frac{\pi}{3}$ and $\alpha = \frac{\pi}{2}$

53. (d) Given, $f(x) = \frac{\sin[x]}{[x]}$; $[x] \neq 0$

$= 0$, $[x] = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0} \frac{\sin(-1)}{-1} = \sin 1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{[x] \to 0} \frac{\sin[x]}{[x]}$$

$$= \lim_{y \to 0} \frac{\sin y}{y} = 1 \qquad \left(\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$$

$\Rightarrow$ LHL $\neq$ RHL

$\Rightarrow$ $\lim_{x \to 0} f(x)$ does not exist.

54. (b) $f(x) = e^{-\frac{1}{x^2}}$; $x \neq 0$

$= 0$; $x = 0$

$$\Rightarrow f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{e^{-\frac{1}{h^2}} - 0}{h}$$

$$= \lim_{h \to 0} \frac{1}{he^{\frac{1}{h^2}}}$$

$$= \lim_{h \to 0} \frac{1}{h\left[1 + \frac{1}{h^2} + \frac{1}{2!\,h^4} + \dots\right]}$$

$$= \lim_{h \to 0} \frac{1}{h + \frac{1}{h} + \frac{1}{2!\,h^3} + \dots}$$

$$= \frac{1}{\infty} = 0$$

55. (a) Given, $\sqrt{x} + \sqrt{y} = 2$ $\quad \dots$(i)

$$\Rightarrow \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

Equation of tangent is $Y - y = -\frac{\sqrt{y}}{\sqrt{x}}(X - x)$

Intercept on the x-axis and y-axis are $x + \sqrt{xy}$ and $y + \sqrt{xy}$ respectively.

$\Rightarrow$ Sum of intercepts $= x + y + 2\sqrt{xy}$

$= (\sqrt{x} + \sqrt{y})^2 = 2^2 = 4;$ $\quad$ [from Eq. (i)]

56. (d) $f(x) = a\log e^{|x| + bx^2 + x}$

$\Rightarrow$ $f(x) = a(|x| + bx^2 + x)$ $\quad (\because |x| = -x, x < 0)$

$= abx^2$, if $x < 0$

$= abx^2 + 2ax$, if $x \geq 0$

$\Rightarrow$ $f'(x) = 2abx$, if $x < 0$

$= 2abx + 2a$, if $x \geq 0$

Given $f(x)$ has the extremums at $x = 1$ and $x = 3$.

$\Rightarrow$ $f'(1) = 0$ and $f'(3) = 0$

$\Rightarrow$ $2ab + 2a = 0$ and $6ab + 2a = 0$

$\Rightarrow$ $a = 0$ and $b \in R$

57. (b) $f(x) = \cos\pi x + 10x + 3x^2 + x^3$ for $-2 \leq x \leq 3$

$\Rightarrow$ $f'(x) = -\pi\sin\pi x + 10 + 6x + 3x^2$

$= 3(x+1)^2 + 7 - \pi\sin\pi x > 0$

$\Rightarrow$ $f(x)$ is increasing for all $x \in R$ and hence also in the interval $[-2, 3]$

$\therefore$ Minimum value of $f(x)$ at $[-2, 3]$

$= f(-2)$

$= \cos(-2\pi) - 20 + 12 - 8$
$= 1 - 20 + 12 - 8 = -15$

58. (a) $f''(x) = 6(x-1)$
$= 6x - 6$
$\Rightarrow \quad f'(x) = 6\left(\frac{x^2}{2}\right) - 6x + C$
$= 3x^2 - 6x + C$

Curve is passing through (2, 1).
$\Rightarrow$ Slope of tangent at (2, 1) is
$= 3 \times 2^2 - 6 \times 2 + C = 12 - 12 + C$
$= C$

But given equation of tangent is
$y = 3x - 5$
$\Rightarrow \quad C = 3$
$\therefore \quad f'(x) = 3x^2 - 6x + 3$
$\Rightarrow \quad f(x) = x^3 - 3x^2 + 3x + k$

It is passing through (2, 1).
$\Rightarrow \quad 1 = 8 - 12 + 6 + k$
$\Rightarrow \quad k = -1$
$\therefore \quad f(x) = x^3 - 3x^2 + 3x - 1$
$= (x-1)^3$

59. (d) Probability of selecting a card which is of any suit
$= \frac{13}{52} = \frac{1}{4}$
$\Rightarrow$ Required probability
$= {}^6C_2 \times {}^4C_2 \times {}^2C_2 \times \left(\frac{1}{4}\right)^6$
$= 90\left(\frac{1}{4}\right)^6$

60. (b) No. of balls $\begin{matrix} y & B & G \\ r & 2r & 20-3r \end{matrix}$

Here, $r = 0, 1, 2, 3, 4, 5, 6$
$\Rightarrow$ Seven possible cases are possible.

61. (a) Abacus

62. (b) Electro-mechanical like as Zu Se Z_3 (Germany) in May, 1941.

63. (b) **Firmware** It is a combination of software and hardware. Computer chips that have data or programs recorded on them are firmware. These chips commonly include the following
ROM PROM EPROM

64. (b) Compiler translates a high level language into Machine language.

65. (a)

66. (b) **Flip-Flop** A Flip-Flop is used to store one bit of information.

67. (c) Register

68. (b) Bytes

69. (d) Shift register.

70. (d) Since, Dynamic and Static are the types of RAM memory and memory is a semi-conductor.

71. (c) Because, 360 KB size of $5\frac{1}{4}$ inch DD. 720 KB size $3\frac{1}{2}$ inch DD or $5\frac{1}{4}$ inch QD and 1.44 MB or 1440 KB size of $3\frac{1}{2}$ inch HD but 1.24 MB or 1240 KB is not valid size of any FDD.

72. (a) Files and infiles as information.

73. (a) **RAM** Random access memory in which data access randomly.

74. (d) Roundtrip time.

75. (b) **DIMM** (Dual Inlime Memory Module), a small circuit board that holds memory chips. It has 64-bitpath with DIMM's, you can install memory one DIMM's at a time.

76. (d) Because, AND, OR and NOT are logic gates.

77. (a) Cards.

78. (a) **MOS** Metal Oxide Semi-conductor

79. (b) **ASCII** American Standard Code for Information Interchange

80. (b) Magnetic tapes

81. (a) Data alone.

82. (c) **EPROM** (Erasable Programmable Read Only Memory) : Erasing and reconstructing the contents of ROM.

83. (c) 1's complement of (0110101) = 1001010
Now, 2's complement of (0110101) = 1001010
$\quad$ + 1
$\quad$ 1001011

84. (c) Number = 1 0 1 0
$\downarrow \downarrow \downarrow \downarrow$
Gray Code = 1 1 1 1
XOR of 1st and 2nd, then 2nd and 3rd and then 3rd and 4th has been taken after retaining the most significant bit.

85. (b) Boolean algebra have only 2 discrete levels : 0 and 1.

86. (b) Digital quantities can take on discrete value over a range while analog quantities can take on any value over a continuous range.

87. (b) The largest decimal number that can be represented using 8 bits is $2^n - 1$, where $n = 8$.
So, $2^8 - 1 = 256 - 1 = 255$

88. (d)

89. (b) $(28)_r = (18)_{16}$
$\Rightarrow \quad 2 \times r^1 + 8 \times r^0 = 1 \times 16^1 + 8 \times 16^0$
$\Rightarrow \quad 2r + 8 = 16 + 8$
$\Rightarrow \quad 2r = 16$
$\Rightarrow \quad r = 8$

90. (c) $(1010)_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
$= 8 + 0 + 2 + 0$
$= 10$
$(100)_2 = 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$
$= 4 + 0 + 0$
$= 4$
So, $(1010)_2 \cdot (100)_2 = 10 \times 4$
$= 40$
$= (101000)_2$

91. (a)

9	$\overline{89}\,\overline{33}\,\overline{04}$	946
9	81	
184	833	
4	736	
1886	9704	
6	11316	
1892		

$\therefore \quad 945^2 < 893304 < 946^2$
Hence, $946^2 - 893304 = 894916 - 893304$
$= 1612$

92. (c) $\frac{9261}{42875} = \frac{27 \times 343}{125 \times 343} = \frac{27}{125} = \left(\frac{3}{5}\right)^3$

Hence, cube root of $\frac{9261}{42875} = \frac{3}{5}$

93. (d) $120393 = 3^3 \times 7^3 \times 13$

⇒ It will become a cube, if it is multiplied by atleast

$13^2 = 169$

94. (d) $(0.000729)^{-3/4} \times (0.09)^{-3/4}$

$= [(0.3)]^{6(-3/4)+2(-3/4)} = (0.3)^{-6} = \left(\frac{10}{3}\right)^6 = \frac{1000000}{729}$

95. (b) If x chocolates of both varieties has been bought, then cost price of $2x$ chocolates.

$\frac{10x}{11} + \frac{10x}{9} = \frac{200x}{99}$

⇒ Cost price per chocolate = ₹ $\frac{100}{99}$

Selling price per chocolate = ₹ 1

⇒ Loss = $\frac{100}{99} - 1 =$ ₹ $\frac{1}{99}$ per chocolate

Hence, loss% = $\frac{1/99}{100/99} \times 100\% = 1\%$

96. (c) Time of 2 rides = 3 h 45 min ...(i)

Time of 1 walk + 1 ride

= 5 h 45 min ...(ii)

∴ Time of 2 walk = 7 h 45min

[by 2 × Eq. (ii) − Eq.(i)]

Solutions (Q. Nos. 97-100)

Room No. \ colour	Yellow	Blue	Green	Pink	
1	Shankutala Gaurang	×	×	×	
2	×	×	×	Dushmanta Jahanvi	Dushmanta + Jahanvi
3	×	Sharmistha Anshu	×	×	
4	×	×	Krishna Sandhya	×	Krishna
	Shankutala	Sharmistha		Dushmanta + Jahanvi	

97. (b) Krishna, 4, green

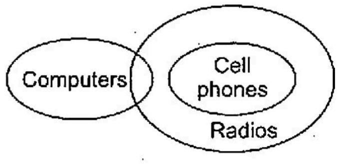
98. (a) Shankutala, 1, yellow

99. (c) Jahanvi, Dushmanta, pink

100. (b) Anshu, Sharmistha, 3.

101. (d) Only (a) and (b) are implicit.

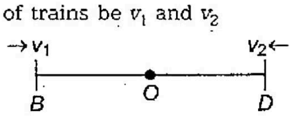
102. (b) According to the statement, Venn diagram is



Conclusions

I.	×
II.	×
III.	×
IV.	✔

103. (b) Let the speed of trains be v_1 and v_2



If x is the distance between Bhubaneswar and Delhi and t is the time after which the two train meet, then

$x = v_1(t + 12)$ ...(i)

$x = v_2(t + 9)$ ...(ii)

$x = (v_1 + v_2)t$ ...(iii)

$v_1 t + 12v_1 = v_2 t + 9v_2 = v_1 t + v_2 t$

⇒ $12v_1 = v_2 t$ and $9v_2 = v_1 t$

$\frac{v_1}{v_2} = \frac{t}{12} = \frac{9}{t}$

⇒ $t^2 = 12 \times 9$

⇒ $\frac{t^2}{12^2} = \frac{3}{4}$

⇒ $\left(\frac{v_1}{v_2}\right)^2 = \frac{3}{4}$

104. (a) Sunday 8 am to next Sunday 8 pm time duration $24 \times 7 + 12 = 180$ h.

In 180 h total gain = 10 min 48 s = 648 s

Gain of 5 min = 3000 s will be obtained in

$\frac{300}{648} \times 180 \text{ h} = \frac{250}{3} \text{ h}$

$= 83\frac{1}{3} \text{ h} = 83 \text{ h } 20 \text{ min}$

Hence, Sunday 8 am + 83 h 20 min

= Wednesday 7:20 pm

105. (c) Number of handshakes

$= {}^{10}C_2 = \frac{10 \times 9}{2 \times 1} = 45$

106. (c) Delhi—Varansi—Patna—Balasore—Bhubaneswar (3, 4, 2, 1)

Different possible routes = 3 × 4 × 2 × 1 = 24

107. (b) Mangoes + Guava = 8

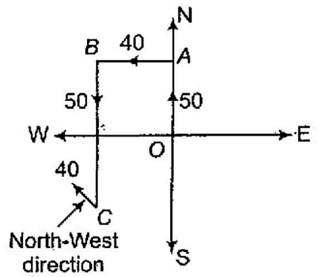
Oranges + Guava = 8

Oranges + Mangoes = 8

∴ 2(Mangoes + Guava + Oranges) = 24

Hence, all fruits = 12

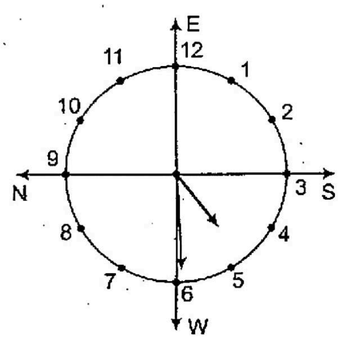
108. (a)



109. (c) Garima → Renu → Malli → Ani → Sangeeta

Hence, Garima won the race.

110. (d) From the below figure hour hand is in South-West direction.



Solutions (Q. Nos. 111-115) We have following information

1. Vakul > Priti > Kranti
2. Rudra ↔ Tarun (Highest and least)

or Sarat ↔ (Uma or Kranti) (Highest and least)

111. (d) R _ _ _ K S T

112. (c) R _ _ _ _ _ T

⇒ Lowest rank of Vakul will be fourth.

113. (a) S R V P K T U

114. (a) RS _ _ _ _ T

| | | |

Possibility; U V P K

115. (c) S _ _ _ V P K

116. (c) E is present in SALUTE but not present in CONSULTATION.

117. (b) Let the nth count of both be the same then counts will be

$$32 + (n-1)(-1) = 1 + (n-1)2$$

$$\Rightarrow \quad 33 - n = 2n - 1$$

$$\Rightarrow \quad 3n = 34$$

$$\Rightarrow \quad n = \frac{34}{3}$$

which is a fraction, so they will not call out the same number.

118. (b) Number of girls = 21 + 8 − 1 = 28

hence, Deepika's position from right = 28 − 12 + 1 = 17th

119. (c) $n(O) = 7$; $n(E) = 8$

$$n(O \cup E) = 15 - 3 = 12$$

Hence, $$n(O \cap E) = n(O) + n(E) - n(O \cup E)$$

$$= 7 + 8 - 12 = 3$$

120. (a) The pattern of the series is

$$P \xrightarrow{+2} R \xrightarrow{+2} T \xrightarrow{+2} V \xrightarrow{+2} \boxed{X}$$

$$3 \xrightarrow{+2} 5 \xrightarrow{+3} 8 \xrightarrow{+4} 12 \xrightarrow{+5} \boxed{17}$$

$$C \xrightarrow{+3} F \xrightarrow{+3} I \xrightarrow{+3} L \xrightarrow{+3} \boxed{O}$$