KIITEE MCA
Solved Paper 2009

Mathematics

1. The number of solutions to the equation $z^2 + \bar{z} = 0$ is
   (a) 2 (b) 3 (c) 4 (d) 1

2. Let $f(x) = -\log_2 x + 3$ and $a \in [1, 4]$, then $f(a)$ is equal to
   (a) [1, 3] (b) [2, 4]
   (c) [1, 2] (d) [1, 9]

3. The function $f(x) = \frac{x}{(e^x - 1)} + \frac{x}{2} + 1$ is
   (a) periodic (b) odd
   (c) even (d) neither odd nor even

4. If $(x + 2y, x - 2y) = xy$, then $f(x, y)$ is equal to
   (a) $\frac{1}{8}(x^2 - y^2)$ (b) $\frac{1}{4}(x^2 - y^2)$
   (c) $\frac{1}{2}(x^2 + y^2)$ (d) $\frac{1}{4}(xy)$

5. Which of the following function is periodic?
   (a) $f(x) = x \cos x$
   (b) $f(x) = \sin(1/x)$
   (c) $f(x) = \cos\sqrt{x}$
   (d) $f(x) = \{x\}$, the fractional part of x

6. The functions f and g are given by $f(x) = (x)$, the fractional part of x and $g(x) = 1/2 \sin[x]n$, where $[x]$ denotes the integral part of x, then the range of $(g \circ f)$ is
   (a) [– 1, 1] (b) {– 1, 1}
   (c) {0} (d) [0, 1]

7. The function $f : R \to R$ given by $f(x) = 3.2 \sin x$ is
   (a) one-one (b) onto
   (c) bijective (d) None of these

8. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & b \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 4 & 4 \end{bmatrix}$. If $A^2 - 2A + 1 = B$, then value of b is (Note that I is identity matrix of order 2)
   (a) 1 (b) 3 (c) – 1 (d) 2

9. The domain of the function $f(x) = \sqrt{2x - 1} + \sqrt{3 - 2x}$ is
   (a) [1/2, 3/2] (b) (1/2, 3/2)
   (c) [1/2, ∞) (d) (– ∞, 3/2)

10. The period of the function $f(x) = \text{cosec}^2 3x + \cot 4x$ is
    (a) π (b) $\pi/8$
    (c) $\pi/4$ (d) $\pi/3$

11. Let $f : R \to R$ be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$, then
    (a) f is both one-one and onto
    (b) f is one-one but not onto
    (c) f is onto but not one-one
    (d) f is neither one-one nor onto

12. The domain of $y = \frac{x}{\sqrt{x^2 - 3x + 2}}$ is
    (a) $R \sim \{1, 2\}$ (b) $(-\infty, 2)$
    (c) $(-\infty, 1) \cup (2, \infty)$ (d) $(1, \infty)$

13. If $f(x - 1) = 2x^2 - 3x + 1$, then $f(x + 1)$ is given by
    (a) $2x^2 + 5x + 1$ (b) $2x^2 + 5x + 3$
    (c) $2x^2 + 3x + 5$ (d) $2x^2 + x + 4$

14. If $y = \log(x)$ and $F = (3, 27)$ the set onto which the set F is mapped contains
    (a) (0, 3) (b) (1, 3)
    (c) (0, 1) (d) (0, 2)

15. If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals to
    (a) $\frac{x}{1 + x^2}$ (b) $\frac{x + \sqrt{x^2 - 4}}{2}$
    (c) $\frac{x - \sqrt{x^2 - 4}}{2}$ (d) $x - \sqrt{x^2 - 4}$

16. Let the sets $A = \{2, 4, 6, 8, \ldots\}$ and $B\,\{3, 6, 9, 12, \ldots\}$ and $n(A) = 200$, $n(B) = 250$, then
    (a) $n(A \cap B) = 67$ (b) $n(A \cap B) = 66$
    (c) $n(A \cap B) = 450$ (d) $n(A \cap B) = 380$

17. Let $t_n = n(n!)$, then $\sum_{n=1}^{15} t_n$ is equal to
    (a) $15! - 1$ (b) $16! - 1$
    (c) $15! + 1$ (d) None of these

18. The number of values of the triple $t(a, b, c)$ for which $a \cos 2x + b \sin^2 x + c = 0$ is satisfied by all real x is
    (a) 0 (b) 2
    (c) 3 (d) infinite

19. The set of real values of x satisfying $|x-1|\leq 3$ and $|x-1|\geq 1$ is

(a) $[2, 4]$ (b) $[-2, 0] \cup [2, 4]$
(c) $(-\infty, 2] \cup [4, \infty)$ (d) None of these

20. A man has 7 friends. The number of ways in which he can invite one or more of his friends to a party is

(a) 132 (b) 116
(c) 127 (d) 130

21. For the function $f(x)=\sqrt{x}, 0\leq x\leq b$, the number c satisfying the mean value theorem is $c=1$, then b is

(a) 0 (b) 4
(c) 2 (d) 3

22. In a geometric progression, if the sum of the first four terms is equal to 15 and the sum of the second, third, fourth and fifth terms is 30, then the sixth term equals to

(a) 16 (b) 32
(c) 48 (d) 64

23. The area of the triangle whose vertices are i, α, β where $i=\sqrt{-1}$ and α, β are the non-real cube roots of unity, is

(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{3\sqrt{3}}{4}$
(c) 0 (d) $\frac{\sqrt{3}}{4}$

24. If z^2 is purely imaginary when z is a complex number of constant modulus, then the number of possible values of z is

(a) 4 (b) infinite
(c) 2 (d) 1

25. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals to

(a) $128\,\omega$ (b) $128\,\omega^2$
(c) $-128\,\omega$ (d) $-128\,\omega^2$

26. The number of ways in which the letters of word ARTICLE can be rearranged so that the odd places are always occupied by consonants is

(a) 576 (b) ${}^4C_3\times 4!$
(c) $2\,(4!)$ (d) None of these

27. Nine hundred distinct n-digit positive numbers are to be formed using only the digits 2, 5, 7. The smallest value of n for which this is possible is

(a) 6 (b) 8
(c) 7 (d) 9

28. Total number of 6-digit numbers in which all the odd digits and only odd digits appear twicely is

(a) $\frac{5}{2}(6!)$ (b) $\frac{1}{2}(6!)$
(c) $6!$ (d) None of these

29. If α, β are non-real numbers satisfying $x^3-1=0$, then the value of $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$ is equal to

(a) 0 (b) λ^3+1
(c) λ^3 (d) None of these

30. The value of the determinant $\begin{vmatrix} {}^5C_0 & {}^5C_3 & 14 \\ {}^5C_1 & {}^5C_4 & 1 \\ {}^5C_2 & {}^5C_5 & 1 \end{vmatrix}$ is

(a) 0 (b) 80
(c) $-(6!)$ (d) None of these

31. The sum of two non-integral roots of $\begin{vmatrix} x & 2 & 5 \\ 3 & x & 3 \\ 5 & 4 & x \end{vmatrix}=0$ is

(a) 5 (b) -18
(c) -5 (d) None of these

32. The sum of $\sum\limits_{r=1}^{n} r\,{}^{2n}C_r$ is equal to

(a) $n\,2^{2n-1}$ (b) $2^{n-1}+1$
(c) 2^{2n-1} (d) None of these

33. $\frac{1}{1!\,(n-1)!}+\frac{1}{3!\,(n-3)!}+\frac{1}{5!\,(n-5)}+\ldots\ldots$ is equal to

(a) $\frac{2^{n-1}}{n!}$ for even values of n only

(b) $\frac{2^{n-1}+1}{n!}-1$ for odd values of n only

(c) $\frac{2^{n-1}}{n!}$ for all $n\in N$

(d) None of the above

34. The coefficient of a^8b^{10} in the expansion of $(a+b)^{18}$ is

(a) ${}^{18}C_8$ (b) ${}^{18}C_{10}$
(c) 2^{18} (d) None of these

35. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^{15}$ the constant term is

(a) ${}^{15}C_6$ (b) $-{}^{15}C_6$
(c) 0 (d) 1

36. If $A=\begin{vmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$, then A^{-1} is equal to

(a) A^T (b) adj (A)
(c) A (d) None of these

37. If $A=\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{bmatrix}$, then the value of $|\text{adj}\,(A)|$ is equal to

(a) 5 (b) 1
(c) 0 (d) None of these

38. If $f(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $f(x+y)$ is equal to

(a) $f(x)\,f(y)$ (b) $f(x)+f(y)$
(c) $f(x)-f(y)$ (d) None of these

39. The middle term in the expansion of $\left(1-\frac{1}{x}\right)^n(1-x)^n$ is

(a) ${}^{2n}C_n$ (b) $-{}^{2n}C_n$
(c) $-{}^{2n}C_{n-1}$ (d) None of these

40. $\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$ has the value

(a) $\sin A \sin B \cos C$ (b) 0
(c) 1 (d) None of these

41. The number of solutions of $|\cos x| = \sin x$, $0 \le x \le 4\pi$ is

(a) 8 (b) 2
(c) 4 (d) None of these

42. The maximum value of the function $y = x(x-1)^2$, $0 \le x \le 2$ is

(a) $\frac{5}{27}$ (b) $\frac{7}{27}$
(c) $\frac{8}{27}$ (d) $\frac{4}{27}$

43. The formula $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$ holds for

(a) $x \in (-1, 0)$ (b) $x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$
(c) $x \in [0, 1]$ (d) $x \in \left(\frac{-\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$

44. The value of $\tan\left\{\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)\right\}$ is

(a) 17/6 (b) 7/16
(c) 6/17 (d) None of these

45. If $\cos^{-1}x > \sin^{-1}x$, then

(a) $x < 0$ (b) $-1 < x < 0$
(c) $0 \le x < \frac{1}{\sqrt{2}}$ (d) $-1 \le x < \frac{1}{\sqrt{2}}$

46. The equation of the line segment AB is $y = x$, if A and B lie on the same side of the line mirror $2x - y = 1$, then image of AB has the equation

(a) $7x - y = 6$ (b) $x + y = 2$
(c) $8x + y = 9$ (d) None of these

47. The point $(-1, 1)$ and $(1, -1)$ are symmetrical about the line

(a) $y + x = 0$ (b) $y = x$
(c) $x + y = 1$ (d) None of these

48. The product of perpendicular drawn from the point $(1, 2)$ to the pair of lines $x^2 + 4xy + y^2 = 0$ is

(a) 9/4 (b) 9/16
(c) 3/4 (d) None of these

49. The centroid of the triangle whose three sides are given by the combined equation $(x^2 + 7xy + y^2)(y - 1) = 0$ is

(a) $\left(\frac{2}{3}, 0\right)$ (b) $\left(\frac{7}{3}, \frac{2}{3}\right)$
(c) $\left(\frac{-7}{3}, \frac{2}{3}\right)$ (d) None of these

50. Two distinct chords drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, where $pq \ne 0$ are bisected by the x-axis, then

(a) $|p| = |q|$ (b) $p^2 = 8q^2$
(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

51. If $A = (5, -1, 1)$, $B = (7, -4, 7)$, $C = (1, -6, 10)$, $D = (-1, -3, 4)$, then $ABCD$ is a

(a) square
(b) rectangle
(c) rhombus
(d) None of these

52. The points $A = (1, 2, -1)$, $B = (2, 5, -2)$, $C = (4, 4, -3)$ and $D = (3, 1, -2)$ are

(a) vertices of a square
(b) vertices of a rectangle
(c) collinear
(d) vertices of a rhombus

53. If $(1, -1, 0)$, $(-2, 1, 8)$ and $(-1, 2, 7)$ are three consecutive vertices of a parallelogram, then the fourth vertex is

(a) $(0, -2, 1)$ (b) $(1, 0, -1)$
(c) $(1, -2, 0)$ (d) $(2, 0, -1)$

54. The length of the latusrectum of the parabola $x = ay^2 + by + c$ is

(a) $a/4$ (b) $1/4a$
(c) $1/a$ (d) $a/3$

55. The equation of the tangent to the $x^2 - 2y^2 = 18$ which is perpendicular to the line $x - y = 0$

(a) $x + y = 3$
(b) $x + y = 3/2$
(c) $x + y + 2 = 0$
(d) $x + y + 3\sqrt{2} = 0$

56. $[\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{c} - \mathbf{a}]$ is equal to

(a) $|\mathbf{a}\ \mathbf{a}\ \mathbf{c}|$ (b) 0
(c) $2|\mathbf{a}\ \mathbf{b}\ \mathbf{c}|$ (d) None of these

57. $\int \frac{1 + \sin x}{1 + \cos x} e^x dx$ is equal to

(a) $\frac{1}{2} e^x \tan\frac{x}{2} + K$ (b) $e^x \sec^2\frac{x}{2} + K$
(c) $e^x \tan x + K$ (d) $e^x \tan\frac{x}{2} + K$

58. The sum of two non-zero numbers is 8. The minimum value of the sum of their reciprocal is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$
(c) $\frac{1}{2}$ (d) None of these

59. The smallest positive integer n, for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is

(a) 8 (b) 12
(c) 16 (d) None of these

60. Let $A = [2, 3, 4, \ldots, 20, 21]$ number is chosen at random from the set A and it is found to be a prime number. The probability that it is more than 10 is

(a) 9/10 (b) 1/5
(c) 1/10 (d) None of these

Computer Awareness

61. The computer can read but not change the information stored in
(a) ROM (b) RAM
(c) KRAM (d) None of these

62. Technician *A* says that a digital signal is either on or off. Technician *B* says that an analog signal changes proportionally to the quantity measured. Who is correct?
(a) Technician *A* only
(b) Technician *B* only
(c) Both technician *A* and *B*
(d) Neither technician *A* nor *B*

63. The binary system used by a digital computer consists of
(a) 10 numbers (b) 5 numbers
(c) 2 numbers (d) 1 number

64. The frequency of an AC signal is determined by the
(a) peak value of the sine wave
(b) number of cycles per unit of time
(c) amplitude of the signal being generated
(d) All of the above

65. One cycle per ...... equals one hertz.
(a) second (b) minute
(c) revolution (d) Any of these

66. Computers rely on ...... for their input signals.
(a) solenoids (b) sensors
(c) actuators (d) All of these

67. Which one of the following networking terms is not associated with the same OSI layer as the others?
(a) Router (b) Packet
(c) TCP (d) IP

68. The largest, fastest, most expensive type of computer is the ...... computer.
(a) personal (b) server
(c) super (d) main frame

69. Acme Corp. sells 1000 different products to over 20000 customers. To record the sales and shipments they would use a ......
(a) word processor (b) project manager
(c) spreadsheet (d) database

70. A single application that combines the major features of several types of the applications is called ......
(a) integrated software (b) a suit
(c) a combo package (d) high-end

71. Input, processing, output and storage are the steps in the
(a) information cycle
(b) information processing cycle
(c) data cycle
(d) data processing cycle

72. GIGO stands for ......
(a) Garbage Input, Garbage Output
(b) Gigabytes In, Gigabytes Out
(c) Garbage In, Garbage Out
(d) None of the above

73. The raw facts are called
(a) data (b) programs
(c) commands (d) user responses

74. VDT refers ......
(a) Vital Data Transfer
(b) Virtual Data Transfer
(c) Video Desk Terminal
(d) Video Display Terminal

75. The term multimedia refers to ......
(a) combination of sound and images with text and graphics
(b) the combination of sound and images only
(c) music only
(d) movies with sound only

76. If a processor has a word size of 32 bits, compared to a processor with a word size of 16 bits, it can process ...... at a time.
(a) twice as much (b) half as much
(c) a fourth as much (d) the same amount

77. If the bus width of a processor is 16 bits, that means that the processor can ...... 16 bits of data at a time.
(a) add (b) transfer
(c) count (d) think with

78. The speed of a printer can be measured in ......
(a) ppm (b) ips
(c) pps (d) None of these

79. Formatting a disk ......
(a) erases all data on the disk
(b) makes a backup copy of the data on the disk
(c) moves the data around on the disk to save space
(d) All of the above

80. To care for data on disks you should do all of the following except
(a) avoid exposing disks to high heat
(b) avoid exposing disks to dust and smoke
(c) avoid bending the disks
(d) keep the disks near magnets to keep the magnetic charge strong

81. A modem is used to ......
(a) change incoming analog signals to digital signals and outgoing digital signals to analog signals
(b) connect two computers using telephone lines
(c) connect a computer to a shared printer
(d) both (a) and (b)

82. A LAN is a ...... Network.
(a) Long Array
(b) Local Area
(c) Land Access
(d) Line Area

83. A computer's BIOS will ......
(a) check for the presence of peripherals like mouse, sound card, scanner
(b) run a check of memory
(c) be loaded first when the computer is powered on
(d) None of the above

84. A backup program ......
(a) makes a copy of files you select
(b) returns you to the previous program
(c) undoes the last change you made
(d) None of the above

85. When a computer is "swapping", it is ......
(a) moving data from the hard drive to the floppy drive
(b) moving data from memory to the swap file on the hard drive
(c) moving data between registers in memory
(d) None of the above

86. A foreground task has more ...... than a background task.
(a) buffers (b) microseconds
(c) registers (d) time slices

87. Checking a computer program for errors is called ......
(a) bugging (b) debugging
(c) correcting (d) syntaxing

88. The computer itself uses ...... language.
(a) natural (b) assembly
(c) machine (d) high-level

89. The term BASIC is an acronym for ......
(a) Balanced Assembly System Integrated Code
(b) Basic All System Internal Code
(c) Beginner's Assembly Syntax Instruction Code
(d) Beginner's All-purpose Symbolic Instruction Code

90. The ...... must decide what a new program is to accomplish.
(a) end user (b) systems analyst
(c) programmer (d) supervisor

Analytical Ability & Logical Reasoning

91. If $a = 3$ and $b = -2$, what is the value of $a^2 + 3ab - b^2$?
(a) 5 (b) -13 (c) 4 (d) -20

92. What is the next-highest prime number after 67?
(a) 68 (b) 69 (c) 71 (d) 73

93. How many 3-inch segments can a 4.5 yard line be divided into?
(a) 15 (b) 45
(c) 54 (d) 64

94. Dave can deliver four newspapers every minute. At this rate, how many newspapers can he deliver in 2 h?
(a) 80 (b) 420
(c) 400 (d) 480

95. If $a = 4$, $b = 3$ and $c = 1$, then $a(b-c)/b(a+b+c)$ is equal to
(a) 4/13 (b) 1/3
(c) 1/4 (d) 1/6

96. Archie's gas tank is 1/3 full. If Archie adds 3 gallons of gas to the tank, it will be 1/2 full. What is the capacity in gallons of Archie's tank?
(a) 28 (b) 12
(c) 16 (d) 18

97. What is 20% of 12/5, expressed as a percentage?
(a) 48% (b) 65%
(c) 72% (d) 76%

98. 90, 84, 79, 75, ...
(a) 62 (b) 72
(c) 71 (d) 70

99. ......, 6, 16, 31, 51
(a) 2 (b) 3
(c) 1 (d) 5

100. If QLMU means SNOW, then JGQR means ......
(a) LION (b) KING
(c) BEST (d) LIST

101. A software engineer has the capability of thinking 100 lines of code in 5 min and can type 100 lines of code in 10 min. He takes a break for 5 min after every 10 min. How many lines of codes will he complete typing after an hour?
(a) 250 (b) 253
(c) 248 (d) 255

102. A two digit number is 4 times the sum of its digits. When 9 is added to the number, the digits will get reversed. Then what is that number?
(a) 10 (b) 11
(c) 14 (d) 12

103. A girl was born on September 6, 1970, which happened to be a Sunday. Her birthday has again fall on Sunday in
(a) 1975 (b) 1976 (c) 1977 (d) 1981

104. There are 19 hockey players in a club. On a particular day, 14 were wearing the hockey shirts prescribed, while 11 were wearing the prescribed hockey pants. None of them was without either hockey pants or hockey shirts. How many were in complete hockey uniform?
(a) 8 (b) 6 (c) 9 (d) 7

Directions (Q. Nos. 105-108) *These four questions are to be answered on the basis of the following information.*

A five members research group is to be chosen from the mathematicians A, B, C and D, and the physicists E, F, G and H. Atleast 3 mathematicians must be in the group. However,
A refuses to work with D.
B refuses to work with E.
F refuses to work with G.
D refuses to work with F.

105. If B is chosen, who else would have to be in the group?
(a) F (b) G (c) A (d) C

106. If B and C are chosen, which of the following is definitely true?
P : A is chosen, Q : D is chosen, R : Either F or G is chosen
(a) P only (b) Q only
(c) R only (d) Q and R only

107. If G is rejected, which other member could not work with the group?
(a) A (b) B (c) D (d) F

108. If H is chosen, which of the following must be true?
P : A must be chosen
Q : B must be chosen
R : G must be chosen
(a) P only (b) Q only
(c) R only (d) P, Q and R

109. If the cost of $\frac{1}{4}$th of kg is ₹ 0.60, then what is the cost of 200 g?

(a) 42 paisa (b) 48 paisa
(c) 40 paisa (d) 50 paisa

110. Bhanu spends 30% of his income on petrol on scooter. $\frac{1}{4}$ of the remaining on house rent and the balance on food. If he spends ₹ 300 on petrol, then what is the expenditure on house rent?

(a) ₹ 525 (b) ₹ 1000
(c) ₹ 675 (d) ₹ 175

111. If the numerator of a fraction is increased by 25% and denominator decreased by 20%, the new value is 5/4. What is the original value?

(a) 3/5 (b) 4/5
(c) 7/8 (d) 3/7

112. The length of a rectangle is increased by 60%. By what % would the width have to be decreased to maintain the same area?

(a) 30% (b) 60%
(c) 75% (d) 37.5%

113. The value of $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\ldots+\frac{17}{5184}+\frac{19}{8100}$ is

(a) 0.90 (b) 0.98
(c) 0.95 (d) None of these

114. A sporting goods store ordered an equal number of white and yellow balls. The tennis ball company delivered 45 extra white balls, making the ratio of white balls to yellow balls 1/5 : 1/6. How many white tennis balls did the store originally order for?

(a) 450 (b) 270
(c) 225 (d) None of these

115. A student's grade in a course is determined by 6 quizzes and one examination. If the examination counts thrice as much as each of the quizzes, what fraction of final grade is determined by the examination?

(a) 1/6 (b) 1/5
(c) 1/3 (d) 1/4

116. A sum of money is divided among A, B and C such that for each rupee A and B gets 65 paise and C gets 35 paise. If C's share is ₹ 560, the sum is .....

(a) ₹ 2400 (b) ₹ 2800
(c) ₹ 1600 (d) ₹ 3800

117. Joe's father will be twice his age 6 years from now. His mother was twice his age 2 years before. If Joe will be 24 two years from now, what is the difference between his father's and mother's age?

(a) 4 (b) 6
(c) 8 (d) 10

118. A traveller walks a certain distance. Had he gone half a kilometre an hour faster, he would have walked it in 4/5 of the time, and had he gone half a kilometre an hour slower, he would have walked $2\frac{1}{2}$ h longer. What is the distance?

(a) 10 km (b) 15 km
(c) 20 km (d) Data insufficient

119. 2 oranges, 3 bananas and 4 apples cost ₹ 15 and 3 oranges, 2 bananas and 1 apple cost ₹ 10. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay?

(a) 10 (b) 8
(c) 15 (d) Cannot be determined

120. A report consists of 20 sheets each of 55 lines and each such line consists of 65 characters. This report is retyped into sheets each of 65 lines such that each line consists of 70 characters. The per cent reduction in the number of sheets is closest to

(a) 20 (b) 5
(c) 30 (d) 35

Answers *with* Solutions

1. (c) Given, $z^2 + \bar{z} = 0$

Let $z = x + iy$

$\Rightarrow x^2 - y^2 + 2ixy + x - iy = 0$

$\Rightarrow (x^2 + x - y^2) + i(2xy - y) = 0 + i0$

On comparing

$\Rightarrow x^2 + x - y^2 = 0$ and $2xy - y = 0$

$\Rightarrow x^2 + x - y^2 = 0$ and $y(2x - 1) = 0$

$\Rightarrow x^2 + x - y^2 = 0$ and $y = 0$ or $x = \frac{1}{2}$

When $y = 0$, then

$x^2 + x = 0$

$\Rightarrow x(x + 1) = 0$

$\Rightarrow x = 0$ or $x = -1$

When $x = \frac{1}{2}$, then

$\frac{1}{4} + \frac{1}{2} - y^2 = 0$

$\Rightarrow y^2 = \frac{3}{4}$

$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$

$0 + i0$, $-1 + i0$, $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ are the required solutions.

So, the number of solutions is 4.

2. (a) $f(x) = -\log_2 x + 3$

$= \log_2 1 - \log_2 x + 3$

$= \log_2 \frac{1}{x} + 3\log_2 2$

$= \log_2 \frac{1}{x} + \log_2 8$

$= \log_2 \frac{8}{x}$

$\Rightarrow f(a) = \log_2 \frac{8}{a}$

Since, $a \in [1, 4]$

$\Rightarrow 1 \le a \le 4$

$\Rightarrow \quad \frac{1}{4} \le \frac{1}{a} \le 1$

$\Rightarrow \quad 2 \le \frac{8}{a} \le 8$

$\Rightarrow \quad \log_2 2 \le \log_2 \frac{8}{a} \le \log_2 8$

$\Rightarrow \quad 1 \le f(a) \le 3$

$\Rightarrow \quad f(a) \in [1, 3]$

3. (c) $f(x) - f(-x)$

$$= \left[\frac{x}{e^x - 1} + \frac{x}{2} + 1\right] - \left\{\frac{-x}{e^{-x} - 1} - \frac{x}{2} + 1\right\}$$

$$= \frac{x}{e^x - 1} + \frac{x}{2} + 1 + \frac{x}{e^{-x} - 1} + \frac{x}{2} - 1$$

$$= \frac{x}{e^x - 1} + x + \frac{xe^x}{1 - e^x}$$

$$= \frac{x}{e^x - 1} + x - \frac{xe^x}{e^x - 1}$$

$$= \frac{x - xe^x}{e^x - 1} + x = \frac{x(1 - e^x)}{e^x - 1} + x = -x + x = 0$$

$\Rightarrow \quad f(x) = f(-x)$

$\Rightarrow f(x)$ is an even function.

4. (a) Put $x + 2y = u$ and $x - 2y = v$

$\Rightarrow \quad 2x = u + v,\ 4y = u - v$

$\Rightarrow \quad x = \frac{u + v}{2},\ y = \frac{u - v}{4}$

Now, from given

$f(x + 2y, x - 2y) = xy$

$$\Rightarrow \quad f(u, v) = \left(\frac{u + v}{2}\right)\left(\frac{u - v}{4}\right) = \frac{u^2 - v^2}{8}$$

$$\Rightarrow \quad f(x, y) = \frac{x^2 - y^2}{8}$$

5. (d) $f(x) = \{x\}$ is a periodic function with period 1.

6. (c) $(gof)\, x = g(f(x))$

$= g(\{x\})$

$= \frac{1}{2}\sin[\{x\}]\, n$

$= \frac{1}{2}\sin 0 = 0$

$\because 0 \le \{x\} < 1$ and $[x] = 0$ when $x \in [0, 1)$

7. (d) Given, $f : R \to R$

$\Rightarrow f(x) = \sin x$ is a many-one function and also into.

8. (b) $A^2 = \begin{bmatrix} 1 & 0 \\ 2 & b \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 + 2b & b^2 \end{bmatrix}$

Now, $\quad A^2 - 2A + I = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 + 2b & b^2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 4 & 2b \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 4 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 2b - 2 & (b - 1)^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 4 & 4 \end{bmatrix}$$

On comparing

$\Rightarrow \quad 2b - 2 = 4$ and $(b - 1)^2 = 4$

$\Rightarrow \quad b = 3$

9. (a) $f(x) = \sqrt{2x - 1} + \sqrt{3 - 2x}$ is defined when $2x - 1 \ge 0$ and $3 - 2x \ge 0$

$\Rightarrow \quad x \ge \frac{1}{2}$ and $x \le \frac{3}{2}$

$\Rightarrow \quad \frac{1}{2} \le x \le \frac{3}{2}$

$\Rightarrow \quad x \in \left[\frac{1}{2}, \frac{3}{2}\right]$

10. (a) Period of $\operatorname{cosec}^2 3x = \frac{\pi}{3}$

and period of $\cot 4x = \frac{\pi}{4}$

$\Rightarrow$ Period of $\operatorname{cosec}^2 3x + \cot 4x$

$= \text{LCM of } \frac{\pi}{3} \text{ and } \frac{\pi}{4}$

$= \frac{\text{LCM}(\pi, \pi)}{\text{HCF}(3, 4)} = \frac{\pi}{1} = \pi$

11. (d) $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}} = 0$, when $x \le 0$

$\Rightarrow f(x) = 0$ holds for negative value of x.

$\Rightarrow f(x)$ is not one-one.

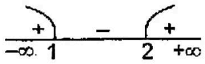
Also, for $x \ge 0$, $f(x) \ge 0$

$\Rightarrow f(x)$ is into (not onto)

12. (c) $y = \frac{x}{\sqrt{x^2 - 3x + 2}}$ will be defined, if

$x^2 - 3x + 2 > 0$

$\Rightarrow \quad (x - 1)(x - 2) > 0$



$\Rightarrow \quad x < 1$ or $x > 2$

Hence, $(-\infty, 1) \cup (2, \infty)$ is the domain.

13. (b) $f(x - 1) = 2x^2 - 3x + 1$

$\Rightarrow \quad f(x + 2 - 1) = 2(x + 2)^2 - 3(x + 2) + 1$

$\Rightarrow \quad f(x + 1) = 2(x^2 + 4x + 4) - 3(x + 2) + 1$

$= 2x^2 + 5x + 3$

14. (b) $F = (3, 27) = (3, 3^3)$

$\Rightarrow \log(x)$ goes from $\log 3$ to $\log 3^3$

i.e., $\log 3$ to $3\log 3$

i.e., from 1 to 3 for base 3

15. (b) $f(x) = y = x + \frac{1}{x}$

$\Rightarrow \quad x^2 + 1 = xy$

$\Rightarrow \quad x^2 - yx + 1 = 0$

$\Rightarrow \quad x = \frac{y + \sqrt{y^2 - 4}}{2} \qquad (\because y \ge 0)$

$\Rightarrow \quad f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$

16. (b) $A = \{2, 4, 6, 8, \ldots\ldots\}$ contains $\left[\frac{200}{3}\right] = 66$

Number divisible by 6 $\Rightarrow n(A \cap B) = 66$

17. (b) $t_n = n(n!) = (n + 1 - 1)(n!) = (n + 1)! - n!$

$$\Rightarrow \sum_{n=1}^{15} t_n = \sum_{n=1}^{15} (n + 1)! - n! = 16! - 1$$

18. (d) $\quad a\cos 2x + b\sin^2 x + c = 0$

$\Rightarrow \quad a(1 - 2\sin^2 x) + b\sin^2 x + c = 0$

$\Rightarrow \quad (a + c) + (b - 2a)\sin^2 x = 0$

$\Rightarrow \quad a + c = 0$ and $(b - 2a) = 0$

$\Rightarrow \quad a = \frac{b}{2} = -c$

$\Rightarrow$ Infinite number of triplets (a, b, c) will satisfy the given relation.

19. (b) $|x-1| \le 3 \Rightarrow -3 \le x-1 \le 3$

$1-3 \le x \le 3+1 \Rightarrow -2 \le x \le 4$ ...(i)

$|x-1| \ge 1 \Rightarrow x-1 \le -1$ or $x-1 \ge 1$

$\Rightarrow x \le 0$ or $x \ge 2$ ...(ii)

Both Eqs. (i) and (ii) are satisfied, if

$x \in [-2, 0] \cup [2, 4]$

20. (c) Number of ways of inviting one or more from 7 friends is

$^7C_1 + {^7C_2} + \ldots\ldots {^7C_7} = 2^7 - 1 = 127$

21. (b) $f(x) = \sqrt{x},\ x \in [0, b]$

$\Rightarrow f'(x) = \dfrac{1}{2\sqrt{x}}$

From mean value theorem, we have

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$\Rightarrow f'(1) = \dfrac{f(b) - f(0)}{b - 0}$ $(\because c = 1)$

$\Rightarrow \dfrac{1}{2} = \dfrac{\sqrt{b}}{b}$

$\Rightarrow \sqrt{b} = 2 \Rightarrow b = 4$

22. (b) If a is the first term and r the common ratio of GP, then

$a + ar + ar^2 + ar^3 = 15$ ...(i)

$ar + ar^2 + ar^3 + ar^4 = 30$ ...(ii)

Dividing Eq. (ii) by Eq. (i), $r = 2$

Putting in Eq. (i), we get $a = 1$

$\Rightarrow t_6 = ar^5 = 32$

23. (d) Vertices are i, α, β

i.e., $(0, 1), \left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$ and $\left(-\dfrac{1}{2}, \dfrac{-\sqrt{3}}{2}\right)$ $\left(\because \begin{array}{l} \alpha = \omega = \dfrac{-1 + i\sqrt{3}}{2} \\ \beta = \omega^2 = \dfrac{-1 - \sqrt{3}}{2} \end{array}\right)$

$$\Rightarrow \text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ -\dfrac{1}{2} & \dfrac{\sqrt{3}}{2} & 1 \\ -\dfrac{1}{2} & -\dfrac{\sqrt{3}}{2} & 1 \end{vmatrix} = \frac{1}{2}\left[-\left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right)\right]$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$$

24. (c) Let $z = a + ib$

$\Rightarrow z^2 = a^2 - b^2 + i(2ab)$

z^2 is purely imaginary, if $a^2 - b^2 = 0$

$\Rightarrow a = \pm b$; so for constant modulus two values of z are possible.

25. (d) $(1 + \omega - \omega^2)^7 = (-2\omega^2)^7 = -128\omega^2$

$(\because 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1)$

26. (d) Number of rearrangements with consonant at odd places $= 4!3! = 144$

27. (c) The number of n digit numbers formed from 2, 5 and 7 will be 3^n.

$3^n \ge 900$

$\Rightarrow n \ge 7$

So, least value of n is 7

28. (a) One odd number will appear twice and rest odd numbers will appear only once.

$\therefore$ Number of numbers $= {^5C_1} \cdot \dfrac{6!}{2!} = \dfrac{5}{2}(6!)$

29. (c) $x^3 - 1 = 0$

$\Rightarrow x = 1, \omega, \omega^2$

$\Rightarrow \alpha = \omega$ and $\beta = \omega^2$

$\Rightarrow 1 + \alpha + \beta = 0$ $(\because 1 + \omega + \omega^2 = 0$ and $\alpha \cdot \beta = 1)$

By using $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} \lambda + 1 & \alpha & \beta \\ \alpha & \lambda + \beta & 1 \\ \beta & 1 & \lambda + \alpha \end{vmatrix} = \begin{vmatrix} \lambda & \alpha & \beta \\ \lambda & \lambda + \beta & 1 \\ \lambda & 1 & \lambda + \alpha \end{vmatrix}$$

$$= \lambda\begin{vmatrix} 1 & \alpha & \beta \\ 1 & \lambda + \beta & 1 \\ 1 & 1 & \lambda + \alpha \end{vmatrix}$$

By using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \lambda\begin{vmatrix} 1 & \alpha & \beta \\ 0 & \lambda + \beta - \alpha & 1 - \beta \\ 0 & 1 - \alpha & \lambda + \alpha - \beta \end{vmatrix}$$

Expand with respect to C_1

$= \lambda[\lambda^2 - (\beta - \alpha)^2 - (1 - \alpha)(1 - \beta)]$

$= \lambda[\lambda^2 - (\beta^2 + \alpha^2 - 2\alpha\beta) - (1 + \alpha\beta - \alpha - \beta)]$

$= \lambda[\lambda^2 - \omega - \omega^2 + 2 - 2 + \omega + \omega^2] = \lambda^3$

30. (d) $\begin{vmatrix} ^5C_0 & ^5C_3 & 14 \\ ^5C_1 & ^5C_4 & 1 \\ ^5C_2 & ^5C_5 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 10 & 14 \\ 5 & 5 & 1 \\ 10 & 1 & 1 \end{vmatrix}$

Expand with respect to R_1

$= 4 + 50 - 630 = -576$

31. (c) $\begin{vmatrix} x & 2 & 5 \\ 3 & x & 3 \\ 5 & 4 & x \end{vmatrix} = 0$

Expand with respect to R_1

$\Rightarrow x(x^2 - 12) - 3(2x - 20) + 5(6 - 5x) = 0$

$\Rightarrow x^3 - 43x + 90 = 0$

$\Rightarrow (x - 5)(x^2 + 5x - 18) = 0$

$\Rightarrow$ Non-integral roots are given by $x^2 + 5x - 18 = 0$

for which sum of roots $= -5$

32. (a) $r \cdot {^{2n}C_r} = r \cdot \dfrac{2n}{r} \cdot {^{2n-1}C_{r-1}} = 2n \cdot {^{2n-1}C_{r-1}}$

$\Rightarrow \sum\limits_{r=1}^{n} r \cdot {^{2n}C_r} = 2n \cdot \sum\limits_{r=1}^{n} {^{2n-1}C_{r-1}}$

$= 2n \cdot [{^{2n-1}C_0} + {^{2n-1}C_1} + {^{2n-1}C_2} + \ldots\ldots + {^{2n-1}C_{n-1}}]$

$= 2n \cdot \dfrac{1}{2}(2^{2n-1}) = n \cdot 2^{2n-1}$

33. (c) $\dfrac{1}{1!(n-1)!} + \dfrac{1}{3!(n-3)!} + \dfrac{1}{5!(n-5)!} + \ldots$

$= \dfrac{1}{n!}[{^nC_1} + {^nC_3} + {^nC_5} + \ldots]$

$= \dfrac{1}{n!} \times \dfrac{1}{2}(2^n) = \dfrac{2^{n-1}}{n!}, \forall n \in N$

34. (b) $\because$ General term

$T_{r+1} = {^{18}C_r}\, a^{18-r} \cdot b^r$

Put $18 - r = 8$

$\Rightarrow r = 10$

So, $T_{11} = {^{18}C_{10}} a^8 \cdot b^{10}$

$\therefore$ Coefficient of $a^8 b^{10}$ in $(a + b)^{18}$ will be ${^{18}C_8} = {^{18}C_{10}}$

35. (b) General term $T_{r+1} = {^{15}C_r}(x^3)^{15-r} \cdot \left(-\dfrac{1}{x^2}\right)^r$

$= {^{15}C_r}\,(-1)^r\, x^{(45-5r)}$

For constant term, put $45 - 5r = 0$, $r = 9$

$= {^{15}C_9}(-1)^9 = -{^{15}C_6}$ $(\because {^nC_r} = {^nC_{n-r}})$

$\Rightarrow$ Constant term is $(-{^{15}C_6})$.

36. (a) Given matrix is

$$A = \begin{bmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$AA^T = \begin{bmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha & 0 \\ -\cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Also, $AA^T = I$

$\Rightarrow \quad A^{-1} = A^T$

37. (b) $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{bmatrix}$ $\qquad (\because |\text{adj}(A)| = |A|^{n-1} \text{ here, } n = 3)$

$\Rightarrow \quad |A| = 1$

$\Rightarrow \quad |\text{adj}(A)| = |A|^2 = 1$

38. (a) $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(x)\,f(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= f(x+y)$

39. (a) $\left(1 - \dfrac{1}{x}\right)^n (1-x)^n = (-1)^n \dfrac{(1-x)^{2n}}{x^n}$

$= \dfrac{(-1)^n}{x^n} \{{}^{2n}C_0 + \ldots + {}^{2n}C_n (1)^n (-x)^n + \ldots + {}^{2n}C_{2n} (-x)^{2n}\}$

Coefficient of constant term (middle term) is $(-1)^n (-1)^n \, {}^{2n}C_n = {}^{2n}C_n$

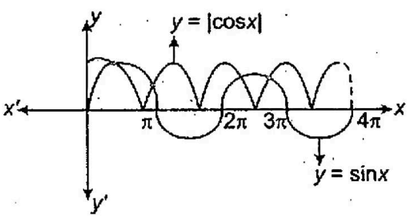
40. (b) $\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$

Expanding along R_1

$= \cos C \sin B \tan A - \sin B \tan A \cos C$

$= 0$

41. (c) $|\cos x| = \sin x, 0 \le x \le 4\pi$



$\Rightarrow \quad x = \dfrac{\pi}{4}, \pi - \dfrac{\pi}{4}, 2\pi + \dfrac{\pi}{4}, 3\pi - \dfrac{\pi}{4}$

i.e., 4 solutions are there.

42. (d) $y = x(x-1)^2 = x^3 - 2x^2 + x$

$\Rightarrow \quad \dfrac{dy}{dx} = 3x^2 - 4x + 1$; for max or min value of y, $\dfrac{dy}{dx} = 0$

$\Rightarrow \quad (3x-1)(x-1) = 0$

$\Rightarrow \quad x = \dfrac{1}{3}, 1$

$\dfrac{d^2y}{dx^2} = 6x - 4;$

At $x = \dfrac{1}{3}$, $\dfrac{d^2y}{dx^2} < 0$,

So, $x = \dfrac{1}{3}$ is local maxima

and max value $= \dfrac{1}{3}\left(\dfrac{1}{3} - 1\right)^2 = \dfrac{4}{27}$

43. (b) Since, $-\dfrac{\pi}{2} \le \sin^{-1}(2x\sqrt{1-x^2}) \le \dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{\pi}{2} \le 2\sin^{-1} x \le \dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{\pi}{4} \le \sin^{-1} x \le \dfrac{\pi}{4}$

$\Rightarrow \quad -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad x \in \left[-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right]$

44. (d) $\cos^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$ is not possible.

As, $\cos^{-1} x$ is defined for $-1 \le x \le 1$.

So, the given expression does not exist.

45. (d) $\cos^{-1} x > \sin^{-1} x$

$\Rightarrow \quad \dfrac{\pi}{2} - \sin^{-1} x > \sin^{-1} x$

$\Rightarrow \quad \dfrac{\pi}{2} > 2\sin^{-1} x$

$\Rightarrow \quad \sin^{-1} x < \dfrac{\pi}{4}$

$\Rightarrow \quad x < \dfrac{1}{\sqrt{2}} \qquad \ldots(i)$

Also, $\sin^{-1} x$ is defined when

$-1 \le x \le 1 \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

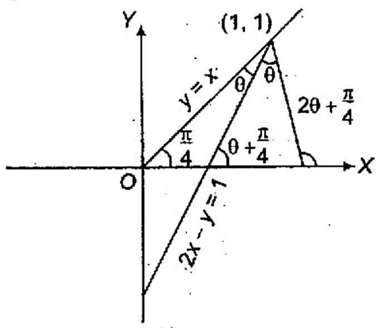
$-1 \le x < \dfrac{1}{\sqrt{2}}$

46. (a) $\tan\theta = \dfrac{2-1}{1+2} = \dfrac{1}{3} \qquad \left(\because \tan\theta = \dfrac{m_1 - m_1}{1 + m_1 m_2}\right)$

where

m_1 = slope of $2x - y = 1$

m_2 = slope of $y = x$



$\Rightarrow \quad \tan 2\theta = \dfrac{\frac{2}{3}}{1 - \frac{1}{9}} = \dfrac{3}{4} \qquad \left(\because \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}\right)$

Slope of required line, $m = \tan\left(2\theta + \dfrac{\pi}{4}\right)$

$$= \frac{\tan^2\theta + 1}{1 - \tan^2\theta}$$

$$= \frac{\frac{3}{4} + 1}{1 - \frac{3}{4}} = 7$$

Hence, required image is

$y - 1 = 7(x - 1)$, hence $(-1, 1)$ becomes $(1, -1)$

$\Rightarrow \quad 7x - y = 6$

47. (b) About $y = x, (\alpha, \beta)$ becomes (β, α) hence $(-1, 1)$ becomes $(1, -1)$.

48. (d) $y^2 + 4xy + x^2 = 0$.

$$\Rightarrow \quad y = \frac{-4x \pm \sqrt{16x^2 - 4x^2}}{2} = -2x \pm \sqrt{3}x$$

$\Rightarrow \quad y + (2 + \sqrt{3})x = 0$ and $y + (2 - \sqrt{3})x = 0$

$\Rightarrow$ Perpendiculars from (1, 2) are

Let $\quad p_1 = \frac{|4 + \sqrt{3}|}{\sqrt{8 + 4\sqrt{3}}}$ and $p_2 = \frac{|4 - \sqrt{3}|}{\sqrt{8 - 4\sqrt{3}}}$

$$p_1p_2 = \frac{13}{\sqrt{16}} = \frac{13}{4}$$

49. (c) $(x^2 + 7xy + y^2)(y - 1) = 0$

Putting $\quad y - 1 = 0$

i.e., $y = 1$ in $\quad x^2 + 7xy + y^2 = 0$

gives $\quad x^2 + 7x + 1 = 0$

$$\Rightarrow \quad x = \frac{-7 \pm \sqrt{45}}{2}$$

So, vertices of triangle are

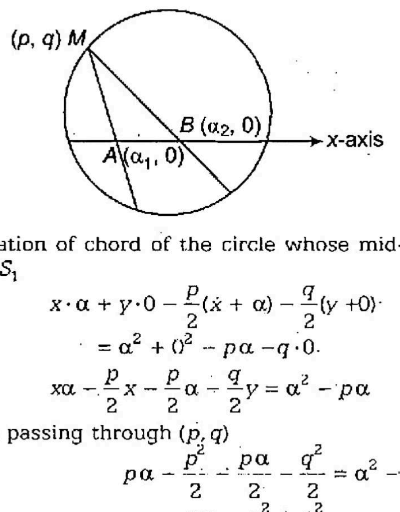
$A(0, 0)$, $B\left(\frac{-7 + \sqrt{45}}{2}, 1\right)$ and $C\left(\frac{-7 - \sqrt{45}}{2}, 1\right)$

$\because$ Centroid $= \left[\frac{(x_1 + x_2 + x_3)}{3}, \frac{(y_1 + y_2 + y_3)}{3}\right]$

$\Rightarrow$ Centroid is $\left(\frac{-7}{3}, \frac{2}{3}\right)$.

50. (d) Given equation of circle is

$$x^2 + y^2 - px - qy = 0$$



Equation of chord of the circle whose mid-point $(\alpha, 0)$ is $T = S_1$

i.e., $\quad x \cdot \alpha + y \cdot 0 - \frac{p}{2}(x + \alpha) - \frac{q}{2}(y + 0)$

$$= \alpha^2 + 0^2 - p\alpha - q \cdot 0.$$

$$\Rightarrow \quad x\alpha - \frac{p}{2}x - \frac{p}{2}\alpha - \frac{q}{2}y = \alpha^2 - p\alpha$$

It is passing through (p, q)

$$\Rightarrow \quad p\alpha - \frac{p^2}{2} - \frac{p\alpha}{2} - \frac{q^2}{2} = \alpha^2 - p\alpha$$

$$\Rightarrow \quad \alpha^2 - p\alpha - p\alpha + \frac{p\alpha}{2} + \frac{p^2 + q^2}{2} = 0$$

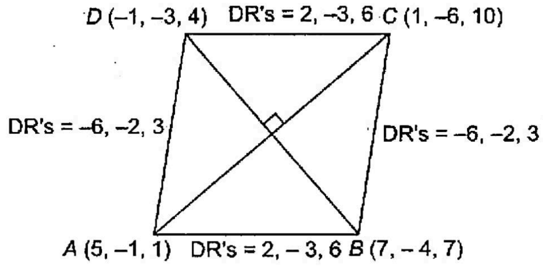
$$\Rightarrow \quad 2\alpha^2 - 3p\alpha + p^2 + q^2 = 0$$

Now, α has two distinct values

$\Rightarrow \quad D > 0$

$\Rightarrow \quad 9p^2 - 8(p^2 + q^2) > 0 \quad \Rightarrow \quad p^2 > 8q^2$

51. (c)



Four sides are equal but angles are not right angles but opposite sides are parallel, hence, it is a parallelogram.

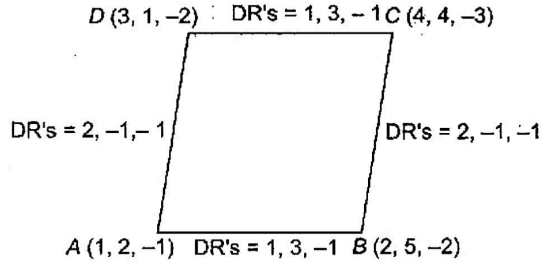
Direction ratios of $AC = 4, 5, -9$ and

Direction ratios of $BD = 8, -1, 3$

and $4 \times 8 + 5 \times (-1) + (-9) \times 3 = 32 - 5 - 27 = 0$

$\Rightarrow$ $ABCD$ is a rhombus.

52. (b)



$AB = \sqrt{1 + 9 + 1} = \sqrt{11} = CD$

$BC = \sqrt{4 + 1 + 1} = \sqrt{6} = DA$

All angles are right angles and opposite sides are equal, so it is a rectangle.

53. (d) If (α, β, γ) is the fourth vertex, then

Mid-point of both diagonals are same. *i.e.*,

$$\left\{\frac{\alpha - 2}{2}, \frac{\beta + 1}{2}, \frac{\gamma + 8}{2}\right\} = \left\{\frac{1 - 1}{2}, \frac{2 - 1}{2}, \frac{7 + 0}{2}\right\}$$

$\Rightarrow \quad \alpha = 2, \beta = 0, \gamma = -1$

$\therefore$ Fourth vertex is $(2, 0, -1)$.

54. (c) Given, $x = ay^2 + by + c$

$$\Rightarrow \quad \frac{1}{a}x = y^2 + \frac{b}{a}y + \frac{c}{a}$$

$$\Rightarrow \quad \left(y + \frac{b}{2a}\right)^2 = \frac{1}{a}\left(x + \frac{b^2 - 4ac}{4a}\right)$$

Latusrectum of parabola is $\frac{1}{a}$

55. (a) Equation of any line perpendicular to $x - y = 0$ is $x + y = a$.

Now, $x + y = a$ is a tangent to $x^2 - 2y^2 = 18$

$\Rightarrow$ Roots of $x^2 - 2(a - x)^2 = 18$ will be equal.

$\Rightarrow$ Discriminant of $x^2 - 4ax + 2a^2 + 18 = 0$ is zero.

i.e., $\quad B^2 - 4AC = 0$

$\Rightarrow \quad 16a^2 = 4(2a^2 + 18)$

$\Rightarrow \quad 4a^2 = 2a^2 + 18$

$\Rightarrow \quad 2a^2 = 18$

$\Rightarrow \quad a = 3$

$\therefore$ Required tangent is $x + y = 3$.

56. (b) $[\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{c} - \mathbf{a}] = [\mathbf{a}, \mathbf{b}, \mathbf{c}] - [\mathbf{a}, \mathbf{b}, \mathbf{c}] = 0$

Alternate method

$[\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{c} - \mathbf{a}]$ $\quad \left[\begin{array}{l}\because (\mathbf{a} \times \mathbf{b}) = -(\mathbf{b} \times \mathbf{a}) \\ (\mathbf{c} \times \mathbf{c}) = 0\end{array}\right]$

$= (\mathbf{a} - \mathbf{b}) \cdot \{(\mathbf{b} - \mathbf{c}) \times (\mathbf{c} - \mathbf{a})\}$

$= (\mathbf{a} - \mathbf{b}) \cdot \{\mathbf{b} \times \mathbf{c} - \mathbf{b} \times \mathbf{a} - \mathbf{c} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}\}$

$= (a-b)\cdot\{b\times c + a\times b - 0 + c\times a\}$
$= (a-b)\cdot\{a\times b + b\times c + c\times a\}$
$= a\cdot(a\times b) + a\cdot(b\times c) + a\cdot(c\times a)$
$= -b\cdot(a\times b) - b\cdot(b\times c) - b\cdot(c\times a)$
$= [aab] + [abc] + [aca] - [bab] - [bbc] - [bca]$
$= 0 + [abc] + 0 - 0 - 0 - [abc]$
$= 0$ $\quad [\because [abc] = [bca]]$

57. (d) $\int \frac{1+\sin x}{1+\cos x} e^x dx$

$$= \int \frac{1+2\cos\frac{x}{2}\sin\frac{x}{2}}{2\cos^2\frac{x}{2}} e^x dx$$

$$= \int \left(\tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2}\right) e^x dx = e^x \tan\frac{x}{2} + K$$

As, $\int e^x (f(x) + f'(x)) dx = e^x f(x) + K$

58. (c) Sum will be minimum when both numbers are equal.

i.e., $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Alternate method

Let x and y be non-zero numbers.

Then, $x + y = 8$ ...(i)

and $\frac{1}{x} + \frac{1}{y} = R$ (let)

$\Rightarrow R = \frac{1}{x} + \frac{1}{(8-x)}$

$\Rightarrow \frac{dR}{dx} = -\frac{1}{x^2} - \frac{1}{(8-x)^2}(-1) = -\frac{1}{x^2} + \frac{1}{(8-x)^2}$

For minimum of R, $\frac{dR}{dx} = 0$

$\Rightarrow (8-x)^2 = x^2 \Rightarrow 8(8-2x) = 0$

$\Rightarrow x = 4$

$\frac{d^2R}{dx^2} = \frac{2}{x^3} + \frac{2}{(8-x)^3}; \left(\frac{d^2R}{dx^2}\right)_{(x=4)} = \frac{2}{64} + \frac{2}{64} = \frac{1}{16} > 0$

So, the minimum value of R at $(x = 4) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

59. (d) $\left(\frac{1+i}{1-i}\right)^n = \left(\frac{2i}{2}\right)^n = (i)^n = 1$ $\quad (\because i^4 = 1)$

$\Rightarrow n = 4$

60. (d) In $A = \{2, 3, 4, \ldots, 20, 21\}$ prime numbers are $B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

P(chosen number is more than 10 given prime)

$\Rightarrow$ Probability $= \frac{4}{8} = \frac{1}{2}$

61. (a) ROM $\rightarrow$ Read Only Memory is a non-volatile memory.

62. (c) Both technicians A and B are correct.

63. (c) 2 numbers. (0 or 1)

64. (d)

65. (d) It may be second, minute or revolution.

66. (b) Sensors.

67. (b) Packets. All other terms router, IP address, TCP are involved in OSI layer as the other model like (TCP/IP).

68. (c) Super computer.

69. (d) They would use a database to record the sales and shipments.

70. (b) A Suit It consists of multiple applications bundled together.

71. (d) Input, processing, output and storage are the four major steps of data processing cycle.

72. (c) GIGO $\rightarrow$ Garbage In Garbage Out.

73. (a) Any raw material is called data.

74. (d) VDT $\rightarrow$ Video Display Terminal.

75. (a) Multimedia is a combination of sound and images with text and graphics.

76. (b) As 32 is twice of 16, so processing speed is half.

77. (b) Transfer.

78. (c) pps $\rightarrow$ point per second.

79. (d) All of the above.

80. (d) To care for the data on disks you should not keep the disks near magnets to keep the magnetic charge strong.

81. (d) Both (a) and (b)

82. (b) LAN $\rightarrow$ Local Area Network

83. (c) A computer's BIOS will be loaded first when the computer is powered on.

84. (a) A backup program makes a copy of files you select.

85. (c) Swapping It is moving data between registors in memory.

86. (c) Registers.

87. (a) Bugging It means checking a computer program for errors.

88. (c) Machine language, because computer understand only Machine language *i.e.,* (0 or 1)

89. (d) BASIC Beginners's All-purpose Symbolic Instruction Code.

90. (b) The system analyst must decide what a new program is to be accomplished.

91. (b) $a^2 + 3ab - b^2 = 9 - 18 - 4 = -13$

92. (c) Prime number after 67 is 71.

93. (c) 4.5 yard $= \frac{9}{2} \times 3 \times 12 = 27 \times 6$ inch

Hence, number of 3 inch segments $= \frac{27\times 6}{3} = 54$

94. (d) In 2h = 120 min, $120 \times 4 = 480$ newspapers will be delivered.

95. (b) $\frac{a(b-c)}{b(a+b+c)} = \frac{4\times 2}{3\times(8)} = \frac{1}{3}$

96. (d) If G is the capacity of tank, then

$\frac{G}{2} - \frac{G}{3} = 3 \Rightarrow \frac{G}{6} = 3 \Rightarrow G = 18$

97. (a) 20% of 12/5 is $\frac{20}{100} \times \frac{12}{5} \times 100\% = 48\%$

98. (b) The pattern of the series is

90 → 84 → 79 → 75 → [72]

−6, −5, −4, −3

99. (c) The pattern of the series is

[1] → 6 → 16 → 31 → 51

+5, +10, +15, +20

100. (d) As,

Q L M U
+2 +2 +2 +2
S N O W

Similarly,

J G Q R
+2 +2 +2 +2
[L I S T]

101. (a) In 1 h 40 min work time is there.
In 15 min 100 lines are tackled, so in 40 min, for the first 30 min, 100 + 100 = 200 lines will be typed, then 4 min thinking will be followed by 5 min typing in which 50 lines will be typed; so, 250 lines are typed an hour.

102. (d) Let ab be the number, then
$10a + b = 4a + 4b$ and $10a + b + 9 = 10b + a$
$\Rightarrow$ $a = 1$ and $b = 2$
$\Rightarrow$ Number is 12.

103. (d)

On September	Day
1970	Sunday
1971	Monday +1
1972 (leap year)	Wednesday +2
1973	Thursday
1974	Friday
1975	Saturday
1976 (leap year)	Monday
1977	Tuesday
1978	Wednesday
1979	Thursday
1980	Saturday
1981	Sunday

104. (b) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$= 14 + 11 - 19 = 6$

105. (b) Keeping in view all the conditions given in the question, B can be chosen in the following two ways.

Mathematicians	Physicists
(I) ABC	FH or GH
(II) BCD	GH

From (I) Two physicists are FH and GH
From (II) only one way of choosing physicists exists.
Hence, if B is chosen, then G will also be chosen.

106. (c) If A is chosen, then mathematician chosen are ABC and physicists chosen are F and H or G and H. If D is chosen, mathematicians chosen are B,C and D physicists chosen are GH. From the above two cases, it is clear that whether A is chosen or D is chosen either F or G is chosen.

107. (c) If G is rejected then, the member selected would be $ABC - FH$.
Hence, if G is rejected D cannot be in the group.

108. (b) If H is chosen, then the group of mathematician will definitely include B.

109. (b) Cost of 200 g $= \frac{60}{250} \times 200 = 48$ paisa

110. (d) House rent per cent is 70% × 25% = 17.5%
As 30% = ₹ 300, so 17.5% = ₹ 175

111. (b) If $\frac{x}{y}$ is original value, then

$$\frac{x \times \frac{5}{4}}{y \times \frac{4}{5}} = \frac{5}{4} \Rightarrow \frac{x}{y} = \frac{5}{4} \times \frac{4}{5} \times \frac{4}{5} = \frac{4}{5}$$

112. (d) If x, y are length and width, then
Area, $A = xy$

If new length $= \frac{8}{5}x$, then new width $= \frac{5}{8}y$ to maintain the area.
$\Rightarrow$ Percentage decrement in width

$$= \frac{y - \frac{5}{8}y}{y} \times 100\% = 37.5\%$$

113. (d) $1 - \left[\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \dots\dots + \frac{17}{5184} + \frac{19}{8100}\right]$

$$= \frac{1}{4} - \left[\frac{5}{36} + \frac{7}{144} + \dots\dots + \frac{17}{5184} + \frac{19}{8100}\right]$$

$$= \frac{1}{9} - \left[\frac{7}{144} + \dots\dots + \frac{17}{5184} + \frac{19}{8100}\right]$$

$$= \frac{1}{4^2} - [\dots] = \frac{1}{10^2} = 0.01$$

$$\Rightarrow \frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \dots\dots + \frac{17}{5184} + \frac{19}{8100} = 1 - 0.01 = 0.99$$

114. (c) Let x be the original amount of white balls ordered, then

$$\frac{x + 45}{x} = \frac{6}{5}$$

$\Rightarrow$ $x = 225$

115. (c) Fraction of examination $= \frac{1 \times 3}{1 \times 3 + 6 \times 1} = \frac{3}{9} = \frac{1}{3}$

116. (c) C's share is ₹ 560, so total money is

$$560 \times \frac{(65 + 35)}{35} = \frac{560 \times 100}{35} = 1600$$

117. (c) If F and M are ages of Joe's father and mother, then
$(F + 6) = 2(22 + 6) \Rightarrow F = 50$
$(M - 2) = 2(22 - 2) \Rightarrow M = 42$
$\Rightarrow$ $F - M = 8$

118. (b) If d is the distance and v the original speed, then

$$\frac{4}{5} \times \frac{d}{v} = \frac{d}{v + \frac{1}{2}}$$

$\Rightarrow$ $4v + 2 = 5v \Rightarrow v = 2$

Also, $\frac{d}{v} + \frac{5}{2} = \frac{d}{v - \frac{1}{2}}$

$\Rightarrow$ $\frac{d}{2} + \frac{5}{2} = \frac{2d}{3}$ (on putting value v)

$\Rightarrow$ $d = 15$ km

119. (c) 2 oranges + 3 bananas + 4 apples cost ₹ 15
3 oranges + 2 bananas + 1 apple cost ₹ 10
∵ 5 of each costs ₹ 25
Hence, 3 of each will cost $\frac{25}{5} \times 3 =$ ₹ 15

120. (a) Number of sheets now required is

$$\frac{20 \times 55 \times 65}{65 \times 70} = \frac{220}{14} = \frac{110}{7}$$

Hence, percentage reduction in sheet $\frac{20 - \frac{110}{7}}{20} \times 100$

$$= \frac{30}{7 \times 20} \times 100 = \frac{150}{7}\% = 21.4\%$$

which is close to 20%.