KIITEE MCA
Solved Paper 2011

Mathematics

1. For real x, let $f(x) = x^3 + 5x + 1$, then
   (a) f is onto R but not one-one
   (b) f is one-one and onto R
   (c) f is neither one-one nor onto R
   (d) f is one-one but not onto R

2. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$, is defined, is
   (a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $[0, \pi]$
   (c) $\left[0, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. Let w denotes the words in the English dictionary. Define the relation R by : $R = \{(x, y) \in w \times w |$ the words x and y have atleast one letter in common$\}$, then R is
   (a) not reflexive, symmetric and transitive
   (b) reflexive, symmetric and not transitive
   (c) reflexive, symmetric and transitive
   (d) reflexive, not symmetric and transitive

4. If $|x + 4| \leq 3$, then the maximum value of $|z + 1|$ is
   (a) 6 (b) 0
   (c) 4 (d) 10

5. If $w = \dfrac{z}{z - \left(\frac{1}{3}\right)}$ and $|w| = 1$, then z, lies on
   (a) a circle (b) an ellipse
   (c) a parabola (d) a straight line

6. If $1, a_1, a_2, \ldots, a_{n-1}$ are nth roots of unity, then $\dfrac{1}{1-a_1} + \dfrac{1}{1-a_2} + \ldots + \dfrac{1}{1-a_{n-1}}$ equals
   (a) $\dfrac{2^n - 1}{n}$ (b) $\dfrac{n-1}{2}$
   (c) $\dfrac{n}{n-1}$ (d) None of these

7. If z_1 and z_2 both satisfy the relation $z + \bar{z} = 2|z - 1|$ and $\arg(z_1 - z_2) = \pi/4$, then $\lim(z_1 + z_2)$ equals
   (a) 0 (b) 1
   (c) 2 (d) 3

8. If a, b, c are positive integers and ω is imaginary cube root of unity and $f(x) = x^{6a} + x^{6b+1} + x^{6c+2}$, then $f(\omega)$ equals
   (a) 0 (b) 1
   (c) -1 (d) None of these

9. If z and w are two non-zero complex numbers such that $|z| = |w|$ and $\arg(z) + \arg(w) = \pi$, then z equals
   (a) $\overline{w}$ (b) w
   (c) $-\overline{w}$ (d) $-w$

10. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n$ represent a real number is
    (a) 4 (b) 8
    (c) 12 (d) 2

11. If $z = \lambda + 3 + i\sqrt{(3 - \lambda^2)}$, $\forall \lambda \in R$, then locus of z is a
    (a) circle
    (b) parabola
    (c) straight line
    (d) None of the above

12. If $z \neq 0$, then $\int_0^{50} \arg(-|z|)\, dx$ equals
    (a) 50
    (b) Not defined
    (c) 0
    (d) 50π

13. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?
    (a) 1 (b) 3
    (c) 5 (d) 7

14. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is
    (a) less than $4ab$
    (b) greater than $-4ab$
    (c) less than $-4ab$
    (d) greater than $4ab$

15. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval
    (a) $(6, \infty)$ (b) $(5, 6]$
    (c) $[4, 5]$ (d) $(-\infty, 4)$

16. If $a > 0$, and discriminant of $ax^2 + 2bx + c < 0$ is greater than zero, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

(a) positive
(b) $(ac - b^2)(ax^2 + 2bx + c)$
(c) negative
(d) 0

17. If l, m, n are pth, qth, rth terms of GP, all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

(a) -1 (b) 2
(c) 1 (d) 0

18. If the system of equations $x + 2ay + az = 0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c are in

(a) GP
(b) HP
(c) satisfy $a + 2b + 3c = 0$
(d) AP

19. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, then only correct statement about the matrix A is

(a) A^{-1} does not exist (b) $A = (-1)I$
(c) A is a zero matrix (d) $A^2 = I$

20. Let $A = \begin{bmatrix} 5 & 5a & a \\ 0 & a & 5a \\ 0 & 0 & 5 \end{bmatrix}$, if $|A^2| = 25$, then $|a|$ is equal to

(a) 1/5 (b) 5
(c) 5^2 (d) 1

21. If $A = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x)$, then A^{-1} is equal to

(a) $f(-x)$ (b) $f(x)$
(c) $-f(x)$ (d) $-f(-x)$

22. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then $\forall n \in N$

(a) $A^{-n} = \begin{pmatrix} 1 & 1 \\ 1 & n \end{pmatrix}$ (b) $\lim_{n \to \infty} \frac{A^{-n}}{n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$
(c) $\lim_{n \to \infty} \frac{A^{-n}}{n} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) None of these

23. The domain of the function $\sqrt{\log_{0.5} x}$ is

(a) $(1, \infty)$ (b) $(0, \infty)$
(c) $(0, 1]$ (d) $(0.5, 1)$

24. If $x = \log_3 5$, $y = \log_{17} 25$, which of the following is correct?

(a) $x < y$ (b) $x = y$
(c) $x > y$ (d) None of these

25. A student has to answer 10 out of 13 questions in an examination such that he must choose atleast 4 questions from first five questions. The number of choices available to him is

(a) 196 (b) 280
(c) 346 (d) 140

26. How many numbers can be formed greater than 1000 but less than 4000 using the digits 0, 2, 3, 4, if repetition is allowed?

(a) 125 (b) 105
(c) 128 (d) 625

27. The probability that a student will obtain grades A, B, C or D are 0.30, 0.35, 0.20 and 0.15 respectively. The probability that he will receive atleast C grade is

(a) 0.65 (b) 0.85
(c) 0.80 (d) 0.20

28. The coefficient of x^p and x^q in the expansion of $(1 + x)^{p+q}$ are

(a) equal
(b) equal with opposite sign
(c) reciprocal of each other
(d) None of the above

29. $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b are

(a) $b = 2$, $a \in R$
(b) $a = 1$, $b \in R$
(c) $a, b \in R$
(d) $a = 1$, $b = 2$

30. Let $f(x) = \int e^x (x-1)(x-2) dx$, then f decreases in the interval

(a) $(-\infty, 2)$
(b) $(-2, -1)$
(c) $(1, 2)$
(d) $(2, \infty)$

31. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ has a local maximum at $x = x_1$ and a local minimum at $x = x_2$, such that $x_2 = x_1^2$, then a is equal to

(a) 0 (b) 1/4
(c) 2 (d) 0 or 2

32. Let $f(x) = \begin{cases} |x^2 - 3x| + a, & 0 \le x \le 3/2 \\ -2x + 3, & x \ge 3/2 \end{cases}$. If $f(x)$ has a local maxima at $x = 3/2$, then

(a) $a \le 0$
(b) $a \le -9/4$
(c) $a \ge 9/4$
(d) None of the above

33. The difference between the greatest and the least values of the function $f(x) = \int_0^x (at^2 + 1 + \cos t)\, dt$, $a > 0$ for $x \in [2, 3]$ is

(a) $\frac{19}{3} a + 1 + (\sin 3 - \sin 2)$
(b) $\frac{18}{3} a + 1 + 2 \sin 3$
(c) $\sin 3 - \sin 2$
(d) 0

34. $\int_{2-a}^{2+a} f(x)\,dx$ is equal to

(where $f(2-a)=f(2+a)$, $\forall\ a \in R$,

(a) $2\int_{a}^{2+a} f(x)dx$

(b) $2\int_{0}^{a} f(x)\,dx$

(c) $2\int_{0}^{2} f(x)dx$

(d) None of the above

35. If $f(x)=\int_0^x \frac{dt}{\{f(t)\}^2}$ and $\int_0^2 \frac{dt}{\{f(t)\}^2}=\sqrt[3]{6}$, then $f(9)$ equals

(a) 0 (b) 1
(c) 2 (d) 3

36. The value of the function $\int_0^{11} [x]^3 dx$, where [.] denotes the greatest integer function is

(a) 0 (b) 14400
(c) 2200 (d) 3025

37. The area bounded by $y=\frac{\sin x}{x}$, x-axis and the ordinates at $x=0$, $x=\pi/4$, is

(a) $=\pi/4$ (b) $<\pi/4$
(c) $>\pi/4$ (d) $<\int_0^{\pi/4} \frac{\tan x}{x}\,dx$

38. The slope of the tangent to a curve $y=f(x)$ at $(x, f(x))$ is $2x+1$. If the curve passes through the point $(1, 2)$, then the area of the region bounded by the curve, the x-axis and the line $x=1$, is

(a) 5/6 sq unit (b) 6/5 sq units
(c) 1/6 sq unit (d) 6 sq units

39. The area enclosed between the curves $y^2=x$ and $y=|x|$ is

(a) 2/3 (b) 1/3
(c) 1/6 (d) 1

40. The value of c for which conclusion of Mean Value Theorem holds for the function $f(x)=\log_e x$ on the interval $[1, 3]$ is

(a) $\log_3 e$ (b) $\log_e 3$
(c) $2\log_3 e$ (d) $\frac{1}{2}\log_e 3$

41. The solution of the differential equation $\frac{dy}{dx}=\frac{x+y}{x}$ satisfying the condition $y(1)=1$, is

(a) $y=x\log x+x^2$ (b) $y=xe^{x-1}$
(c) $y=\log x+x$ (d) $y=x+x\log x$

42. The differential equation which represents the family of curves $y=c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants is

(a) $y''=y'y$ (b) $yy''=y'$
(c) $yy''=(y')^2$ (d) $y'=y^2$

43. The function $f(x)=\frac{x}{2}+\frac{2}{x}$ has a local minimum at

(a) $x=2$ (b) $x=-2$
(c) $x=0$ (d) $x=1$

44. The image of P (a, b) on $y=-x$ is Q and the image of Q on the line $y=x$ is R, then the mid-point of PR is

(a) $(a+b, b+a)$ (b) $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$
(c) $(a-b, b-a)$ (d) $(0, 0)$

45. If the distance of any point (x, y) from the origin is defined as $d(x, y)=|x|+|y|$, then the locus of $d(x, y)=1$ is a

(a) circle of area π sq units (b) square of area 1 sq unit
(c) square of area 2 sq units (d) None of these

46. The pair of lines represented by $3ax^2+5xy+(a^2-2)y^2=0$ are perpendicular to each other for

(a) two values of a (b) for all a
(c) for one value of a (d) for no values of a

47. In an ellipse, the distance between its focii is 6 and minor axis is 8, then its eccentricity is

(a) 3/5 (b) 1/2
(c) 4/5 (d) $1/\sqrt{5}$

48. The parabola has the origin at its focus and $x=2$ as the directrix, then the vertex of the parabola is at

(a) (0, 1) (b) (1, 0)
(c) (2, 0) (d) (0, 2)

49. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals

(a) 3 (b) -1
(c) 1 (d) -3

50. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2+y^2+2x+4y-3=0$, is

(a) (3, 4) (b) (3, −4)
(c) (−3, −4) (d) (−3, 4)

51. Let the lines $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$, then (α, β) equals

(a) (−6, 7) (b) (5, −15)
(c) (−5, 5) (d) (6, −17)

52. A particle acted by constant forces $4\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ is displaced from the point $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ to the point $5\mathbf{i}+4\mathbf{j}+\mathbf{k}$. The total work done by the forces is

(a) 30 units (b) 40 units
(c) 50 units (d) 20 units

53. Let $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $\mathbf{c}=x\mathbf{i}+(x-2)\mathbf{j}-\mathbf{k}$. If the vector $\mathbf{c}$ lies in the plane of $\mathbf{a}$ and $\mathbf{b}$, then x equals

(a) −2 (b) −4
(c) 0 (d) 1

54. The non-zero vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ related by $\mathbf{a}=8\mathbf{b}$ and $\mathbf{c}=-7\mathbf{b}$, then the angle between $\mathbf{a}$ and $\mathbf{c}$ is

(a) 0 (b) $\pi/2$
(c) π (d) $\pi/4$

55. If $|\mathbf{a}|=2$ and $|\mathbf{b}|=3$ and $\mathbf{a}\cdot\mathbf{b}=0$, then $|\mathbf{a}\times(\mathbf{a}\times(\mathbf{a}\times(\mathbf{a}\times\mathbf{b})))|$ is equal to

(a) 48 (b) $-48\,\mathbf{b}$
(c) $48\,\mathbf{a}$ (d) -48

56. If $\sin(\pi\cos x)=\cos(\pi\sin x)$, then x equals

(a) $\frac{1}{2}\sin^{-1}(3/4)$ (b) $\frac{1}{2}\cos^{-1}(3/4)$
(c) $-\frac{1}{2}\sin^{-1}(1/4)$ (d) $-\frac{1}{2}\cos^{-1}(3/4)$

57. If $\cot^{-1}[\sqrt{\cos\alpha}]+\tan^{-1}[\sqrt{\cos\alpha}]=x$, then $\sin x$ equals

(a) $\tan\alpha$
(b) $\cot^2(\alpha/2)$
(c) 1
(d) $\cot(\alpha/2)$

58. The function $f(x)=\tan^{-1}(\sin x+\cos x)$ is an increasing function in

(a) $(0, \pi/2)$
(b) $(-\pi/2, \pi/2)$
(c) $(\pi/4, \pi/2)$
(d) $(-\pi/2, \pi/4)$

59. If two tangents drawn from a point P to the parabola $y^2=4x$ are at right angles, then the locus of P is

(a) $x=1$ (b) $2x+1=0$
(c) $x=-1$ (d) $2x-1=0$

60. Let $f: R\to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$, then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}$ is equal to

(a) 1 (b) 2/3
(c) 3/2 (d) 3

Computer Awareness

61. A collection of program that controls how your computer system runs and processes information is called

(a) operating system (b) compiler
(c) interpreter (d) office

62. Conditional results after execution of an instruction in a micro processor is stored in

(a) register
(b) accumulator
(c) flag register
(d) flag register part of Program Status Word

63. Which software allows users to perform calculation on rows and columns of data?

(a) Word Processing
(b) Presentation Graphics
(c) Database Management Systems
(d) Electronic Spreadsheet

64. The operating system does all of the following EXCEPT

(a) provide a way for the user to interact with the computer
(b) manage the central processing unit (CPU)
(c) manage memory and storage
(d) enable users to perform a specific task such as document editing

65. The unique signal, generated by a device, that tells the operating system that it is in need of immediate attention is called an

(a) action
(b) event
(c) interrupt
(d) activity

66. Which of the following is the correct sequence of actions that takes place during the boot-up process?

(a) Load operating system? Activate BIOS? Perform POST? Check configuration settings.
(b) Activate BIOS? Perform POST? Load operating system? Check configuration settings.
(c) Perform POST? Load operating system? Activate BIOS? Check configuration settings.
(d) Activate BIOS? Check configuration settings? Perform POST? Load operating system.

67. Using Windows Explorer, a plus (+) sign in front of a folder indicates

(a) an open folder
(b) the folder contains subfolders
(c) a text file
(d) a graphics file

68. A computerized system consists of

(a) Hardware, Data, Procedure, Processing, People
(b) Hardware, Programs, Data, Processing, Networks
(c) Hardware, Programs, Data, Networks, People
(d) Hardware, Software, Procedure, Data, People

69. Which one of following helps a user in locating information over internet?

(a) URL
(b) Search engine
(c) Network
(d) None of the above

70. The only language understood by a digital computer is called

(a) Assembly language (b) High level language
(c) English language (d) Binary language

71. The ability of a computer to execute multiple programs by using multiple processors simultaneously is known as

(a) Multitasking (b) Multiprocessing
(c) Multiprogramming (d) Multithreading

72. A goal of data mining includes which of the following?

(a) To explain some observed event or condition.
(b) To confirm that data exists.
(c) To analyze data for expected relationship.
(d) To create a new data warehouse.

73. When data changes in multiple lists and all lists are not updated, this causes

(a) data redundancy (b) information overload
(c) duplitcate data (d) data inconsistency

74. The purpose of the primary key in a database is to

(a) unlock the database
(b) provide a map of the data
(c) uniquely identify a record
(d) establish constraints on database operations

75. Personal logs or journal entries posted on the Web are known as
(a) listservs (b) webcasts
(c) blogs (d) subject directories

76. Which of the following places the common data elements in order from the smallest to the largest?
(a) Character, file, record, field, database.
(b) Character, record, field, database, file.
(c) Character, field, record, file, database.
(d) Bit, byte, character, record, field, file, database.

77. Which of the following would most likely not be a symptom of a virus?
(a) Existing program files and icons disappear.
(b) The CD-ROM stops functioning.
(c) The Web browser opens to an unusual home page.
(d) Odd messages or images are displayed on the screen.

78. A machine having 64MB memory runs a executable which is 300MB on disk. This is achieved by
(a) use of FAR pointers
(b) page swapping
(c) saving some variables on another machine on network
(d) cannot be run on the machine

79. What causes "Thrashing" of a program?
(a) The constant swapping of program due to page faults.
(b) The inability of a program to get access to a network resource.
(c) A near overflow/underflow of a variable.
(d) Assessing a memory area not allocated to the process.

80. DCOM and CORBA are
(a) specifications which enable faster downloads on the net.
(b) specifications that allow objects to be accessed in a location independent manner.
(c) parallel implementations of XML by Microsoft and Sun respectively.
(d) specifications to store objects on disk, for later retrieval.

81. Testing based on External Specifications without knowledge of how the system is constructed is
(a) Black Box Testing
(b) White Box Testing
(c) Stress Testing
(d) Performance Testing

82. Which is a typical page layout program out of the following software products?
(a) Adobe Photoshop (b) Adobe PageMaker
(c) Macromedia FreeHand (d) Macromedia Director

83. Which of the following are normally used to initialize a computer system's hardware?
(a) Volatile memory (b) External mass memory
(c) Static memory (d) Random access memory

84. Graphical diagrams used to represent different multiple perspectives of a system include
(a) use-case, class and state diagrams
(b) state, interaction and derivative diagrams
(c) interaction, relationship and class diagrams
(d) deployment, relationship and use-case diagrams

85. Which kind of lock includes a keypad that can be used to control access into areas?
(a) Cipher (b) Warded
(c) Device (d) Tumbler

86. Which of the following groups consist of only output devices?
(a) Scanner, Printer, Monitor
(b) Keyboard, Printer, Monitor
(c) Mouse, Printer, Monitor
(d) Plotter, Printer, Monitor

87. Which is the part of a computer that one can touch and feel?
(a) Hardware (b) Software
(c) Programs (d) Output

88. One of the following is a target group for the marketing of Internet Banking.
(a) All the customers
(b) All the educated customers
(c) All the computer educated customers
(d) Only creditors

89. Which of the following companies do not manufacture chips?
(a) Microsoft (b) Motorola
(c) Intel (d) HP

90. In which of the following the flow is in both the directions?
(a) Single linked list (b) Double linked list
(c) Queue (d) None of these

Analytical Ability & Logical Reasoning

91. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/h and the time of flight increased by 30 min. The duration of the flight is
(a) 1 h (b) 2 h
(c) 3 h (d) 4 h

92. A alone can do a piece of work in 6 days and B alone in 8 days. A and B undertook to do it for ₹ 3200. With the help of C, they completed the work in 3 days. How much is to be paid to C?
(a) ₹ 375 (b) ₹ 400
(c) ₹ 600 (d) ₹ 800

93. Two trains running in opposite directions cross a man standing on the platform in 27 s and 17 s, respectively and they cross each other in 23 s. The ratio of their speeds is
(a) 1 : 3 (b) 3 : 2
(c) 3 : 4 (d) None of these

94. Ravi and Kumar are working on an assignment. Ravi takes 6 h to type 32 pages on a computer, while Kumar takes 5 h to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?
(a) 7 h 30 min (b) 8 h
(c) 8 h 15 min (d) 8 h 25 min

95. How much time will it take for an amount of ₹ 450 to yield ₹ 81 as interest at 4.5% per annum of simple interest?
(a) 3.5 yr (b) 4 yr
(c) 4.5 yr (d) 5 yr

96. The captain of a cricket team of 11 members in 26 yr old and the wicket keeper is 3 yr older. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. What is the average age of the team?
(a) 23 yr (b) 24 yr
(c) 25 yr (d) None of these

97. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 sq m, then what is the width of the road?
(a) 2.91 m (b) 3 m
(c) 5.82 m (d) None of these

98. Two pipes A and B can fill a tank in 15 min and 20 min, respectively. Both the pipes are opened together but after 4 min, pipe A is turned off. What is the total time required to fill the tank?
(a) 10 min 20 s (b) 11 min 45 s
(c) 12 min 30 s (d) 14 min 40 s

99. The difference between the place value and the face value of 6 in the numeral 856973 is
(a) 973 (b) 6973
(c) 5994 (d) None of these

100. Shyam invested an amount of ₹ 8000 in a fixed deposit scheme for 2 yr at compound interest rate 5% per annum. How much amount will Shyam get on maturity of the fixed deposit?
(a) ₹ 8600 (b) ₹ 8620
(c) ₹ 8820 (d) None of these

101. A and B started a partnership business investing some amount in the ratio of 3 : 5. C joined them after six months with an amount equal to that of B. In what proportion should the profit at the end of one year be distributed among A, B and C?
(a) 3 : 5 : 2 (b) 3 : 5 : 5
(c) 6 : 10 : 5 (d) Data inadequate

102. A boatman goes 2 km against the current of the stream in 1 h and goes 1 km along the current in 10 min. How long will it take to go 5 km in stationary water?
(a) 40 min (b) 1 h
(c) 1 h 15 min (d) 1 h 30 min

103. A wheel that has 6 cogs is meshed with a larger wheel of 14 cogs. When the smaller wheel has made 21 revolutions, then the number of revolutions made by the larger wheel is
(a) 4 (b) 9
(c) 12 (d) 49

104. Standing on a platform, Amit told Sunita that Aligarh was more than 10 km but less than 15 km from there. Sunita knew that it was more than 12 but less than 14 km from there. If both of them were correct, which of the following could be the distance of Aligarh from the platform?
(a) 11 km (b) 12 km
(c) 13 km (d) 14 km

105. How many bricks, each measuring $25\text{ cm} \times 11.25\text{ cm} \times 6\text{ cm}$, will be needed to build a wall of $8\text{ m} \times 6\text{ m} \times 22.5\text{ cm}$?
(a) 5600 (b) 6000
(c) 6400 (d) 7200

106. A man walks 5 km toward South and then turns to the right. After walking 3 km he turns to the left and walks 5 km. Now in which direction is he from the starting place?
(a) West (b) South
(c) North-East (d) South-West

107. Pointing to a photograph Bajpai said, "He is the son of the only daughter of the father of my brother." How Bajpai is related to the man in the photograph?
(a) Nephew (b) Brother
(c) Father (d) Maternal Uncle

Directions (Q. Nos. 108-111) *Read the following passage and solve the questions based on it.*

Six friends are sitting in a circle and are facing the centre of the circle. Deepa is between Prakash and Pankaj. Priti is between Mukesh and Lalit. Prakash and Mukesh are opposite to each other.

108. Who is sitting opposite to Prakash?
(a) Mukesh (b) Deepa
(c) Pankaj (d) Lalit

109. Who is just right to Pankaj?
(a) Deepa (b) Lalit
(c) Prakash (d) Priti

110. Who are the neighbours of Mukesh?
(a) Prakash and Deepa
(b) Deepa and Priti
(c) Priti and Pankaj
(d) Lalit and Priti

111. Who is sitting opposite to Priti?
(a) Prakash (b) Deepa
(c) Pankaj (d) Lalit

112. Arrange the words given below in a meaningful sequence.
1. Windows 2. Walls 3. Floor
4. Foundation 5. Roof 6. Room
(a) 4, 5, 3, 2, 1, 6
(b) 4, 2, 1, 5, 3, 6
(c) 4, 1, 5, 6, 2, 3
(d) 4, 3, 5, 6, 2, 1

113. If GOLD is coded as HOME, COME is coded as DONE and CORD is coded as DOSE, how would you code SONS?
(a) TPOT (b) TOOT
(c) TOOS (d) TONT

114. In the series 2, 6, 18, 54,..., what will be the 8th term?
(a) 4370 (b) 4374
(c) 7443 (d) 7434

115. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?
(a) 810 (b) 1440
(c) 2880 (d) 50400

116. Two cylindrical blocks have their diameters in the ratio 3 : 1 and their heights in the ratio 1 : 3. Their volumes would, thus, be in the ratio of

(a) 3 : 1 (b) 3 : 4
(c) 1 : 2 (d) 2 : 3

117. On simplifying $\sqrt{12}+2\sqrt{48}+5\sqrt{147}-45\sqrt{3}$, we get

(a) 1 (b) 2
(c) 3 (d) 0

118. Twenty tickets are numbered from 1 to 20 and one of them is drawn at random, the probability that number is divisible by 3 or 5 is

(a) 1/5 (b) 9/20
(c) 3/5 (d) 4/5

119. What should come in the place of the question mark (?) in the following letter series?

BXJ ETL HPN KLP ?

(a) NHR
(b) MHQ
(c) MIP
(d) NIR

120. A clock is set right at 5 am. The clock loses 16 min in 24 h. What will be the true time when the clock indicates 10 pm on the 4th day?

(a) 9 am (b) 11 pm
(c) 11 am (d) 9 pm

Answers *with* Solutions

1. (b) Given, $f(x)=x^3+5x+1$

On differentiating w.r.t. x, we get

$$f'(x)=3x^2+5>0,\ \forall x\in R$$

Since, $f(x)$ is an increasing function, so it is one-one.
Since, $f(x)$ is an odd polynomial, so it will give all values of real number.
Hence, $f(x)$ is one-one and onto on R.

2. (c) Given, $f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$

Here, 4^{-x^2} is defined for $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ ...(i)

$\cos^{-1}\left(\frac{x}{2}-1\right)$ is defined for $-1\le\frac{x}{2}-1\le 1$

$$\Rightarrow\quad 0\le\frac{x}{2}\le 2$$

$$\Rightarrow\quad 0\le x\le 4 \qquad\text{...(ii)}$$

$\log(\cos x)$ is defined for $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ ...(iii)

∴ From Eqs. (i), (ii) and (iii), we get

$$\left[0,\frac{\pi}{2}\right)$$

3. (b) Given, $R=\{(x,y)\in w\times w$: the words x and y have atleast one letter in common$\}$

(i) **Reflexive**
It is clear that $(x,x)\in R,\ \forall x\in w$
$\Rightarrow$ R is reflexive.

(ii) **Symmetric**
Let $(x,y)\in R$, then $(y,x)\in R$
[∵ x,y have atleast one letter in common]
$\Rightarrow$ R is symmetric.

(iii) **Transitive**
Let x = INDIA, y = MUMBAI, z = ZUHU
If $(x,y)\in R$ and $(y,z)\in R$
$\Rightarrow$ $(x,z)\notin r$
$\Rightarrow$ R is not transitive.

4. (a) Now, $|z+1|=|(z+4)-3|\le|z+4|+|3|$

$$\le 3+3=6$$

$$\therefore\ 0\le|z+1|\le 6$$

5. (d) Given, $w=\dfrac{z}{z-\frac{1}{3}i}$

$$\Rightarrow\quad |w|=\frac{|z|}{\left|z-\frac{1}{3}i\right|}$$

$$\Rightarrow\quad 1=\frac{|x+iy|}{\left|x+\left(y-\frac{1}{3}\right)i\right|}\qquad(\because |w|=1)$$

$$\Rightarrow\quad 1=\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+\left(y-\frac{1}{3}\right)^2}}$$

$$\Rightarrow\quad x^2+y^2+\frac{1}{9}-\frac{2y}{3}=x^2+y^2$$

$$\Rightarrow\quad \frac{2y}{3}=\frac{1}{9}$$

$$\Rightarrow\quad y=\frac{1}{6}$$

Hence, it is a straight line.

6. (b) Since, $1,a_1,a_2,\ldots\ldots,a_{n-1}$ are the nth roots of unity.

$$\therefore\quad x^n-1=(x-1)(x-a_1)(x-a_2)\ldots\ldots(x-a_{n-1})$$

$$\Rightarrow\quad (1+x+x^2+\ldots+x^{n-1})=(x-a_1)(x-a_2)\ldots(x-a_{n-1})$$

On taking log both sides, we get

$$\log(1+x+x^2+\ldots+x^{n-1})$$
$$=\log(x-a_1)+\log(x-a_2)+\ldots+\log(x-a_{n-1})$$

On differentiating w.r.t. x, we get

$$\frac{(1+2x+3x^2+\ldots+(n-1)x^{n-2})}{1+x+x^2+\ldots+x^{n-1}}=\frac{1}{x-a_1}+\frac{1}{x-a_2}+\ldots+\frac{1}{x-a_{n-1}}$$

Put $x=1$, we get

$$\Rightarrow\quad \frac{1+2+3+\ldots+n-1}{1+1+\ldots+1}=\frac{1}{1-a_1}+\frac{1}{1-a_2}+\ldots+\frac{1}{1-a_{n-1}}$$

$$=\frac{(n-1)(n)}{2\times n}$$

$$=\frac{n-1}{2}$$

7. (c) Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$

Given, $z+\bar{z}=2|z-1|$

Since, z_1 and z_2 satisfy

$$\therefore\quad z_1+\bar{z}_1=2|z_1-1|$$

$$\Rightarrow\quad 2x_1=2|(x_1-1)+iy_1|$$

$$\Rightarrow\quad x_1^2=(x_1-1)^2+y_1^2$$

$$\Rightarrow\quad 0=1-2x_1+y_1^2\qquad\text{...(i)}$$

Similarly, for z_2

$$0=1-2x_2+y_2^2\qquad\text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$2(x_2 - x_1) + y_1^2 - y_2^2 = 0$$

$$\Rightarrow \quad 2(x_2 - x_1) + (y_1 - y_2)(y_1 + y_2) = 0 \quad \ldots\text{(iii)}$$

Also, $\arg(z_1 - z_2) = \frac{\pi}{4}$

$$\therefore \quad \arg(x_1 + iy_1 - (x_2 + iy_2)) = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \frac{y_1 - y_2}{x_1 - x_2} = 1$$

$$\Rightarrow \quad x_1 - x_2 = y_1 - y_2$$

$\therefore$ From Eq. (iii), $2(x_2 - x_1) + (x_1 - x_2)(y_1 + y_2) = 0$

$$\Rightarrow \quad y_1 + y_2 = 2$$

$$\therefore \quad \text{Im}(z_1 + z_2) = y_1 + y_2 = 2$$

8. (a) Given, $f(x) = x^{6a} + x^{6b+1} + x^{6c+2}$

$$\therefore \quad f(\omega) = \omega^{6a} + \omega^{6b+1} + \omega^{6c+2}$$

$$= 1 + 1\cdot\omega + 1\cdot\omega^2 \quad (\because \omega^{3n} = 1)$$

$$= 1 + \omega + \omega^2 = 0$$

9. (c) Given, $|z| = |w| \Rightarrow |z| = |\overline{w}|$

Now, $\arg(z) + \arg(\overline{w}) = \pi$

$\Rightarrow \quad \arg(z) - \arg(\overline{w}) = \pi$

$\therefore \quad z + \overline{w} = 0$

$\Rightarrow \quad z = -\overline{w}$

10. (d) $$\left(\frac{1+i}{1-i}\right)^n = \left(\frac{(1+i)\times(1+i)}{(1-i)(1+i)}\right)^n$$

$$= \left(\frac{1 - 1 + 2i}{1+1}\right)^n = (i)^n \quad (\because i^2 = -1)$$

Hence, least integer value of n is 2.

11. (a) Let $z = x + iy$

Given, $z = \lambda + 3 + i\sqrt{(3 - \lambda^2)}, \quad \forall \lambda \in R$

$\therefore \quad x + iy = (\lambda + 3) + i\sqrt{3 - \lambda^2}$

$\Rightarrow \quad x = \lambda + 3$ and $y = \sqrt{3 - \lambda^2}$

$\Rightarrow \quad (x-3)^2 = \lambda^2$ and $\lambda^2 = 3 - y^2$

$\therefore \quad (x-3)^2 = 3 - y^2$

$\Rightarrow \quad (x-3)^2 + y^2 = 3$

Hence, it represents the equations of circle. Whose centre at $(3, 0)$ and radius is $\sqrt{3}$.

12. (d) Let $z = x + iy$

$$\therefore \quad \int_0^{50} \arg(-|z|)\,dx$$

$$= \int_0^{50} [\arg(|z|) + \pi]\,dx$$

$$= \int_0^{50} [0 + \pi]\,dx$$

$$= \pi\,[x]_0^{50}$$

$$= 50\pi$$

13. (a) Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$

$$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \ \forall x \in R$$

Hence, $f(x)$ is an increasing function, $\forall x \in R$.

So, number of real solutions is 1.

14. (b) Since, roots of the equation $bx^2 + cx + a = 0$ are imaginary.

$$\therefore \quad c^2 - 4ab < 0 \Rightarrow c^2 < 4ab \quad \ldots\text{(i)}$$

Also, $E = 3b^2x^2 + 6bcx + 2c^2$

$$= 3(bx + c)^2 - c^2$$

From Eq. (i), $-c^2 > -4ab$

$$\Rightarrow 3(bx + c)^2 - c^2 > 3(bx + c)^2 - 4ab$$

$$\Rightarrow \quad E > -4ab + 3(bx + c)^2$$

$$= E > -4ab$$

15. (d) Since, both roots are less than 5.

$$\therefore \quad D \ge 0, af(5) > 0, 5 > -\frac{b}{2a}$$

Given, $x^2 - 2kx + k^2 + k - 5 = 0$

(i) $D \ge 0, (-2k)^2 - 4 \times 1 \times (k^2 + k - 5) \ge 0$

$\Rightarrow \quad -4(k - 5) \ge 0$

$\Rightarrow \quad k \le 5$

(ii) $af(5) > 0$

$1(25 - 10k + k^2 + k - 5) > 0$

$\Rightarrow \quad k^2 - 9k + 20 > 0$

$\Rightarrow \quad (k - 4)(k - 5) > 0$

$\Rightarrow \quad k < 4$ or $k > 5$

(iii) $5 > \frac{-(-2k)}{2(1)}$

$\Rightarrow \quad k < 5$

From Eqs. (i), (ii) and (iii), we get $k < 4$

i.e., $(-\infty, k)$

16. (c) $$\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$$

$$= (ax + b)[b^2x + bc - acx - bc]$$

$$- (bx + c)[abx + ac - abx - b^2]$$

$$= (ax + b)(b^2 - ac)x - (bx + c)(ac - b^2)$$

$$= (b^2 - ac)(ax^2 + bx + bx + c)$$

$$= (b^2 - ac)(ax^2 + 2bx + c) \quad \ldots\text{(i)}$$

But it is given $ax^2 + 2bx + c < 0$ and $4b^2 - 4ac > 0$

$$\therefore \quad ax^2 + 2bx + c < 0 \text{ and } b^2 - ac > 0 \quad \ldots\text{(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$\therefore \quad \Delta < 0$

17. (d) $T_p = aR^{p-1} = l$

$\Rightarrow \quad \log a + (p - 1)\log R = \log l$

Similarly,

$\log a + (q - 1)\log R = \log m \quad (\because T_q = m)$

and $\log a + (r - 1)\log R = \log n \quad (\because T_r = n)$

$$\therefore \begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log a + (p-1)\log R & p & 1 \\ \log a + (q-1)\log R & q & 1 \\ \log a + (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log a & p & 1 \\ \log a & q & 1 \\ \log a & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \log a \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$$

Use $C_2 \to C_2 - C_3$

$$= 0 + 0 = 0$$

18. (b) Given systems

$$x + 2ay + az = 0, x + 3by + bz = 0$$

and $x + 4cy + cz = 0$

has non-zero solution.

$$\therefore \quad \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

Appling $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

Expand w.r.t. C_1

$\Rightarrow \quad 1(3b-2a)(c-a)-(b-a)(4c-2a)=0$

$\Rightarrow \quad 3bc-3ab-2ac+2a^2-[4bc-2ab-4ac+2a^2]=0$

$\Rightarrow \quad -bc-ab+2ac=0$

$\Rightarrow \quad \frac{2ac}{a+c}=b$

Hence, a, b and c are in HP.

19. (d) Given, $A=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\therefore \quad A^2=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I,$

i.e., A is also known as involution matrix.

20. (a) Given, $A=\begin{bmatrix} 5 & 5a & a \\ 0 & a & 5a \\ 0 & 0 & 5 \end{bmatrix}$

$|A|=5(5a)=25a$

$\therefore \quad |A^2|=25 \Rightarrow |A|^2=25$

$\Rightarrow \quad |25a|^2=25$

$\Rightarrow \quad a^2=\frac{1}{25} \Rightarrow a=\frac{1}{5}$

21. (a) Given, $A=\begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}=f(x)$

$\therefore \quad |A|=1(\cos^2 x+\sin^2 x)=1$

$\operatorname{adj}(A)=\begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}^T=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{-1}=\frac{\operatorname{adj}(A)}{|A|}\begin{bmatrix} +\cos x & -\sin x & 0 \\ +\sin x & +\cos x & 0 \\ 0 & 0 & +1 \end{bmatrix}$

$=f(-x)$

22. (b) Given, $A=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow \quad A^{-1}=\frac{1}{1}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow \quad A^{-2}=\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Similarly, $A^{-n}=\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

$$\lim_{n\to\infty}\frac{A^{-n}}{n}=\lim_{n\to\infty}\begin{bmatrix} \frac{1}{n} & \frac{0}{n} \\ -1 & \frac{1}{n} \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

$=\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

23. (c) We know $\log_a x$ is defined

if $\quad 0<a<1,\ \log_a x \geq 0$, for $0<x\leq 1$

$\log_a x<0$, for $x>1$

Let $\quad f(x)=\sqrt{\log_{0.5} x}$

For $f(x)$ to be defined, $\log_{0.5} x \geq 0$

Here, base 0.5 which is less than 1.

$\therefore \quad 0<x\leq 1$

24. (c) Given, $\quad x=\log_3 5, y=\log_{17} 25$

It is clear that $x>y$.

25. (d) $\therefore$ Required number of ways $={}^5C_4 \times {}^8C_6$

$=5\times\frac{8\times 7}{2\times 1}$

$=140$

26. (c) In first place may be 2 or 3.

$\therefore$ Required number of ways $=2\times 4\times 4\times 4=128$

27. (b) Given, $\quad P(A)=0.30, P(B)=0.35, P(C)=0.20$

and $\quad P(D)=0.15$

$\therefore$ Required probability $=P(C \text{ grade})+P(B \text{ grade})$ $+P(A \text{ grade})$

$=0.20+0.35+0.30$

$=0.85$

28. (a) The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are

${}^{p+q}C_p$ and ${}^{p+q}C_q$

But $\quad {}^{p+q}C_p={}^{p+q}C_p \quad (\because {}^nC_r={}^nC_{n-r})$

29. (b) Given, $\quad \lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$

$\therefore \quad \lim_{x\to\infty}\left[\left(1+\frac{ax+b}{x^2}\right)^{\frac{x^2}{ax+b}}\right]^{\frac{2(ax+b)}{x}}=e^2$

$\Rightarrow \quad e^{\lim_{x\to\infty}\frac{2(ax+b)}{x}}=e^2$

$\Rightarrow \quad \lim_{x\to\infty}\frac{2(ax+b)}{x}=2$

$\Rightarrow \quad 2a=2$

$\Rightarrow \quad a=1$ and $b\in R$

30. (c) Given, $\quad f(x)=\int e^x (x-1)(x-2)\,dx$

On differentiating w.r.t. x, we get

$f'(x)=e^x(x-1)(x-2)$

For $f(x)$ to be decreasing, $f'(x)<0$

$\Rightarrow \quad 1<x<2$

31. (d) Given, $\quad f(x)=2x^3-9ax^2+12a^2x+1$

$\Rightarrow \quad f'(x)=6x^2-18ax+12a^2$

Put $\quad f'(x)=0$

$\Rightarrow \quad 6(x^2-3ax+2a^2)=0$

$\Rightarrow \quad (x-a)(x-2a)=0$

$\Rightarrow \quad x=a, 2a$

Now, $\quad f''(x)=12x-18a$

At $x=a$, $f''(a)=12a-18a=-6a<0$, maxima

$\therefore \quad x_1=a$

and at $x=2a$, $f''(2a)=24a-18a=6a>0$, minima

$\therefore \quad x_2=2a$

Also, given $x_2=x_1^2$

$\therefore \quad 2a=a^2 \Rightarrow a(a-2)=0 \Rightarrow a=0$ or 2

32. (d) Given, $f(x)=\begin{cases} |x^2-3x|+a, & 0\leq x\leq 3/2 \\ -2x+3, & x\geq 3/2 \end{cases}$

$=\begin{cases} 3x-x^2+a, & 0\leq x\leq 3/2 \\ -2x+3, & x\geq 3/2 \end{cases}$

$\Rightarrow \quad f'(x) = \begin{cases} 3-2x, & 0 \le x < 3/2 \\ -2, & x > 3/2 \end{cases}$

For local maxima,

LHD, $f'(x) > 0 \Rightarrow 3 - 2x > 0$

$\Rightarrow \quad x < \frac{3}{2}$

RHD, $f'(x) < 0 \Rightarrow -2 < 0$

Hence, for every value of a, $f(x)$ has local maxima at $x = 3/2$

33. (a) Given, $f(x) = \int_0^x (at^2 + 1 + \cos t)\,dt$

Here, we see that $ax^2 + 1 + \cos x$ is an increasing function.

$\therefore$ Greatest value is

$$f(3) = \int_0^3 (at^2 + 1 + \cos t)\,dt$$
$$= \left[\frac{at^3}{3} + t + \sin t\right]_0^3$$
$$= \frac{27}{3}a + 3 + \sin 3$$

Similarly, least value is

$$f(2) = \frac{8}{3}a + 2 + \sin 2$$

$\therefore$ Required difference $= f(3) - f(2)$

$$= \frac{27}{3}a + 3 + \sin 3 - \left(\frac{8}{3}a + 2 + \sin 2\right)$$
$$= \frac{19}{3}a + 1 + (\sin 3 - \sin 2)$$

34. (d) Given, $f(2-a) = f(2+a), \forall a \in R$

$\therefore$ Function is symmetrical about the line $x = 2$

$$\therefore \quad \int_{2-a}^{2+a} f(x)\,dx = 2\int_2^{2+a} f(x)\,dx$$

35. (d) Let $\quad f(t) = \frac{1}{(3t)^{2/3}}$

$$\therefore \int_0^2 \frac{dt}{\{f(t)\}^2} = \int_0^2 \frac{dt}{(3t)^{2/3}}$$
$$= \left[3\,(3t)^{1/3} \times \frac{1}{3}\right]_0^2$$
$$= 6^{1/3} = \sqrt[3]{6}$$

which is true.

$$\therefore \quad f(9) = \int_0^9 \frac{dt}{(3t)^{2/3}}$$
$$= \left[3 \times (3t)^{1/3} \times \frac{1}{3}\right]_0^9$$
$$= [(3 \times 9)^{1/3} - 0]$$
$$= 3$$

36. (d) Given $\int_0^{11} [X]^3\,dx$

$$= \int_0^1 [x]^3\,dx + \int_1^2 [x]^3\,dx + \int_2^3 [x]^3\,dx + \ldots + \int_{10}^{11} [x]^3\,dx$$
$$= \int_0^1 (0)^3\,dx + \int_1^2 (1)^3\,dx + \int_2^3 (2)^3\,dx + \ldots + \int_{10}^{11} (10)^3\,dx$$
$$= 0 \cdot (1-0) + (1)^3\,(2-1) + (2)^3\,(3-2) + \ldots + (10)^3\,(11-10)$$
$$= 0 + 1^3 + 2^3 + 3^3 + \ldots + 10^3 \qquad \left(\Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2\right)$$
$$= \left[\frac{10\,(10+1)}{2}\right]^2 = (55)^2 = 3025$$

37. (d) $\therefore$ Required area

$$= \int_0^{\pi/4} \frac{\sin x}{x}\,dx < \int_0^{\pi/4} \frac{\tan x}{x}\,dx$$

38. (a) Given, $\frac{dy}{dx} = (2x+1)$

$\Rightarrow \quad dy = (2x+1)\,dx$

On integrating, we get

$$y = x^2 + x + C$$

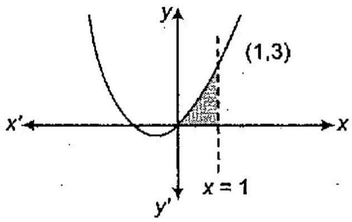
At point (1, 2)

$2 = 1 + 1 + C$

$\Rightarrow \quad C = 0$

$\therefore \quad y = x^2 + x$

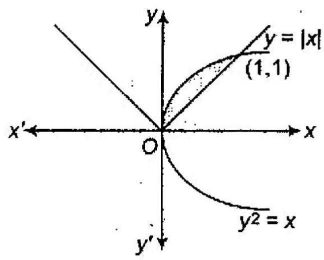
$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 = y + \frac{1}{4}$



$\therefore$ Required area $= \int_0^1 y\,dx$

$$= \int_0^1 (x^2 + x)\,dx$$
$$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$$
$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}\text{ sq unit}$$

39. (c) Given curves are $y^2 = x$ and $y = |x|$



$\therefore \quad |x|^2 = x$

$\Rightarrow \quad x = 0, 1$

$\Rightarrow \quad y = 0, 1$

$\therefore$ Required area $= \int_0^1 (y_2 - y_1)\,dx$

$$= \int_0^1 (\sqrt{x} - x)\,dx$$
$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$$
$$= \frac{2}{3} - \frac{1}{2} - 0 - 0$$
$$= \frac{1}{6}$$

40. (c) Given, $f(x) = \log_e x$, $f'(x) = \frac{1}{x}$

By using Lagrange Mean Value theorem,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
$$\frac{1}{c} = \frac{\log_e 3 - \log_e 1}{3 - 1} = \frac{\log_e 3}{2}$$

$\Rightarrow \quad c = 2\log_3 e$

41. (d) Given, $\frac{dy}{dx} = \frac{x+y}{x}$ (homogeneous equation)

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{x+vx}{x}$

$\Rightarrow \quad v + x\frac{dv}{dx} = 1 + v$

$\Rightarrow \quad dv = \frac{1}{x}dx$

On integrating both sides, we get

$v = \log x + C$

$\Rightarrow \quad \frac{y}{x} = \log x + C$

At $\quad y(1) = 1$

$\therefore \quad \frac{1}{1} = \log 1 + C \Rightarrow C = 1$

$\therefore \quad \frac{y}{x} = \log x + 1$

$\Rightarrow \quad y = x\log x + x$

Alternate method

$\frac{dy}{dx} = 1 + \frac{y}{x} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = 1$

which linear differential equation.

$\text{IF} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Solution is

$y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot 1dx + C = \log x + C$

At $(y = 1) \Rightarrow x = 1$

$\frac{1}{1} = \log 1 + C \rightarrow C = 1$

$\Rightarrow \quad y/x = \log x + 1$

$\Rightarrow \quad y = x\log x + x$

42. (c) Given, $y = C_1 e^{C_2 x}$

$\Rightarrow \quad \frac{dy}{dx} = C_1 C_2 e^{C_2 x} = C_2 y$

$\Rightarrow \quad \frac{d^2y}{dx^2} = C_2\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} \times \frac{1}{y}\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

$\Rightarrow \quad yy'' = (y')^2$

43. (a) Given, $f(x) = \frac{x}{2} + \frac{2}{x}$

$f'(x) = \frac{1}{2} - \frac{2}{x^2}$

Put $\quad f'(x) = 0$

$\Rightarrow \quad \frac{1}{2} - \frac{2}{x^2} = 0$

$\Rightarrow \quad x^2 = 4 \Rightarrow x = \pm 2$

Now, $\quad f''(x) = \frac{4}{x^3}$

At $x = 2$, $f''(2) = \frac{4}{8} > 0$, local minima

At $x = -2$, $f''(-2) = \frac{4}{-8} < 0$, local maxima

44. (d) The image of $P(a, b)$ on $y = -x$ is

$\frac{x_1 - a}{1} = \frac{y_1 - b}{1} = -\frac{2(a+b)}{1^2 + 1^2}$

$\Rightarrow \quad x_1 - a = y_1 - b = -(a+b)$

$\Rightarrow \quad x_1 = -b, y_1 = -a$

i.e., $\quad Q(-b, -a)$

Now, in image of $Q(-b, -a)$ on the line $x - y = 0$ is

$\frac{x_2 + b}{1} = \frac{y_2 + a}{-1} = -\frac{2(-b+a)}{1^2 + 1^2}$

$\Rightarrow \quad x_2 + b = \frac{y_2 + a}{-1} = b - a$

$\therefore \quad x_2 = -a, \ y_2 = -b$

$\therefore \quad R(-a, -b)$

$\therefore$ Mid-point of PR is $\left(\frac{a-a}{2}, \frac{b-b}{2}\right)$ *i.e.*, (0, 0).

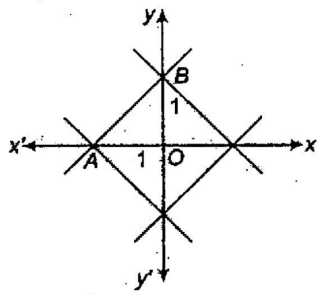
45. (c) Given, $d(x, y) = |x| + |y|$

Also, $\quad d(x, y) = 1$

$\therefore \quad 1 = |x| + |y|$

$1 = \pm x \pm y$

Hence, locus of point is a square.



In ΔAOB,

$AB^2 = 1^2 + 1^2 \Rightarrow AB = \sqrt{2}$

Area of square $= (AB)^2 = 2$ sq units

46. (a) Given pair of lines are perpendicular.

$\therefore \quad A + B = 0$

$\Rightarrow \quad 3a + (a^2 - 2) = 0$

$\Rightarrow \quad a^2 + 3a - 2 = 0$

$\Rightarrow \quad a = \frac{-3 \pm \sqrt{9+8}}{2(1)} = \frac{-3 \pm \sqrt{17}}{2}$

Hence, two values of a exist.

47. (a) Given, $2ae = 6$ and $2b = 8$

$\Rightarrow \quad ae = 3$ and $b = 4$

$\Rightarrow \quad a\sqrt{1 - \frac{b^2}{a^2}} = 3$

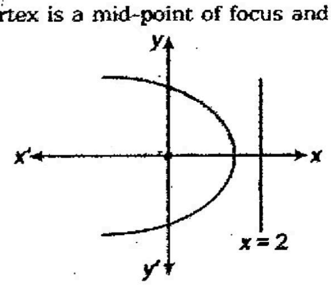
$\Rightarrow \quad a^2\left(1 - \frac{b^2}{a^2}\right) = 9$

$\Rightarrow \quad a^2 - 16 = 9 \Rightarrow a^2 = 25$

$\Rightarrow \quad a = 5$

$\therefore \quad 5 \times e = 3 \Rightarrow e = \frac{3}{5}$

48. (b) We know vertex is a mid-point of focus and directrix.



$\therefore$ Vertex of a parabola $= \left(\frac{0+2}{2}, 0\right) = (1, 0)$

49. (d) Let another equation be $ax + by$.

$\therefore \quad (3x + 4y)(ax + by) = 6x^2 - xy + 4cy^2$

$\Rightarrow \quad 3ax^2 + (4a + 3b)xy + 4by^2 = 6x^2 - xy + 4cy^2$

On equating both sides, we get

$3a = 6, 4a + 3b = -1, 4b = 4c$

$\Rightarrow \quad a = 2, 3b = -1 - 4 \times 2$

$\Rightarrow \quad b = -3$

$\therefore \quad c = b = -3$

50. (c) Given, $x^2 + y^2 + 2x + 4y - 3 = 0$

On differentiating w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} + 2 + 4\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(y + 2) = -1 - x$$

$$\Rightarrow \quad \frac{dy}{dx_{(1,0)}} = \frac{-1-1}{0+2} = -1$$

$\therefore$ Equation of normal at $P(1, 0)$ is

$$y - 0 = -\frac{1}{dy/dx}(x - 1)$$

$\Rightarrow \quad y = 1(x - 1)$

$\Rightarrow \quad x - y = 1$, which is the equation of diameter.

$\therefore$ Point of intersection of $x - y = 1$ and

$$x^2 + y^2 + 2x + 4y - 3 = 0 \text{ is}$$

$$x^2 + (x - 1)^2 + 2x + 4(x - 1) - 3 = 0$$

$$\Rightarrow \quad x^2 + x^2 - 2x + 1 + 2x + 4x - 4 - 3 = 0$$

$$\Rightarrow \quad 2x^2 + 4x - 6 = 0$$

$$\Rightarrow \quad x^2 + 2x - 3 = 0$$

$$\Rightarrow \quad (x + 3)(x - 1) = 0 \Rightarrow x = 1, -3$$

$$\therefore \quad y = 0, y = -4$$

Hence, other point is $(-3, -4)$.

Alternate method

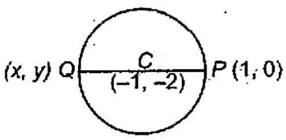
The equation of circle is $x^2 + y^2 + 2x + 4y - 3 = 0$

Centre $C = (-1, -2)$ and given one point $P(1, 0)$ of the diameter of circle. Let (x, y) be the other point of the diameter.

Then, $\frac{x+1}{2} = -1, \frac{y+0}{2} = -2$

$\Rightarrow \quad x = -3, y = -4$

Hence, $\quad = (-3, -4)$



51. (a) Since, the line lies on the given plane.

It means the given plane passing through $(2, 1, -2)$.

$\therefore \quad 2 + 3(1) - \alpha(-2) + \beta = 0$

$\Rightarrow \quad 2\alpha + \beta = -5 \quad \ldots(i)$

Also, normal to the plane is perpendicular to the line.

$\therefore \quad 1 \times 3 + 3 \times (-5) - \alpha \times (2) = 0$

$\Rightarrow \quad 2\alpha = -12 \Rightarrow \alpha = -6$

$\therefore$ From Eq. (i),

$2 \times (-6) + \beta = -5$

$\Rightarrow \quad \beta = 7$

$\therefore \quad (\alpha, \beta) = (-6, 7)$

52. (b) Here, $\mathbf{F} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k} + 3\mathbf{i} + \mathbf{j} - \mathbf{k}$

$= 7\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

$\mathbf{d} = 5\mathbf{i} + 4\mathbf{j} + \mathbf{k} - (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$

$= 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

$\therefore$ Work done $= \mathbf{F} \cdot \mathbf{d}$

$= (7\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$= 28 + 4 + 8$

$= 40$ units

53. (a) Given, $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$

and $\quad \mathbf{c} = x\mathbf{i} + (x - 2)\mathbf{j} - \mathbf{k}$

Since, the vector $\mathbf{c}$ lies in the plane of $\mathbf{a}$ and $\mathbf{b}$.

$\therefore \quad \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$ *i.e.*, coplanar

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

$$1(1 - 2x + 4) - 1(-1 - 2x) + 1(x - 2 + x) = 0$$

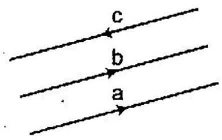
$$\Rightarrow \quad 5 - 2x + 1 + 2x + 2x - 2 = 0$$

$$\Rightarrow \quad 2x + 4 = 0$$

$$\Rightarrow \quad x = -2$$

54. (c) Given, $\mathbf{a} = 8\mathbf{b}$ and $\mathbf{c} = -7\mathbf{b}$

Since, $\mathbf{a}$ and $\mathbf{c}$ are parallel vectors but opposite in direction.



Hence, angle between $\mathbf{a}$ and $\mathbf{c}$ is $180°$.

55. (a) Given, $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 3$ and $\mathbf{a} \cdot \mathbf{b} = 0$

$\therefore |\mathbf{a} \times (\mathbf{a} \times (\mathbf{a} \times (\mathbf{a} \times \mathbf{b})))|$

$= |\mathbf{a} \times (\mathbf{a} \times [(\mathbf{a} \cdot \mathbf{b})\,\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\,\mathbf{b}])|$

$= |\mathbf{a} \times (\mathbf{a} \times [0 - \mathbf{b}\,|\mathbf{a}|^2])| = |\mathbf{a} \times (\mathbf{a} \times (-4\mathbf{b}))|$

$= 4\,|\mathbf{a} \times (\mathbf{a} \times \mathbf{b})| = 4\,|(\mathbf{a} \cdot \mathbf{b})\,\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\,\mathbf{b}|$

$= 4\,|0 - \mathbf{b}|\mathbf{a}|^2| = |4\,|-4\,\mathbf{b}| = 4 \times 4 \times |\mathbf{b}|$

$= 4 \times 4 \times 3 = 48$

56. (a) Given, $\sin(\pi \cos x) = \cos(\pi \sin x)$

$$\therefore \quad \sin(\pi \cos x) = \sin\left(\frac{\pi}{2} + \pi \sin x\right)$$

$$\Rightarrow \quad \pi \cos x = \frac{\pi}{2} + \pi \sin x$$

$$\Rightarrow \quad \cos x - \sin x = \frac{1}{2}$$

$$\Rightarrow \quad \cos^2 x + \sin^2 x - 2 \sin x \cos x = \frac{1}{4}$$

$$\Rightarrow \quad 1 - \frac{1}{4} = \sin 2x \Rightarrow \frac{3}{4} = \sin 2x$$

$$\Rightarrow \quad x = \frac{1}{2}\sin^{-1}\frac{3}{4}$$

57. (c) Given, $\cot^{-1}[\sqrt{\cos \alpha}] + \tan^{-1}[\sqrt{\cos \alpha}] = x$

$$\therefore \quad \tan^{-1}\left[\frac{1}{\sqrt{\cos \alpha}}\right] + \tan^{-1}[\sqrt{\cos \alpha}] = x$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\frac{1}{\sqrt{\cos \alpha}} + \sqrt{\cos \alpha}}{1 - \frac{1}{\sqrt{\cos \alpha}} \times \sqrt{\cos \alpha}}\right] = x$$

$$\Rightarrow \quad \tan^{-1}(\infty) = x$$

$$\Rightarrow \quad x = \frac{\pi}{2}$$

$$\therefore \sin \quad x = \sin\frac{\pi}{2} = 1$$

Alternate method We know that,

$$\tan^{-1} x + \cot^{-1} x = \frac{\lambda}{2} : x \in R$$

$$\therefore \quad x = \cot^{-1}\sqrt{\cos \alpha} + \tan^{-1}\sqrt{\cos \alpha} = \frac{\pi}{2}$$

$$\Rightarrow \sin x = \sin\frac{\pi}{2} = 1$$

58. (d) Given, $f(x) = \tan^{-1}(\sin x + \cos x)$

$\therefore \quad f(x) = \tan^{-1}\left(\frac{1}{\sqrt{2}}\sin\left(x + \frac{\pi}{4}\right)\right)$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{1}{1 + \frac{1}{2}\sin^2\left(x + \frac{\pi}{4}\right)}\left[\frac{1}{\sqrt{2}}\cos\left(x + \frac{\pi}{4}\right)\right]$$

For $f(x)$ to be increasing function,

$\therefore \quad f'(x) > 0$

$\Rightarrow \quad \cos\left(x + \frac{\pi}{4}\right) > 0$

$\Rightarrow \quad x \in \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$

Hence, option (d) satisfies.

59. (c) Let $P(h, k)$ be the point of intersection of tangents at points $Q(t_1^2, 2t_1)$ and $R(t_2^2, 2t_2)$ on the parabola $y^2 = 4x$.

Then, $h = t_1 t_2$ and $k = t_1 + t_2$. Equation of tangents at Q and R are $t_1 y = x + t_1^2$ and $t_2 y = x + t_2^2$ respectively.

Since, two lines will be perpendicular.

$\therefore \quad \frac{1}{t_1} \times \frac{1}{t_2} = -1 \Rightarrow t_1 t_2 = -1$

$\therefore \quad h = -1$

Hence, locus of a point is $x = -1$.

60. (a) Let $f(x) = \log x$

$\therefore \quad \lim_{x \to \infty} \frac{f(3x)}{f(x)} = \lim_{x \to \infty} \frac{\log 3x}{\log x} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$

By L' Hospital rule

$$= \lim_{x \to \infty} \frac{\frac{1}{3x} \times 3}{\frac{1}{x}} = 1, \text{ which is true.}$$

$\therefore \quad \lim_{x \to \infty} \frac{f(2x)}{f(x)} = \lim_{x \to \infty} \frac{\log 2x}{\log x}$

$$= \lim_{x \to \infty} \frac{\frac{1}{2x} \times 2}{\frac{1}{x}} = 1$$

61. (a)	**62.** (b)	**63.** (d)	**64.** (d)	**65.** (c)	**66.** (b)
67. (b)	**68.** (d)	**69.** (a)	**70.** (d)	**71.** (b)	**72.** (c)
73. (a)	**74.** (c)	**75.** (c)	**76.** (a)	**77.** (d)	**78.** (b)
79. (a)	**80.** (b)	**81.** (d)	**82.** (b)	**83.** (d)	**84.** (c)
85. (b)	**86.** (d)	**87.** (a)	**88.** (a)	**89.** (b)	**90.** (b)

91. (a) Let the speed of flight be x km/h.

Then, $\quad \frac{600}{x - 200} - \frac{600}{x} = \frac{30}{60}$

$\Rightarrow \quad \frac{600 \times x - 600(x - 200)}{x(x - 200)} = \frac{1}{2}$

$\Rightarrow \quad \frac{600x - 600x + 120000}{x(x - 200)} = \frac{1}{2}$

$\Rightarrow \quad \frac{120000}{x^2 - 200x} = \frac{1}{2}$

$\Rightarrow \quad x^2 - 200x = 240000$

$\Rightarrow \quad x^2 - 200x - 240000 = 0$

$\Rightarrow \quad x = \frac{+200 \pm \sqrt{(-200)^2 - 4 \times 1 \times (-240000)}}{2 \times 1}$

$\Rightarrow \quad x = \frac{+200 \pm \sqrt{40000 + 960000}}{2}$

$\Rightarrow \quad x = \frac{+200 \pm \sqrt{1000000}}{2}$

$\Rightarrow \quad x = \frac{+200 \pm 1000}{2}$

$\Rightarrow \quad x = 600$ km/h

$\therefore$ Duration of the flight $= \frac{600}{600}$

$= 1$h

92. (b) A's 1 day work $= \frac{1}{6}$

B's 1 day work $= \frac{1}{8}$

$\therefore C$'s 1 day work $= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right)$

$= \frac{1}{3} - \left(\frac{4 + 3}{24}\right)$

$= \frac{1}{3} - \frac{7}{24}$

$= \frac{1}{24}$

$\therefore$ Ratio of work among A, B and C

$= \frac{1}{6} : \frac{1}{8} : \frac{1}{24}$

$= 4 : 3 : 1$

Hence, C's amount $= \frac{1}{(4 + 3 + 1)} \times 3200$

$=$ ₹ 400

93. (b) Let the speed of two trains be x m/s and y m/s respectively.

$\therefore$ Length of first train $= 27x$ m

and length of second train $= 17y$ m

Then, $\quad \frac{27x + 17y}{x + y} = 23$

$\Rightarrow \quad 27x + 17y = 23x + 23y$

$\Rightarrow \quad 4x = 6y$

$\Rightarrow \quad \frac{x}{y} = \frac{3}{2}$

$\Rightarrow \quad x : y = 3 : 2$

94. (c) Ravi's 1 h work $= \frac{32}{6}$ pages

Kumar's 1 h work $= \frac{40}{5}$ pages

$= 8$ pages

(Ravi + Kumar)'s 1 h work $= \frac{32}{6} + 8$

$= \frac{80}{6}$

$= \frac{40}{3}$ pages

$\because \frac{40}{3}$ pages will be completed in 1 h

$\therefore$ 110 pages will be completed in $\frac{3}{40} \times 110 = 8\frac{1}{4}$ h

$= 8$ h 15 min

95. (b) Simple interest $= \frac{P \times r \times t}{100}$

$81 = \frac{450 \times 4.5 \times t}{100}$

$\Rightarrow \quad t = \frac{81 \times 100}{450 \times 4.5}$

$\Rightarrow \quad t = 4$ yr

96. (a) Given, captain's age $= 26$ yr

Wicket keeper's age $= 26 + 3$

$= 29$ yr

Suppose, the average age of the team = x years

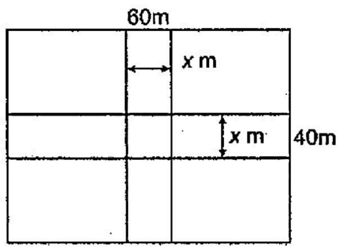
Then, $11x - 26 - 29 = 9(x-1)$

$\Rightarrow$ $11x - 26 - 29 = 9x - 9$

$\Rightarrow$ $2x = 46$

$\Rightarrow$ $x = 23$ yr

97. (b) Suppose, width of the road = x metres



Area of the rectangular park $= 60 \times 40$
$= 2400\ m^2$

Area of the two concrete cross roads
$= 60 \times x + 40 \times x - x \times x$
$= 60x + 40x - x^2$
$= 100x - x^2$

Then,

$$2400 - (100x - x^2) = 2109$$

$\Rightarrow$ $2400 - 100x + x^2 = 2109$

$\Rightarrow$ $x^2 - 100x + 2400 - 2109 = 0$

$\Rightarrow$ $x^2 - 100x + 291 = 0$

$\Rightarrow$ $x^2 - 3x - 97x + 291 = 0$

$\Rightarrow$ $x(x-3) - 97(x-3) = 0$

$\Rightarrow$ $(x-97)(x-3) = 0$

$\Rightarrow$ $x = 97$ (not valid) and $x = 3$ m

98. (d) Pipe A, 1 min work $= \frac{1}{15}$

Pipe B, 1 min work $= \frac{1}{20}$

Pipe $(A + B)$, 4 min work $= 4 \times \left(\frac{1}{15} + \frac{1}{20}\right)$

$= 4 \times \left(\frac{4+3}{60}\right)$

$= 4 \times \frac{7}{60}$

$= \frac{7}{15}$

Remaining work $= 1 - \frac{7}{15}$

$= \frac{8}{15}$

$\because$ Pipe B fills $\frac{1}{20}$ part in 1 min

$\therefore$ Pipe B fills $\frac{8}{15}$ part in $20 \times \frac{8}{15} = 10$ min 40 s

Hence, required time = 4 min + 10 min 40 s
= 14 min 40 s

99. (c) Face value = 6000

Numeral value = 6

Hence, required difference = 6000 − 6
= 5994

100. (c) Required amount $= P\left(1 + \frac{r}{100}\right)^t$

$= 8000\left(1 + \frac{5}{100}\right)^2$

$= 8000\left(\frac{21}{20}\right)^2$

$= 8000 \times \frac{21}{20} \times \frac{21}{20} =$ ₹ 8820

101. (c) Profit ratio among A, B and C

$= 3x \times 12 : 5x \times 12 : 5x \times 6$
$= 36x : 60x : 30x$
$= 6x : 10x : 5x$
$= 6 : 10 : 5$

102. (c) Boatman upstream speed = 2 km/h

Boatman downstream speed $= \frac{1}{10} \times 60$
= 6 km/h

$\therefore$ Stationary water speed $= \frac{1}{2}(6 + 2)$ km/h
= 4 km/h

Hence, required time $= \frac{5}{4}$ h

$= \frac{5}{4} \times 60 = 75$ min

= 1h 15 min

103. (b) Let the number of revolutions made by the larger wheel be x.

Then, $6 \times 21 = 14 \times x$

$\Rightarrow$ $x = \frac{6 \times 21}{14}$ $\Rightarrow$ $x = 9$

104. (c) According to Amit ⟶ 11, 12, 13, 14 km

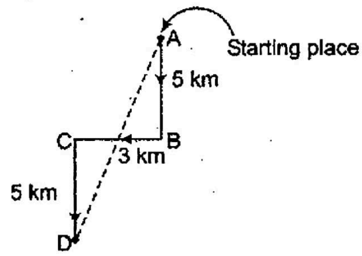
According to Sunita ⟶ 13 km

Hence, required distance = 13 km

105. (c) Required number of bricks $= \frac{8 \times 100 \times 6 \times 100 \times 22.5}{25 \times 11.25 \times 6}$

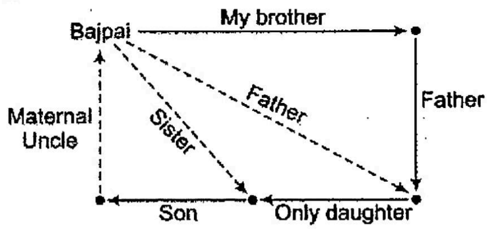
= 6400

106. (d) Man's walking direction are as follows



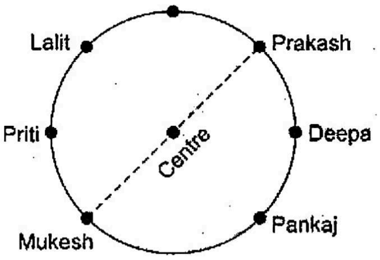
Hence, man in South-West direction from his starting place.

107. (d)



Hence, Bajpai is related as maternal uncle of Man.

Solutions (Q. Nos. 108-111)
Six friends seating arrangement is as follows



108. (a) Mukesh

109. (a) Deepa

110. (c) Priti and Pankaj

111. (b) Deepa

112. (b) The meaningful sequence of the words are
Foundation → Walls → Windows → Roof → Floor → Room
(4) (2) (1) (5) (3) (6)

113. (c) As,

$$\begin{array}{cccc} G & O & L & D \\ +1\downarrow & +0\downarrow & +1\downarrow & +1\downarrow \\ H & O & M & E \end{array} \quad \begin{array}{cccc} C & O & M & E \\ +1\downarrow & +0\downarrow & +1\downarrow & +0\downarrow \\ D & O & N & E \end{array} \text{ and } \begin{array}{cccc} C & O & R & D \\ +1\downarrow & +0\downarrow & +1\downarrow & +1\downarrow \\ D & O & S & E \end{array}$$

Similarly,

$$\begin{array}{cccc} S & O & N & S \\ +1\downarrow & +0\downarrow & +1\downarrow & +0\downarrow \\ T & O & O & S \end{array}$$

114. (b) The pattern of series is

$$2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \xrightarrow{\times 3} 54 \xrightarrow{\times 3} 162 \xrightarrow{\times 3} 486 \xrightarrow{\times 3} 1458 \xrightarrow{\times 3} 4374$$

115. (d) In 'CORPORATION' word vowels are (A, I, O, O , O) and consonants are (C, R, P, R, T, N)

$\therefore$ Required number of ways $= \dfrac{7!}{2!} \times \dfrac{5!}{3!}$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{2 \times 1} \times \frac{5 \times 4 \times 3!}{3!} = 50400$$

(Here, we consider vowels as one unit)

116. (a) We know that volume of a cylinder $= \pi r^2 h$

Hence, required ratio $= \dfrac{\pi \cdot \left(\frac{3}{2}\right)^2 \times 1}{\pi \cdot \left(\frac{1}{2}\right)^2 \times 3}$

$$= \frac{9/4 \times 1}{1/4 \times 3} = \frac{9}{3} = 3:1$$

117. (d) $\sqrt{12} + 2\sqrt{48} + 5\sqrt{147} - 45\sqrt{3}$

$$= \sqrt{4 \times 3} + 2\sqrt{16 \times 3} + 5\sqrt{49 \times 3} - 45\sqrt{3}$$
$$= 2\sqrt{3} + 8\sqrt{3} + 35\sqrt{3} - 45\sqrt{3}$$
$$= 45\sqrt{3} - 45\sqrt{3}$$
$$= 0$$

118. (b) From number 1 to 20, number divisible by 3 or 5
$= 3, 5, 6, 9, 10, 12, 15, 18, 20 = 9$

Hence, required probability $= \dfrac{9}{20}$

119. (a) The pattern of series is

$$B \xrightarrow{+3} E \xrightarrow{+3} H \xrightarrow{+3} K \xrightarrow{+3} N$$
$$X \xrightarrow{-4} T \xrightarrow{-4} P \xrightarrow{-4} L \xrightarrow{-4} H$$
$$J \xrightarrow{+2} L \xrightarrow{+2} N \xrightarrow{+2} P \xrightarrow{+2} R$$

120. (b) Time from 5 am of a particular day to 10 pm on the 4th day is 89 h. Now, the clock loses 16 min in 24 h or in other words we can say that 23 h 44 min of this clock is equal to 24 h of the correct clock.

or $\left(23 + \dfrac{44}{60}\right) \Rightarrow \dfrac{356}{15}$ h of this clock $= 24$ h of the correct clock.

$\therefore$ 89 h of this clock

$$= \left(\frac{24 \times 15}{356} \times 89\right) \text{h of the correct clock}$$
$$= 90 \text{ h of the correct clock}$$

or 89 h of this clock $= 90$ h of the correct clock. Therefore, it is clear that in 89 h. This clock loses 1 h and hence, the correct time is 11 pm when this clock shows 10 pm.