KIITEE MCA

Solved Paper 2012

Mathematics

1. The function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is

(a) an odd function (b) an even function
(c) a periodic function (d) None of these

2. Consider the following relations

$R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational numbers $w\}$; $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \mid m, n, p\right.$ and q are integers such that $n, q \neq 0$ and $\left. qm = pm\right\}$. Then,

(a) R is an equivalence relation but S is not an equivalence relation
(b) Neither R nor S is an equivalence relation
(c) S is an equivalence relation but R is not an equivalence relation
(d) R and S both are equivalence relation

3. If $f : [0, \infty) \to [0, \infty)$ and $f(x) = \frac{x}{1+x}$, then f is

(a) one-one and onto
(b) one-one but not onto
(c) onto but not one-one
(d) Neither one-one nor onto

4. $\lim_{h \to 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$, given that $f'(2) = 6$ and $f'(1) = 4$

(a) does not exist (b) is equal to $-3/2$
(c) is equal to $3/2$ (d) is equal to 3

5. Range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}$; $x \in R$ is

(a) $(1, \infty)$ (b) $\left(1, \frac{11}{7}\right)$
(c) $\left(1, \frac{7}{3}\right]$ (d) $\left(1, \frac{7}{5}\right)$

6. The conjugate of a complex number is $\frac{1}{i-1}$, then that complex number is

(a) $\frac{1}{i-1}$ (b) $-\frac{1}{i-1}$
(c) $\frac{1}{i+1}$ (d) $\frac{-1}{i+1}$

7. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

(a) the imaginary axis (b) an ellipse
(c) a circle (d) the real axis

8. If z is a complex number such that $|z| = 1$, $z \neq 1$, then real part of $\frac{z-1}{z+1}$ is

(a) $\frac{1}{|z+1|^2}$ (b) $\frac{1}{|z+1|^2}$
(c) $\frac{\sqrt{2}}{|z+1|^2}$ (d) 0

9. Locus of point z, so that z, i and iz are collinear, is

(a) a straight line (b) a circle
(c) an ellipse (d) a rectangular hyperbola

10. The number of roots of the equation $|z|^2 - 5|z| + 1 = 0$ is

(a) 0 (b) 2
(c) 4 (d) infinite

11. Consider the system of linear of equations

$$x_1 + 2x_2 + x_3 = 3,$$
$$2x_1 + 3x_2 + x_3 = 3$$

and $3x_1 + 5x_2 + 2x_3 = 1$, the system has

(a) infinite number of solutions
(b) exactly 3 solutions
(c) a unique solution
(d) no solution

12. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(a) atleast 7 (b) 5
(c) 6 (d) less than 4

13. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $\det A^2 = 25$, then $|\alpha|$ is

(a) 1 (b) 1/5
(c) 5 (d) 5^2

14. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define Tr(A) = Sum of diagonal elements of A and $|A|$ = Determinant of A.
Statement I : tr(A) = 0; Statement II : $|A| = 1$.

(a) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

15. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2005} P$ is equal to

(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2005 \\ 2005 & 2005 \end{bmatrix}$

16. The largest integer k such that 3^k divides $2^{3^n} + 1$, $n \in N$, is

(a) 2 (b) n
(c) $n-1$ (d) $n+1$

17. The remainder left out when $8^{2n} - 62^{(2n+1)}$ is divided by 9 is

(a) 0 (b) 2
(c) 7 (d) 8

18. The letters of the word 'COCHIN' are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word 'COCHIN' is

(a) 360 (b) 192 (c) 96 (d) 48

19. For positive integer n, $10^{n-2} > 81n$, if

(a) $n > 5$ (b) $n \geq 5$
(c) $n < 5$ (d) $n > 6$

20. For all integers $n \geq 1$, which of the following is divisible by 9?

(a) $8^n + 1$ (b) $3^{2n} + 3n + 1$
(c) $4^n - 3n - 1$ (d) None of these

21. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

(a) $x = -a$ (b) $x = -\frac{a}{2}$
(c) $x = 0$ (d) $x = \frac{a}{2}$

22. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) 4

23. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then c is equal to

(a) $\frac{(a_2^2 + b_2^2 - a_1^2 - b_1^2)}{2}$ (b) $(a_1^2 - a_2^2 + b_1^2 - b_2^2)$
(c) $\frac{(a_1^2 + a_2^2 + b_1^2 - b_2^2)}{2}$ (d) $\sqrt{a_1^2 + b_1^2 - a_2^2 + b_2^2}$

24. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then, the locus of the centroid of the ΔPAB as P moves on the circle is

(a) a parabola (b) a circle
(c) an ellipse (d) a pair of straight lines

25. The number of integer values of m for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

(a) 2 (b) 0
(c) 4 (d) 10

26. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$
(c) $-\frac{2}{9}$ (d) $-\frac{3}{2}$

27. If $2x - y + 2z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$, then the value of λ such that the system of equations has no solution, is

(a) 3 (b) 1
(c) 0 (d) -3

28. Domain of definition of the function $f(x) = \sqrt{\sin^{-1} 2x + \frac{\pi}{6}}$ for real valued x is

(a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(c) $\left[-\frac{1}{2}, \frac{1}{9}\right]$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

29. If $\lim_{x \to 0} \frac{[(a-n)nx - \tan x]\sin x}{x^2} = 0$, where n is a non-zero real number, then a is equal to

(a) 0 (b) $\frac{n+1}{n}$
(c) n (d) $n + \frac{1}{n}$

30. The number of values of x in $[0, 3\pi]$ such that $2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 1 (b) 2
(c) 4 (d) 6

31. If $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$, then $\mathbf{b}$ is

(a) $\mathbf{i} - \mathbf{j} + \mathbf{k}$ (b) $2\mathbf{j} - \mathbf{k}$
(c) $\mathbf{i}$ (d) $2\mathbf{i}$

32. The value of 'a' so that the volume of the parallelopiped formed by $\mathbf{i} + a\mathbf{j} + \mathbf{k}$, $\mathbf{j} + a\mathbf{k}$ and $a\mathbf{i} + \mathbf{k}$ becomes minimum, is

(a) -3 (b) 3
(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

33. Let $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 3\mathbf{k}$. If $\mathbf{u}$ is unit vector, then the maximum value of the scalar triple product $[\mathbf{uvw}]$ is

(a) -1 (b) $\sqrt{10} + \sqrt{6}$
(c) $\sqrt{59}$ (d) $\sqrt{60}$

34. If $\mathbf{a}$ and $\mathbf{b}$ are two unit vectors such that $\mathbf{a} + 2\mathbf{b}$ and $5\mathbf{a} - 4\mathbf{b}$ are perpendicular to each other, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is

(a) 45°
(b) 60°
(c) $\cos^{-1}\left(\frac{1}{3}\right)$
(d) $\cos^{-1}\left(\frac{2}{7}\right)$

35. If the vectors $\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$, $\mathbf{c} = \lambda\mathbf{i} + \mathbf{j} + \mu\mathbf{k}$ are mutually orthogonal, then (λ, μ) is equal to

(a) $(-3, 2)$ (b) $(2, -3)$
(c) $(-2, 3)$ (d) $(3, -2)$

36. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx$, where $a > 0$, is

(a) π (b) $a\pi$
(c) $\frac{\pi}{2}$ (d) 2π

37. $\frac{d^2x}{dy^2}$ is equal to

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
(c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

38. Which of the following functions is differentiable at $x = 0$

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$
(c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$

39. If $D^* f(x) = \lim_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}$, where $f^2(x) = \{f(x)\}^2$, then $D^*(\tan x)$ is equal to

(a) $\sec^2 x$ (b) $2\sec^2 x$
(c) $\tan x \sec^2 x$ (d) $2\tan x \sec^2 x$

40. Let $f(x) = \int e^x (x-1)(x-2)\, dx$. Then, f decreases in the interval

(a) $(-\infty, -2)$ (b) $(-2, -1)$
(c) $(1, 2)$ (d) $(2, \infty)$

41. $\int \left(\frac{\log x - 1}{1 + (\log x)^2}\right)^2 dx$ is equal to

(a) $\frac{xe^x}{1+x^2} + C$ (b) $\frac{x}{(\log x)^2 + 1} + C$
(c) $\frac{\log x}{(\log x)^2 + C}$ (d) $\frac{x}{x^2+1} + C$

42. Let $f(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t} dt$. Then, $f(e)$ equals to

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) 2

43. If y is a function of x and $\log(x + y) = 2xy$, then the value of $y'(0)$ is equal to

(a) 1 (b) -1 (c) 2 (d) 0

44. If $f(x)$ is differentiable and $\int_0^{t^2} xf(x)\, dx = \frac{2}{5} t^5$, then $f\left(\frac{4}{25}\right)$ equals to

(a) $\frac{2}{5}$ (b) $-\frac{5}{2}$ (c) 1 (d) $\frac{5}{2}$

45. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sin \sqrt{t}\, dt}{x^3}$ is

(a) 0 (b) 2/9
(c) 1/3 (d) 2/3

46. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines family of circles with

(a) variable radii and a fixed centre at (0, 1)
(b) variable radii and a fixed centre at (0, − 1)
(c) fixed radius 1 and variable centres along the x-axis
(d) fixed radius 1 and variable centres along the y-axis

47. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4, is

(a) $\frac{1}{15}$ (b) $\frac{14}{15}$
(c) $\frac{1}{5}$ (d) $\frac{4}{5}$

48. If $P(B) = \frac{3}{4}, P(A \cap B \cap \overline{C}) = \frac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$, then $P(B \cap C)$ is

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$
(c) $\frac{1}{15}$ (d) $\frac{1}{9}$

49. How many different nine-digit numbers can be formed from the number 223355888 by rearranging it's digits, so that the odd digits occupy even positions?

(a) 16 (b) 36 (c) 60 (d) 180

50. Suppose n (≥ 3) persons are sitting in a row. Two of them are selected at random. The probability that they are not together, is

(a) $1-\frac{2}{n}$ (b) $\frac{2}{n-1}$
(c) $1-\frac{1}{n}$ (d) None of these

51. The function $f: R \to R$ given by $f(x) = 3 - 2\sin x$ is

(a) one-one (b) onto
(c) bijective (d) None of these

52. Which of the following functions is an odd function?

(a) $f(x) = \cos x$ (b) $y = 2^{-x^2}$
(c) $y = 2^{x - x^4}$ (d) None of these

53. For real x, let $f(x) = x^3 + 5x + 1$, then

(a) f is one-one but not onto R
(b) f is onto R but not one-one
(c) f is one-one and onto R
(d) f is neither one-one nor onto R

54. The number i^i is

(a) real and positive (b) real and negative
(c) pure imaginary (d) None of these

55. Let $\arg(\overline{z}) + \arg(-z) = \begin{cases} \pi, & \text{if } \arg(z) < 0 \\ -\pi, & \text{if } \arg(z) > 0 \end{cases}$

The value of

$\sqrt{\{\arg(z) + \arg(-\overline{z}) - 2\pi\}\{\arg(-z) + \arg(\overline{z})\}}$

$\forall z = x + iy, x, y > 0$ is

(a) π (b) $-\pi$
(c) 0 (d) Not defined

56. Let $z = \cos\theta + i\sin\theta$. Then, the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is

(a) $\frac{1}{\sin 2°}$

(b) $\frac{1}{2\sin 2°}$

(c) $\frac{1}{6\sin 2°}$

(d) $\frac{1}{4\sin 2°}$

57. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the value of k is

(a) $\frac{1}{8}$ (b) 8
(c) 4 (d) $\frac{1}{4}$

58. The number of complex numbers satisfying the equation $|z| = 2$ and $|z| = |z-1|$ is

(a) 2 (b) 1
(c) 0 (d) infinite

59. Let $f: R \to R$ be such that $f(1) = 3$ and $f^1(1) = 6$. Then, $\lim_{x \to 0} \left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals to

(a) 1 (b) $\frac{1}{e^2}$
(c) e^2 (d) e^3

60. If $z = 4 + i\sqrt{7}$, then the value of $z^3 - 4z^2 - 9z + 91$ equals to

(a) 0 (b) 1 (c) 2 (d) −1

Computer Awareness

61. The ALU of computer normally contains a number of high speed storage elements called

(a) semiconductor memory (b) registers
(c) hard disk (d) magnetic disk

62. Offline device is

(a) a device which is not connected to CPU
(b) a device which is connected to CPU
(c) a direct access storage device
(d) an I/O device

63. A proxy server is used for which of the following?

(a) To provide security against unauthorised users
(b) To process client requests for web pages
(c) To process client requests for database access
(d) To provide TCP/IP

64. When data changes in multiple list and all lists are not updated, this causes

(a) data redundancy (b) information overload
(c) duplicate data (d) data inconsistency

65. Which of the following would most likely not be a symptom of a virus?

(a) Existing program files and icons disappear
(b) The CD-ROM stops functioning
(c) The web browser opens to an unusual home page
(d) Odd messages or images are displayed on the screen

66. Any data or instruction entered into the memory of a computer is considered as

(a) storage (b) output
(c) input (d) information

67. The computer code for interchange of information between terminals is

(a) ASCII (b) BCD (c) BCDIC (d) Hollerith

68. The two broad categories of software are

(a) word processing and spreadsheet
(b) transaction and application
(c) windows and mac OS
(d) system and application

69. A peripheral device used in a word processing system is

(a) floppy disk (b) magnetic card reader
(c) CRT (d) All of these

70. The analytical engine developed during first generation of computers used as a memory unit

(a) RAM (b) floppies
(c) counter wheels (d) punch cards

71. The daily processing of corrections to customer accounts best exemplifies the processing mode of

(a) time sharing (b) real time processing
(c) batch processing (d) offline processing

72. Which of the following terms could be used to describe the concurrent processing of computer programs via CRTs, on one computer system?

(a) Time sharing (b) Online processing
(c) Interactive processing (d) All of these

73. The most widely used commercial programming computer language is

(a) BASIC (b) COBOL
(c) FORTRAN (d) PASCAL

74. Which term identifies a specific computer on the web and the main page of the entire site?

(a) URL (b) Website address
(c) Hyperlink (d) Domain name

75. In excel, any set of characters containing a letter hyphen or space is considered

(a) a formula (b) a text
(c) a name (d) a title

76. Computers can be classified in which of the hierarchical orders?

(a) PC, Large, Super Micro, Super Computer
(b) Super Micro, PC, Large, Super Computer
(c) PC, Super Micro, Large, Super Computer
(d) Large, Super Micro, Super Computer, PC

77. In the binary language, each letter of the alphabet, each number and each special character is made up of a unique combination of

(a) eight bytes (b) eight kilobytes
(c) eight characters (d) eight bits

78. Which part of operating system manages the essential peripherals, such as the keyboard, screen, disk drives and parallel and serial port?

(a) Basic input / Output system
(b) Secondary input / Output system
(c) Peripheral input / Output system
(d) Marginal input / Output system

79. Word processing spreadsheet and photo editing are examples of

(a) application software
(b) system software
(c) operating system software
(d) platform software

80. The ability to recover and read deleted or damaged files from a criminal's computer is an example of law enforcement speciality called

(a) robotics (b) simulation
(c) computer forensics (d) animation

81. The most frequently used instructions of a computer programme are likely to be fetched from

(a) the hard disk (b) cache memory
(c) RAM (d) registers

82. Two or more computers connected to each other for sharing information form a

(a) server (b) modem
(c) switch (d) network

83. Unwanted repetitious messages, such as unsolicited bulk e-mail is known as

(a) spam (b) trash
(c) calibri (d) courier

84. A set of rules that computer on a network use to communicate with each other are called

(a) regulation (b) data flow
(c) protocol (d) network connectivity

85. Which of the following contains data descriptions and defines the name, data type and length of each field in the database?

(a) Data dictionary (b) Data table
(c) Data record (d) Data field

86. To delete an incorrect character in a document, which of the following is performed to erase to the right of the insertion point?

(a) Press the left mouse key
(b) Double click the right mouse key
(c) Press the Backspace key
(d) Press the delete key

87. The blinking symbol on the computer screen is called the

(a) mouse (b) screen saver
(c) cursor (d) keyboard

88. Software, such as viruses, worms and trojan horses, that has a malicious intent, is known as

(a) spyware (b) adware
(c) spam (d) malware

89. After a picture has been taken with a digital camera and processed appropriately, the actual print of the picture is considered

(a) data (b) output
(c) input (d) the process

90. The PC (Pesonal Computer) and the Apple Macintosh are examples of two different

(a) platforms
(b) applications
(c) programs
(d) storage devices

Analytical Ability & Logical Reasoning

91. Free notebooks were distributed equally among children of a class. The number of notebooks each child got was one-eighth of the number of children. Had the number of the children been half, each child would have got 16 notebooks. How many notebooks were distributed in total?

(a) 512 (b) 500 (c) 450 (d) 412

92. A man works for 2 days and then rests for one day, then works for 2 days and rests for one day and so on. For everyday he works and earns ₹ 100. How much will he earn from Monday to Saturday?

(a) ₹ 200 (b) ₹ 300 (c) ₹ 400 (d) ₹ 500

93. in climbing a round pole of 80 m height, a monkey climbs 5 m in a minute and slips 2 m in the alternate minute. To get to the top of the pole, the monkey would take

(a) 51 min (b) 54 min (c) 58 min (d) 61 min

94. Ramesh and Kunal start walking to meet one another from places 25 km apart. If Ramesh walks at the rate of 2 km/h and Kunal at 3 km/h, how many hours will it be before they meet?

(a) 4 h 20 min (b) 5 h
(c) 10 h (d) 12 h 30 min

95. Leena took a loan of ₹ 1200 with simple interest for as many years as the rate of interest. If she paid ₹ 432 as interest at the end of the loan period, what was the rate of interest?

(a) 3.6 (b) 6
(c) 18 (d) Cannot be determined

96. A sum of money amounts to ₹ 9800 after 5 yr and ₹ 12005 after 8 yr at the same rate of simple interest. The rate of interest per annum is

(a) 5% (b) 8%
(c) 12% (d) 15%

97. A boy multiplies 987 by a certain number and obtains 559981 as his answer. If in the answer, both 9's are wrong but the other digits are correct, then correct answer will be

(a) 556581 (b) 555681
(c) 555181 (d) 553681

98. The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is

(a) 6 (b) 4
(c) 2 (d) 1

99. To fill a tank, 25 buckets of water is required. How many buckets of water will be required to fill the same tank, if the capacity of the bucket reduced to two-fifth of its present?

(a) $62\frac{1}{2}$ (b) $60\frac{1}{2}$
(c) 60 (d) 62

100. Three numbers which are coprime to each other are such that the product of the first two is 551 and that of the last two is 1073. Find the sum of three numbers.

(a) 89 (b) 85
(c) 81 (d) 75

101. Three years ago, the average age of a family of five members was 17 yr. A baby having been born, the average age of the family is the same as what was three years ago. The present age of the baby is

(a) 6 months (b) 9 months
(c) 1 yr (d) 2 yr

102. The distance between two stations A and B is 300 km. A train leaves the station A with a speed of 40 km/h. At the same time another train departs from the station B with a speed of 50 km/h. How much time will these two trains take to cross each other?

(a) 3 h 40 min (b) 3 h 20 min
(c) 2 h 20 min (d) 3 h 45 min

103. Early morning after sunrise, Rajesh was standing in front of his house in such a way that his shadow was falling exactly behind him. He starts walking straight and walks 5 m. He turns to his left and walks 3 m and again turning to his left walks 2 m. Now, in which direction is he from his starting point?

(a) South-West
(b) North-West
(c) South-East
(d) North-East

104. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if atleast one black ball is to be included in the draw?

(a) 32 (b) 48
(c) 64 (d) 96

105. A machine P can print one lakh books in 8 h; machine Q can print the same number of books in 10 h while machine R can print them in 12 h. All the machines are started at 9:00 am while machine P is closed at 1:00 am and the remaining two machines complete work. Approximately, at what time will the work (to print one lakh books) be finished?

(a) 11:30 am (b) 12 noon
(c) 12:30 pm (d) 1:00 pm

106. In three annual examinations, of which the aggregate marks of each was 500, a student secured average marks 45% and 55% in the first and the second yearly examinations, respectively. To secure 60% average total marks, it is necessary for him in third yearly examination to secure marks

(a) 300 (b) 350
(c) 400 (d) 450

107. On sports day, if 30 children were made to stand in a column, then 16 columns could be formed. If 24 children were made to stand in a column, then how many columns could be formed?

(a) 45 (b) 29
(c) 22 (d) 20

108. A shop gives 10% discount on the purchase of an item. If paid for in cash immediately, a further discount of 12% is given. If the original price of the item is ₹ 250, what is the price of the article, if a cash purchase is made?

(a) ₹ 200 (b) ₹ 195
(c) ₹ 198 (d) ₹ 190

109. The number of revolutions of a wheel of diameter 40 cm makes in travelling a distance of 176 m, is

(a) 140 (b) 150
(c) 160 (d) 200

110. A horse is tied at the corner of a rectangular field, whose length is 20 m and width is 16 m with a rope whose length is 14 m. Find the area which the horse can graze?

(a) 156 sq m (b) 154 sq m
(c) 164 sq m (d) 144 sq m

111. In a two-digit number, unit's digit exceeds its ten's digit by 2 and that the product of the given number and the sum of its digits is equal to 144. What is the number?

(a) 46 (b) 42
(c) 26 (d) 24

112. Divide 48 into two parts such that 7 times the first part, added to 5 times the second part is 246. Then, the smallest part is

(a) 2 (b) 3
(c) 4 (d) 5

113. If 'M $ N' means 'M is the father of N', 'M ≠ N' means 'M is the sister of N' and 'M * N' means 'M is the brother of N', then what is the relation of C with A in A ≠ B $ C * D?

(a) Niece (b) Nephew
(c) Aunt (d) Data inadequate

114. The students of a class are divided into two groups A and B. If Sangita is included in the group A, then her rank is 7th from the top and if she is included in the group B, her rank is 13th from the top. If the students of both groups are brought together, what will be the rank of Sangita?

(a) 20 (b) 19
(c) 21 (d) None of these

115. It is given that M is either greater than or equal to P. P is smaller than Q and Q is not greater than R. Which of the following is definitely true?

(a) M is either greater than or equal to R
(b) M is either greater than or equal to Q
(c) R is greater than P
(d) R is either greater than or equal to P

116. Four friends in the sixth grade were sharing a pizza. They decided that the oldest friend would get the extra piece. Ram is two months older than Gagan, who is three months younger than Neeraj. Rehan is one month older than Gagan. Who should get the extra piece of pizza?

(a) Ram (b) Gagan
(c) Neeraj (d) Rehan

117. In how many different ways can the letters of the word 'OPTICAL' be arranged, so that the vowels always come together?

(a) 120 (b) 720
(c) 4320 (d) 2160

118. In a certain code language, 'AUTHORITY' is written as 'YTUROHTIA'. How will 'DESIGNATE' be written in that code language?

(a) ESENGATDI (b) ESEGNITAD
(c) ESENGITAD (d) ESNEIGTDA

119. At the baseball game, Henry was sitting in seat 253. Maria was sitting to the right of Henry in seat 254. In the seat to the left of Henry was John. Alex was sitting to the left of John. Which seat is Alex sitting in?

(a) 251 (b) 254
(c) 255 (d) 256

120. The maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencil, is

(a) 1911
(b) 1001
(c) 910
(d) 91

Answer *with* Explanations

1. (b) Given function, $f(x)=\dfrac{x}{e^x-1}+\dfrac{x}{2}+1=\dfrac{x(e^x+1)}{2(e^x-1)}+1$

Now, $f(-x)=\dfrac{-x}{e^{-x}-1}-\dfrac{x}{2}+1$

$$=\frac{x}{1-e^{-x}}-\frac{x}{2}+1$$

$$=\frac{xe^x}{e^x-1}-\frac{x}{2}+1$$

$$=x\left\{\frac{e^x}{e^x-1}-\frac{1}{2}\right\}+1$$

$$=x\left\{\frac{2e^x-e^x+1}{2(e^x-1)}\right\}+1$$

$$=\frac{x(e^x+1)}{2(e^x-1)}+1=f(x)$$

which is an even function. $\{\because f(-x)=f(x)\}$

2. (c) Since, the relation R defined as

$R=\{(x, y) \mid x, y$ are real numbers and $x=wy$ for some rational number $w\}$

(i) **Reflexive** $xRx \Rightarrow xw\,x$, $\therefore w=1\in$ rational number

So, the relation R is reflexive.

(ii) **Symmetric** $xRy \nRightarrow yRx$ as $0R1$

$\Rightarrow 0\cdot(1)$ but $1R0 \Rightarrow 1=w\cdot(0)$

which is not true for any rational number.

So, the relation R is not symmetric.

Thus, R is not equivalence relation.

Now for S; (i) **Reflexive** $\dfrac{m}{n}R\dfrac{m}{n}\Rightarrow mn=mn$ (true)

So, the relation S is reflexive.

(ii) **Symmetric** $\dfrac{m}{n}R\dfrac{p}{q}\Rightarrow mq=np\Rightarrow np=mq=\dfrac{p}{q}R\dfrac{m}{n}$

So, the relation S is symmetric.

(iii) **Transitive** $\dfrac{m}{n}R\dfrac{p}{q}$ and $\dfrac{p}{q}R\dfrac{r}{s}\Rightarrow mq=np$ and $ps=rq$

$$mq\cdot ps=np\cdot rq$$

$$\Rightarrow ms=nr$$

$$\Rightarrow \frac{m}{n}=\frac{r}{s}$$

$$\Rightarrow \frac{m}{n}R\frac{r}{s} \quad \text{(transitive)}$$

So, S is an equivalence relation.

3. (b) $f:[0,\infty)\to[0,\infty)$

and $f(x)=\dfrac{x}{1+x}$

$\because$ $f(x_1)=f(x_2)$

$\Rightarrow \dfrac{x_1}{1+x_1}=\dfrac{x_2}{1+x_2}$

$\Rightarrow x_1x_2+x_1=x_2+x_1x_2$

$\Rightarrow x_1=x_2$

Hence, $f(x)$ is one-one.

Let $y=\dfrac{x}{1+x}\Rightarrow y+yx=x$

$\Rightarrow y=x(1-y)$

$\Rightarrow x=\dfrac{y}{1-y}$

Here, range of $f(x)\in R-\{1\}$

and codomain of $f(x)$ is $[0,\infty)$

Hence, $f(x)$ is not onto.

4. (d) Given, $\lim\limits_{h\to 0}\dfrac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$

and $f'(2)=6$, $f'(1)=4$

$$=\lim_{h\to 0}\frac{f\{1+(h+1)^2\}-f(2)}{f\left\{\frac{5}{4}+\left(h-\frac{1}{2}\right)^2\right\}-f(1)}$$

(by L'hospital rule)

$$=\lim_{h\to 0}\frac{[f'\{1+(h+1)^2\}-0]\cdot 2(h+1)}{\left[f'\left\{\frac{5}{4}-\left(h-\frac{1}{2}\right)^2\right\}-0\right]\cdot 2\cdot\left(h-\frac{1}{2}\right)}$$

$$=\frac{f'(1+1)\cdot 2\cdot(0+1)}{f'\left(\frac{5}{4}-\frac{1}{4}\right)(-2)\cdot\left(0-\frac{1}{2}\right)}=\frac{f'(2)\cdot 2}{f'(1)\cdot 1}$$

$$=\frac{6\times 2}{4}=3$$

5. (c) Given, $f(x)=\dfrac{x^2+x+2}{x^2+x+1}$; $x\in R$

Let $y=\dfrac{x^2+x+2}{x^2+x+1}$

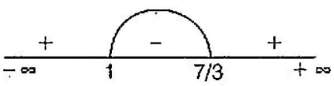
$\Rightarrow x^2y+yx+y=x^2+x+2$

$\Rightarrow x^2(y-1)+x(y-1)+(y-2)=0$

For real values of x,

$$b^2-4ac\ge 0$$

$$\Rightarrow \quad (y-1)^2 - 4(y-1)(y-2) \geq 0$$
$$\Rightarrow \quad (y-1)^2 - \{(y-1) - (4y-8) \geq 0$$
$$\Rightarrow \quad (y-1)(-3y+7) \geq 0$$
$$\Rightarrow \quad (y-1)\left(y-\frac{7}{3}\right) \leq 0$$



$\therefore$ Range of $f(x) \in \left(1, \frac{7}{3}\right)$

6. (d) Let $z = \frac{1}{i-1}$, firstly convert it in $A + iB$ form.

$$= \frac{i+1}{(i-1)(i+1)} = \frac{i+1}{i^2-1} = -\frac{1}{2} - \frac{i}{2}$$

Now, $\bar{z} = -\frac{1}{2} + \frac{i}{2} = \frac{1}{2}\frac{(i-1)}{(i+1)} \times (i+1) = \frac{1}{2}\frac{(i^2-1)}{(i+1)}$

$$= \frac{-1}{i+1} \text{ (which is required conjugate)}$$

7. (a) Given, $|z^2 - 1| = |z|^2 + 1$ (let $z = x + iy$)

$$\Rightarrow \quad |(x+iy)^2 - 1| = |x+iy|^2 + 1$$
$$\Rightarrow \quad |x^2 - y^2 + 2ixy - 1| = |x+iy|^2 + 1$$
$$\Rightarrow \quad |(x^2 - y^2 - 1) + 2ixy| = |x+iy|^2 + 1$$
$$\Rightarrow \quad \sqrt{(x^2-y^2-1)^2 + 4x^2y^2} = x^2 + y^2 + 1$$

Now, squaring on both sides, we get

$$(x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$$
$$\Rightarrow (x^2 - y^2)^2 + 1 - 2(x^2 - y^2) + 4x^2y^2 = (x^2 + y^2)^2 + 1 + 2(x^2 + y^2)$$
$$\Rightarrow x^4 + y^4 - 2x^2y^2 + 1 - 2x^2 + 2y^2 + 4x^2y^2 = x^4 + y^4 + 2x^2y^2 + 1 + 2x^2 + 2y^2$$
$$\Rightarrow 1 - 2x^2 + 2y^2 + 2x^2y^2 = 1 + 2x^2 + 2y^2 + 2x^2y^2$$
$$\Rightarrow 4x^2 = 0$$
$$\Rightarrow \quad x = 0, \text{ i.e., } y\text{-axis or the imaginary axis.}$$

8. (d) Given, $|z| = 1$, $z \neq 1$

$$\Rightarrow \quad |z|^2 = 1$$
$$\Rightarrow \quad x^2 + y^2 = 1 \quad \ldots\text{(i)}$$

Now, $\frac{z-1}{z+1}$ (let $z = x + iy$)

$$= \frac{(x+iy)-1}{(x+iy)+1} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$
$$= \frac{(x^2-1) + y^2 + i(xy + y - xy + y)}{(x+1)^2 + y^2}$$
$$= \frac{(x^2+y^2-1) + i \cdot 2y}{(x^2+y^2+2x+1)} = \frac{(x^2+y^2-1)}{(x+1)^2+y^2} + i \cdot \frac{2y}{(x^2+y^2+2x+1)}$$

$\therefore$ Its real part $= \frac{x^2+y^2-1}{(x^2+y^2)+2x+1} = \frac{1-1}{1+2x+1} = 0$

[from Eq. (i)]

9. (b) If the points z, i and iz are collinear, then area of triangle formed by these points should be 0.

Let $z = x + iy$

Then, $\begin{vmatrix} x & y & 1 \\ 0 & 1 & 1 \\ -y & x & 1 \end{vmatrix} = 0$ $\left[\begin{array}{l}\because i = 0 + i \cdot 1 \\ \text{and } iz = ix - y\end{array}\right]$

Apply $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$,

$$\begin{vmatrix} x & y & 1 \\ -x & 1-y & 0 \\ -y-x & x-y & 0 \end{vmatrix} = 0$$

Expand w.r.t. C_3

$$-x(x-y) + (x+y)(1-y) = 0$$
$$\Rightarrow \quad -x^2 + xy + x - xy + y - y^2 = 0$$
$$\Rightarrow \quad -x^2 - y^2 + x + y = 0$$
$$\Rightarrow \quad x^2 + y^2 - x - y = 0$$

So, the locus of z form a circle with centre at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and radius is $\frac{1}{\sqrt{2}}$.

10. (c) Given fraction,

$$|z|^2 - 5|z| + 1 = 0$$
$$\Rightarrow \quad |z| = \frac{+5 \pm \sqrt{25 - 4 \cdot 1 \cdot 1}}{2} = \frac{5 \pm \sqrt{21}}{2}$$
$$\Rightarrow \quad = \frac{5 \pm 4.59}{2} = \frac{9.59}{2} \text{ or } \frac{0.41}{2}$$
$$\therefore \quad z = \pm \frac{9.59}{2} \text{ or } \pm \frac{0.41}{2}$$

So, the given fraction have four roots.

11. (d) The system of given equations

$$x_1 + 2x_2 + x_3 = 3,$$
$$2x_1 + 3x_2 + x_3 = 3$$
and
$$3x_1 + 5x_2 + 2x_3 = 1$$

Augmented matrix,

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 1 & 3 \\ 3 & 5 & 2 & 1 \end{array}\right]$$

Use $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$,

$$\sim\left[\begin{array}{ccc|c}1 & 2 & 1 & 3\\ 0 & -1 & -1 & -3\\ 0 & -1 & -1 & -8\end{array}\right]$$

Use $R_3 \to R_3 - R_2$,

$$\sim\left[\begin{array}{ccc|c}1 & 2 & 1 & 3\\ 0 & -1 & -1 & -3\\ 0 & 0 & 0 & -5\end{array}\right]$$

Here, $f[A:B]=3$

and $f[A]=2$

i.e., $f[A]< f[A:B]$

So, the system is in consistent and have no solution.

12. (c) The number of 3×3 matrices (non-singular) with four entries as 1 and all other entries as 0, is exactly 6. which is

$$\begin{bmatrix}1&0&0\\0&1&0\\0&1&1\end{bmatrix},\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix},\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix} \text{ and } \begin{bmatrix}1&0&0\\0&1&1\\0&0&1\end{bmatrix}$$

13. (b) Given, $A=\begin{bmatrix}5&5\alpha&\alpha\\0&\alpha&5\alpha\\0&0&5\end{bmatrix}$ and $A^2=25$

Now, $|A|=\begin{vmatrix}5&5\alpha&\alpha\\0&\alpha&5\alpha\\0&0&5\end{vmatrix}=5\cdot\alpha\cdot 5=25\alpha$

$\Rightarrow \quad |A|^2=(25\alpha)^2$

$\Rightarrow \quad A^2=625\alpha^2 \qquad \{\because |A|^2=A^2\}$

$\Rightarrow \quad 25=625\alpha^2$

$\Rightarrow \quad \alpha^2=\dfrac{1}{25}$

$\therefore \quad \alpha=\dfrac{1}{5}$

14. (c) Let $A=\begin{bmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{bmatrix}$

Now, $A^2=A\cdot A=\begin{bmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{bmatrix}\times\begin{bmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{bmatrix}$

$$=\begin{bmatrix}\frac12+\frac12&\frac12-\frac12\\ \frac12-\frac12&\frac12+\frac12\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I$$

So, $\operatorname{tr}(A)=\dfrac{1}{\sqrt2}-\dfrac{1}{\sqrt2}=0$

and $|A|=\begin{vmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{vmatrix}=-\dfrac12-\dfrac12=-1\neq 1$

So, only Statement I is correct.

15. (a) Given, $P=\begin{bmatrix}\frac{\sqrt3}{2}&\frac12\\ -\frac12&\frac{\sqrt3}{2}\end{bmatrix}$, $A=\begin{bmatrix}1&1\\0&1\end{bmatrix}$ and $Q=PAP^T$

$Q=PAP^T \quad\Rightarrow\quad Q^{2005}=(PAP^T)^{2005}$

$=P^{2005}A^{2005}(P^T)^{2005}$

$=(PP^T)^{2005}A^{2005}$

$=\begin{bmatrix}1&0\\0&1\end{bmatrix}^{2005}\begin{bmatrix}1&2005\\0&1\end{bmatrix}$

$=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1&2005\\0&1\end{bmatrix}$

$=\begin{bmatrix}1&2005\\0&1\end{bmatrix}$

$$\left\{\begin{array}{l}\because A=\begin{bmatrix}1&1\\0&1\end{bmatrix}\\ \Rightarrow A^2=\begin{bmatrix}1&2\\0&1\end{bmatrix}\\ \cdots\cdots\cdots\\ \Rightarrow A^n=\begin{bmatrix}1&n\\0&1\end{bmatrix}\end{array}\right\}$$

$\therefore \quad P^TQ^{2005}P=(P^TP)\,Q^{2005}$

$=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1&2005\\0&1\end{bmatrix}$

$=\begin{bmatrix}1&2005\\0&1\end{bmatrix}$

16. (a) Given expression, $2^{3^n}+1,\ n\in N$

Put $n=1$, $2^{3^n}+1=2^3+1=9$

$n=2$, $2^9+1=512+1=513$

....

....

which is divisible by 9.

$\therefore$ The largest integer $k=9$

17. (b) We know that, when we divide $(x-1)^{2n}$ and $(x-1)^{2n+1}$ by x its given remainder 1 and -1, respectively.

Now, $8^{2n}-62^{(2n+1)}=(9-1)^{2n}-(63-1)^{2n+1}$

$=(9-1)^{2n}-(9\cdot 7-1)^{2n+1}$

$=1-(-1)$

($\because$ divided by 9 leaves remainders)

$=1+1=2$

18. (c) Firstly, setting the alphabet of word 'COCHIN' in alphabetical order : CCHINO

For CC = 4!, CH = 4!, CI = 4!, CN = 4!

Now, for words 'COCHIN' $= 4! + 4! + 4! + 4!$
$= 24 + 24 + 24 + 24 = 96$

So, the number of words that appear before the word 'COCHIN' = 96

19. (b) $10^{n-2} > 81n$, here $n \in I^+$

Now $\frac{10^n}{100} > 81n$

$\Rightarrow \quad 10^n > 8100n$

Put $n = 5$,

$\Rightarrow \quad 10^5 > 8100(5)$

$\Rightarrow \quad 100000 > 40500$ (ture)

$\therefore \quad n \geq 5$

20. (c) (a) $8^n + 1 = (9-1)^n + 1$

$= \{{}^nC_0\, 9^n - {}^nC_1\, 9^{n-1} + {}^nC_2\, 9^{n-2} \ldots\} + 1$

$= \{9^n - n\, 9^{n-1} + {}^nC_2\, 9^{n-2} - \ldots\} + 1$

$= \{9^n - n\, 9^{n-1} + {}^nC_2\, 9^{n-2} - \ldots + (-1)^n\} + 1$

= not possible

(b) $3^{2n} + 3n + 1 = 9^n + 3n + 1 =$ not possible

(c) $4^n - 3n - 1$

Put $n = 1, 2, 3 \ldots$

$n = 1$, $4^n - 3n - 1 = 0$, which is divisible by 9.

$n = 2$, $4^n - 3n - 1 = 9$, which is divisible by 9.

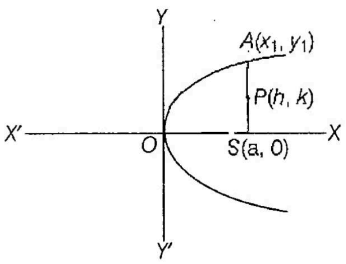
$n = 3$, $4^n - 3n - 1 = 54$, which is divisible by 9

So, $4^n - 3n - 1$ is divisible by 9

21. (c) Equation of parabola is

$$y^2 = 4ax \quad \ldots(i)$$

whose focus, $S = (a, 0)$



Now, mid-piont $P = \left\{\frac{x_1 + a}{2}, \frac{y}{2}\right\}$

$\Rightarrow \quad (h, K) = \left(\frac{x_1 + a}{2}, \frac{y}{2}\right)$

$\Rightarrow \quad x_1 = 2h - a$ and $y_1 = 2K$

which satisfy Eq. (i).

$\Rightarrow \quad y_1^2 = 4ax_1$

$\Rightarrow \quad (2K)^2 = 4a(2h - a)$

$\Rightarrow \quad 4K^2 = 8a\left(h - \frac{a}{2}\right)$

$\Rightarrow \quad K^2 = 2a\left(h - \frac{a}{2}\right)$

$\therefore$ Locus of mid-point is

$$y^2 = 2a\left(x - \frac{a}{2}\right)$$

which is another parabola.

Whose directrix,

$$x - \frac{a}{2} = -\frac{a}{2}$$

$\Rightarrow \quad x = 0$, *i.e.*, y-axis.

22. (b) The given curves,

$y = |x| - 1$

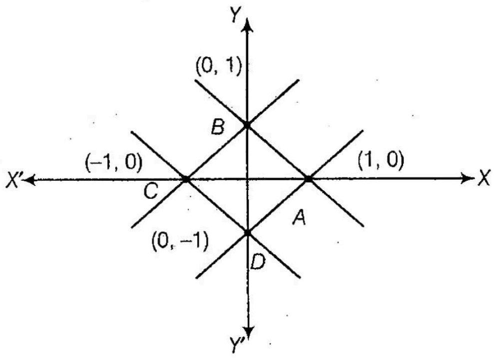
and $y = -|x| + 1$

$\Rightarrow \quad y = x - 1$ and $y = -x - 1$

$y = -x + 1$ and $y = x + 1$

(i) $x - y = 1$ (ii) $x + y = -1$

(iii) $x + y = 1$ (iv) $x - y = -1$



$\therefore$ Required area of $ABCD$

= Area of square with side $\sqrt{2}$ unit

$= (\sqrt{2})^2 = 2$

23. (a) Since, the distance from the points (a_1, b_1) and (a_2, b_2) to $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$ is equal.

Let $P_1 = \frac{|(a_1 - a_2)a_1 + (b_1 - b_2)b_1 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$

and $P_2 = \frac{|(a_1 - a_2)a_2 + (b_1 - b_2)b_2 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$

By given condition; $P_1 = P_2$

$\Rightarrow \quad \frac{|a_1^2 - a_1a_2 + b_1^2 - b_1b_2 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}} = \frac{|a_1a_2 - a_2^2 + b_1b_2 - b_2^2 + c|}{\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}}$

$\Rightarrow \quad a_1^2 = a_1a_2 + b_1^2 - b_1b_2 + c$

$$= -a_1a_2 + a_2^2 - b_1b_2 + b_2^2 - c$$

(taking –ve sign)

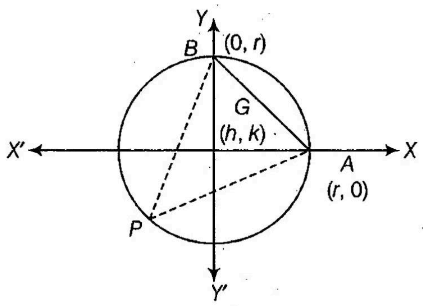
$$\Rightarrow \quad 2c = a_2^2 + b_2^2 - a_1^2 - b_1^2$$

$$\therefore \quad c = \frac{(a_2^2 + b_2^2 - a_1^2 - b_1^2)}{2}$$

24. (b) If (h, k) is the centroid of the ΔPAB, then

$$h = \frac{r(1+\cos\theta)}{3}, K = \frac{r(1+\sin\theta)}{3}$$

$$\Rightarrow \left(h - \frac{r}{3}\right)^2 + \left(K - \frac{r}{3}\right)^2 = \left(\frac{r}{3}\right)^2 \quad (\because \sin^2\theta + \cos^2\theta = 1)$$



Hence, locus of (h, k) is

$$\left(x - \frac{r}{3}\right)^2 + \left(y - \frac{r}{3}\right)^2 = \left(\frac{r}{3}\right)^2$$

which is a circle.

25. (b) The equation of lines

$$3x + 4y = 9 \quad \text{...(i)}$$

and $$y = mx + 1 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$3x + 4(mx + 1) = 9$$

$$\Rightarrow \quad 3x + 4mx + 4 = 9$$

$$\Rightarrow \quad (3 + 4m)x = 5$$

$$\therefore \quad x = \frac{5}{3 + 4m}$$

Here, 5 is a prime, which is divisible by 1 and itself and also here no integer value of m for which $(3 + 4m)$ becomes 1 or 5.

Hence, no integer value of m for which x-coordinate is also an integer.

26. (b) The equation of lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = r_1 \quad \text{...(i)}$$

and $$\frac{x-3}{1} = \frac{y-K}{2} = \frac{z-0}{1} = r_2 \quad \text{...(ii)}$$

From Eq. (i), the point $(2r_1 + 1, 3r_1 - 1, 4r_1 + 1)$ and from Eq. (ii), the point $(r_2 + 3, 2r_2 + K, r_2)$ are coincide.

$$\therefore \quad 2r_1 + 1 = r_2 + 3$$

$$\Rightarrow \quad 2r_1 - r_2 = 2 \quad \text{...(iii)}$$

Again, $$3r_1 - 1 = 2r_2 + K$$

$$\Rightarrow \quad 3r_1 - 2r_2 = 1 + K \quad \text{...(iv)}$$

and $$4r_1 + 1 = r_2$$

$$\Rightarrow \quad 4r_1 - r_2 = -1 \quad \text{...(v)}$$

From Eqs. (iii) and (v),

$$2r_1 = -3 \Rightarrow r_1 = -\frac{3}{2}$$

and $$-3 - r_2 = 2 \Rightarrow r_2 = -5$$

Put these values in Eq. (iv),

$$1 + K = -\frac{9}{2} + 10 = \frac{11}{2}$$

$$\therefore \quad K = \frac{9}{2}$$

27. (b) The given system of equations

$$2x - y + 2z = 2$$
$$x - 2y + z = -4$$
$$x + y + \lambda z = 4$$

Augmented matrix

$$[A:B] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 2 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right]$$

Use operation : $R_1 \to \frac{R_1}{2}$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/2 & 1 & 1 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right]$$

Use operation : $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/2 & 1 & 1 \\ 0 & -3/2 & 0 & -5 \\ 0 & 3/2 & \lambda - 1 & 3 \end{array}\right]$$

Use operation : $R_3 \to R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1/2 & 1 & 1 \\ 0 & -3/2 & 0 & -5 \\ 0 & 0 & \lambda - 1 & -2 \end{array}\right]$$

Since, the system have no solution *i.e.,* in consistent, for this $f[A] < f[A:B]$.

$\therefore$ When $\lambda = 1$, then $f(A) = 2$

and $$f[A:B] = 3$$

28. (a) Given, $f(x) = \sqrt{\sin^{-1}2x + \frac{\pi}{6}}$

Here, $$\sin^{-1}2x + \frac{\pi}{6} \geq 0$$

$$\Rightarrow \quad \sin^{-1}2x \geq -\frac{\pi}{6}$$

$$\Rightarrow \quad 2x \geq \sin\left(-\frac{\pi}{6}\right)$$

$\Rightarrow \quad 2x \geq -\sin\frac{\pi}{6}$

$\Rightarrow \quad 2x \geq -\frac{1}{2} \qquad \{\because \sin(-\theta) = -\sin\theta\}$

$\Rightarrow \quad x \geq -\frac{1}{4} \qquad \ldots(\text{i})$

Since, $\sin^{-1} x$ lie between $x \in (-1, 1)$.

$\Rightarrow \quad 2x \leq 1$

$\Rightarrow \quad x \leq \frac{1}{2} \qquad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$$-\frac{1}{4} \leq x \leq \frac{1}{2}$$

29. (d) $\lim_{x \to 0} \frac{\{(a-n)nx - \tan x\} \cdot \sin x}{x^2} = 0;\ n \in R - \{0\}$

$\Rightarrow \lim_{x \to 0} \frac{\{(a-n)nx - \tan x\}}{x} \cdot \lim_{x \to 0} \frac{\sin x}{x} = 0$

$\Rightarrow \lim_{x \to 0} \frac{\{(a-n)nx - \tan x\}}{x} \cdot 1 = 0$

Now, using L'hospital rule,

$$\lim_{x \to 0} \frac{(a-n)n - \sec^2 x}{1} = 0$$

$\Rightarrow \quad (a-n)n - \sec^2 0 = 0$

$\Rightarrow \quad (a-n)n = 1$

$\Rightarrow \quad a - n = \frac{1}{n}$

$\therefore \quad a = n + \frac{1}{n}$

30. (c) $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow 2\sin^2 x + 6\sin x - \sin x - 3 = 0$

$\Rightarrow 2\sin x(\sin x + 3) - 1(\sin x + 3) = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \quad \sin x \neq -3$

and $\quad \sin x = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \quad x = n\pi + (-1)^n \cdot \frac{\pi}{6}$

Put $\quad n = 0, 1, 2, \ldots$

At $n = 0$; $\quad x = \frac{\pi}{6}$

At $n = 1$; $\quad x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

At $n = 2$; $\quad x = 2\pi + \frac{\pi}{6} = \frac{13\pi}{6}$

At $n = 3$; $\quad x = 3\pi - \frac{\pi}{6} = \frac{17\pi}{6}$

which lies between $x \in (0, 3\pi)$

$\therefore$ Total number of values = 4

31. (c) Given, $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

$\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$

Let $\quad \mathbf{b} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

$\therefore \quad \mathbf{a} \cdot \mathbf{b} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = 1$

$\Rightarrow \quad x + y + z = 1 \qquad \ldots(\text{i})$

and $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$

$= (z - y)\mathbf{i} + (x - z)\mathbf{j} + (y - x)\mathbf{k} = \mathbf{j} - \mathbf{k}$

On comparing both sides

$z - y = 0 \qquad \ldots(\text{ii})$

$y - x = -1 \qquad \ldots(\text{iii})$

$x - z = 1 \qquad \ldots(\text{iv})$

From Eqs. (i) and (ii),

$x + 2z = 1 \qquad \ldots(\text{v})$

From Eqs. (iv) and (v)

$x + \{2(x-1)\} = 1$

$\Rightarrow \quad 3x = 3 \Rightarrow x = 2$

$\therefore \quad z = 0$ and $y = 0$

Hence, $\quad \mathbf{b} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \mathbf{i}$

32. (c) Given, $\quad \mathbf{u} = \mathbf{i} + a\mathbf{j} + \mathbf{k}$

$\mathbf{v} = \mathbf{j} + a\mathbf{k}$

$\mathbf{w} = a\mathbf{i} + \mathbf{k}$

Now, volume of parallelopiped = $[\mathbf{u}\ \mathbf{v}\ \mathbf{w}]$

$$A = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$A = 1 + a(a^2 - 1)$

$A = a^3 - a + 1 \qquad \ldots(\text{i})$

Differential w.r.t. 'a'

$\frac{dA}{da} = 3a^2 - 1 \qquad \ldots(\text{ii})$

For minimum value of a,

Put $\quad \frac{dA}{da} = 0 \Rightarrow 3a^2 - 1 = 0 \Rightarrow a = \pm\frac{1}{\sqrt{3}}$

Now, $\quad \frac{d^2A}{da^2} = 6a$

At $\quad \left(a = \frac{1}{\sqrt{3}}\right), \frac{d^2A}{da^2} = \frac{6}{\sqrt{3}} \qquad$ (minimum)

At $\quad \left(a = -\frac{1}{\sqrt{3}}\right), \frac{d^2A}{da^2} = -\frac{6}{\sqrt{3}} \qquad$ (maximum)

Hence, volume of parallelopiped becomes minimum, if $a = \frac{1}{\sqrt{3}}$

33. (c) Given, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$

and $\mathbf{w} = \mathbf{i} + 3\mathbf{k}$

Here, **v** is the unit vector

i.e., $|\mathbf{u}| = 1$

Now; $[\mathbf{u}\ \mathbf{v}\ \mathbf{w}] = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) \le |\mathbf{u}||\mathbf{v} \times \mathbf{w}|$ ...(i)

$\{\because \mathbf{a} \cdot \mathbf{b} \le |\mathbf{a}||\mathbf{b}|\}$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 3\mathbf{i} - 7\mathbf{j} + (-\mathbf{k})$$

$\therefore$ $|\mathbf{v} \times \mathbf{w}| = \sqrt{9 + 49 + 1} = \sqrt{59}$ [from Eq. (i)]

$[\mathbf{u}\ \mathbf{v}\ \mathbf{w}] \le 1 \cdot \sqrt{59} = \sqrt{59}$

which is the required minimum value.

34. (b) Given that **a** and **b** are unit vectors.

$\therefore$ $|\mathbf{a}| = |\mathbf{b}| = 1$ ...(i)

Also given that, $(\mathbf{a} + 2\mathbf{b})$ and $(5\mathbf{a} - 4\mathbf{b})$ are perpendicular to each other, then

$(\mathbf{a} + 2\mathbf{b}) \cdot (5\mathbf{a} - 4\mathbf{b}) = 0$

$\Rightarrow$ $5\mathbf{a} \cdot \mathbf{a} + 10\mathbf{b} \cdot \mathbf{a} - 4\mathbf{a} \cdot \mathbf{b} - 8\mathbf{b} \cdot \mathbf{b} = 0$

$\Rightarrow$ $5(1) + 10\mathbf{a} \cdot \mathbf{b} - 4\mathbf{a} \cdot \mathbf{b} - 8(1) = 0$

$(\because \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}, \mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{b} = 1)$

$\Rightarrow$ $6\mathbf{a} \cdot \mathbf{b} - 3 = 0$

$\Rightarrow$ $\mathbf{a} \cdot \mathbf{b} = \dfrac{1}{2}$

Let θ be the angle between **a** and **b**, then

$|\mathbf{a}||\mathbf{b}| \cos\theta = \dfrac{1}{2}$

$\Rightarrow$ $1 \cdot 1 \cdot \cos\theta = \cos\dfrac{\pi}{3}$

$\therefore$ $\theta = 60°$

35. (a) Given vectors,

$\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \lambda\mathbf{i} + \mathbf{j} + \mu\mathbf{k}$

are mutually orthogonal.

Then, $\mathbf{a} \cdot \mathbf{c} = 0$

$\Rightarrow$ $(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (\lambda\mathbf{i} + \mathbf{j} + \mu\mathbf{k}) = 0$

$\Rightarrow$ $\lambda - 1 + 2\mu = 0$

$\Rightarrow$ $\lambda + 2\mu = 1$ ...(i)

and $\mathbf{c} \cdot \mathbf{b} = 0$

$(\lambda\mathbf{i} + \mathbf{j} + \mu\mathbf{k}) \cdot (2\mathbf{i} + 4\mathbf{j} + \mathbf{k}) = 0$

$\Rightarrow$ $2\lambda + 4 + \mu = 0$

$\Rightarrow$ $2\lambda + \mu = -4$ ...(ii)

From Eqs. (i) and (ii),

$$\begin{aligned} 4\lambda + 2\mu &= -8 \\ \lambda + 2\mu &= 1 \\ \hline 3\lambda &= -9 \Rightarrow \lambda = -3 \end{aligned}$$

From Eq. (ii),

$-6 + \mu = -4$

$\Rightarrow$ $\mu = 2$

$\therefore$ $(\lambda, \mu) = (-3, 2)$

36. (c) Let $I = \displaystyle\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx, \quad a > 0$ ...(i)

$$I = \int_{-\pi}^{\pi} \frac{\cos^2(\pi - \pi - x)}{1 + a^{(\pi - \pi - x)}} dx = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^{-x}} dx$$

$$= \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1 + a^x} dx \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$2I = \int_{-\pi}^{\pi} \frac{(1 + a^x)\cos^2 x\, dx}{(1 + a^x)} = \int_{-\pi}^{\pi} \cos^2 x\, dx$$

$$2I = 2\int_0^{\pi} \cos^2 x\, dx$$

$(\because \cos^2 x$ is an even function)

$$I = \frac{1}{2}\int_0^{\pi} (1 + \cos^2 x)\, dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \frac{1}{2}\left[\pi + \frac{\sin 2x}{2}\right] = \frac{1}{2}(\pi - 0) = \frac{\pi}{2}$$

37. (d) $\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dx}{dy}\right) = \dfrac{d}{dx}\left(\dfrac{dx}{dy}\right) \cdot \dfrac{dx}{dy}$

$$= \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1} \cdot \left(\frac{dx}{dy}\right)$$

$$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d^2y}{dx^2} \cdot \frac{dx}{dy}$$

$$= -\left(\frac{d^2y}{dx^2}\right) \cdot \left(\frac{dy}{dx}\right)^{-3}$$

38. (d) Let $f(x) = \sin(|x|) - |x|$

Now, $Rf'(0) = \displaystyle\lim_{h \to 0} \frac{f(0 + h) - f(0)}{h}$

$$= \lim_{h \to 0} \frac{\sin|h| - |h| - 0}{h}$$

$$= \lim_{h \to 0} \frac{\sin h}{h} - 1 \qquad \left(\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$$

$= 1 - 1 = 0$

and $Lf'(0) = \displaystyle\lim_{h \to 0} \frac{f(0 - h) - f(0)}{-h}$

$$= \lim_{h \to 0} \frac{\sin|-h| - |-h| - 0}{-h}$$

$$= \lim_{h \to 0} \frac{\sin h - h}{-h}$$

$$= \lim_{h \to 0} -\frac{\sin h}{h} + 1$$

$$= -1 + 1 = 0$$

$\because \quad Lf'(0) = Rj'(0)$

Hence, $f(x)$ is differentiable at $x = 0$.

39. (d) If $D^* f(x) = \lim_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}$

where, $f^2(x) = \{f(x)\}^2$

Now, $D^*(\tan x) = \lim_{h \to 0} \frac{\tan^2(x+h) - \tan^2 x}{h}$

$$= \lim_{h \to 0} \frac{[\tan(x+h) + \tan x][\tan(x+h) - \tan x]}{h}$$

$$= \lim_{h \to 0} \frac{\sin(2x+h) \cdot \sin h}{\cos^2(x+h) \cdot \cos x \cdot h}$$

$$= \lim_{h \to 0} \frac{\sin(2x+h)}{\cos^2(x+h) \cdot \cos^2 x} \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{\sin 2x}{\cos^2 x \cdot \cos^2 x} \cdot 1 = \frac{2 \sin x \cdot \cos x}{\cos^2 x \cdot \cos^2 x}$$

$$= 2 \tan x \cdot \sec^2 x$$

40. (c) Given, $f(x) = \int e^x (x-1)(x-2)\, dx$

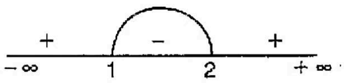
Now, $\quad f'(x) = e^x (x-1)(x-2)$

For decreasing funtion; $f'(x) < 0$

$\Rightarrow \quad e^x (x-1)(x-2) < 0$

Here, e^x never negative, $\forall\, x \in R$

$\therefore \quad (x-1)(x-2) < 0$



$\therefore f$ decreases in the interval $(1, 2)$.

41. (b) Let $I = \int \left(\frac{\log x - 1}{1 + (\log x)^2}\right)^2 dx$, put $\begin{cases} \log x = t \Rightarrow x = e^t \\ dx = x\, dt \end{cases}$

$$I = \int \left(\frac{t-1}{1+t^2}\right)^2 e^t\, dt = \int \frac{(1+t^2-2t)}{(1+t^2)^2} e^t \cdot dt$$

$$= \int \left\{\frac{e^t}{1+t^2} - \frac{2t \cdot e^t}{(1+t^2)^2}\right\} dt$$

$$= \int \frac{e^t}{1+t^2} dt - 2\left\{\int \frac{t \cdot e^t}{(1+t^2)^2} dt\right\}$$

$$= \frac{1}{1+t^2} \cdot e^t - \int \frac{-2te^t \cdot dt}{(1+t^2)^2} - 2\int \frac{t \cdot e^t\, dt}{(1+t^2)^2}$$

$$= \frac{e^t}{1+t^2} + 2\int \frac{te^t}{(1+t^2k)^2} dt - 2\int \frac{te^t}{(1+t^2)^2} dt$$

$$= \frac{x}{1 + (\log x)^2} + C$$

42. (a) Given that, $f(x) = f(x) + f\left(\frac{1}{x}\right)$

and $\quad f(x) = \int_1^x \frac{\log t}{1+t} dt$

$\therefore \quad f(e) = f(e) + f\left(\frac{1}{e}\right)$

$$= \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$$

Put $t = \frac{1}{t}$ in second integration, we get

$$= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log\left(\frac{1}{t}\right)}{1+\frac{1}{t}} d\left(\frac{1}{t}\right)$$

$$= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{-\log t}{t+1} \times t\left(-\frac{1}{t^2}\right) dt$$

$$= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$$

$$= \int_1^e \frac{\log t}{1+t} \cdot \left(\frac{1+t}{t}\right) dt$$

$$= \int_1^e \frac{\log t}{t} dt$$

Put $z = \log t,\ dz = \frac{dt}{t}$

$$= \int_0^1 z\, dz = \left[\frac{z^2}{2}\right]_0^1$$

$$= \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

43. (a) Given that, $\log(x + y) = 2xy$ ...(i)

Put $\quad x = 0;$

$\Rightarrow \quad \log\{0 + y(0)\} = 2 \cdot 0 \cdot y(0) = 0$

$\Rightarrow \quad y(0) = e^0 = 1$ ...(i)

Now, differentiating Eq. (i) w.r.t. x, we get

$$\frac{1}{(x+y)}\{1 + y'\} = 2(y + xy')$$

Put $x = 0;$

$$\frac{1}{0 + y'(0)}\{1 + y'(0)\} = 2\{y(0) + 0\}$$

$\Rightarrow \quad 1 + y'(0) = 2y'(0)(1) = 2y'(0)$

$\Rightarrow \quad y'(0) = 1$

44. (a) Given that, $\int_0^{t^2} x f(x)\, dx = \frac{2}{5} t^5$

Differentiating both sides w.r.t. t by Leibnitz rule,

$\Rightarrow \quad t^2 f(t^2) \cdot 2t - 0 = \frac{2}{5} \cdot 5t^4$

$\Rightarrow \quad 2t^3 f(t^2) = 2t^4$

$\Rightarrow \quad f(t^2) = t$

Put $\left(t = \frac{2}{5}\right)$, we get

$$f\left(\frac{4}{25}\right) = \frac{2}{5}$$

45. (d) $\lim_{x \to 0} \int_0^{x^2} \frac{\sin \sqrt{t}}{x^3} dt \qquad \left(\text{form } \frac{0}{0}\right)$

$= \lim_{x \to 0} \frac{\sin x \cdot 2x - 0}{3x^2} \qquad$ (by Leibnitz rule)

$= \lim_{x \to 0} \frac{2}{3}\left(\frac{\sin x}{x}\right)$

$= \frac{2}{3}(1) \qquad \left(\because \lim_{\theta \to 0} \frac{\sin a}{\theta} = 1\right)$

$= \frac{2}{3}$

46. (c) Given, $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

Separating the variable,

$$\int \frac{y}{\sqrt{1-y^2}} dy = \int dx$$

$\Rightarrow \quad \int \frac{-t\,dt}{t} = \int dx$

$\{\text{put}, t^2 = 1 - y^2, 2t\,dt = -2y\,dy \Rightarrow -t\,dt = y\,dy\}$

$\Rightarrow \quad \int -dt = \int dx$

$\Rightarrow \quad -t = x + c$

$\Rightarrow \quad x + \sqrt{1-y^2} + c = 0$

$\Rightarrow \quad x + c = -\sqrt{1-y^2}$

$\Rightarrow \quad (x^2 + c^2 + 2xc) = 1 - y^2$

$\Rightarrow \quad x^2 + y^2 + 2cx + (c^2 - 1) = 0$

Radius $= \sqrt{c^2 - c^2 + 1} = 1$

and centre $= (-c, 0)$ *i.e.*, along x-axis.

47. (d) $\therefore$ Required probability $= \frac{(5+4+3)}{{}^6C_2}$

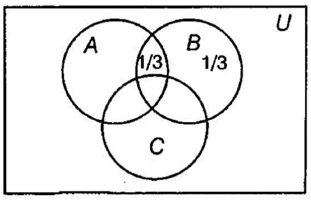
$= \frac{12}{15} = \frac{4}{5}$

48. (a) If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \bar{C}) = \frac{1}{3}$

and $\quad P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$

$\therefore \quad P(B) = \frac{1}{3} + \frac{1}{3} + P(B \cap C)$

$\Rightarrow \quad P(B \cap C) = \frac{3}{4} - \frac{2}{3} = \frac{9-8}{12}$



$\therefore \quad P(B \cap C) = \frac{1}{12}$

49. (c) $\bar{1} \quad \bar{2} \quad \bar{3} \quad \bar{4} \quad \bar{5} \quad \bar{6} \quad \bar{7} \quad \bar{8} \quad \bar{9}$

Here, four digits (3, 3, 5, 5) in which 5 and 3 two times repeats arranging in even places $= \frac{4!}{2!\,2!}$

and here, five digits (2, 2, 8, 8, 8) in which 2 repeat two times and 8 repeat three times arranging in odd places

$$= \frac{5!}{2!\,3!}$$

$\therefore$ Required number of ways $= \frac{4!}{2!\,2!} \times \frac{5!}{2!\,3!}$

$= \frac{24}{2 \cdot 2} \times \frac{120}{2 \cdot 6}$

$= 6 \times 10 = 60$

50. (a) Two person selected in n (≥ 3) person $= {}^nC_2$

The number of ways in which two selected persons are together is $(n-1)$.

$\therefore$ The probability that the selected persons are together

$$= \frac{(n-1)}{{}^nC_2} = \frac{(n-1)}{\frac{n(n-1)}{2}} = \frac{2}{n}$$

Hence, required probability $= 1 - \frac{2}{n}$

51. (a) Given, $f : R \to R$ and $f(x) = 3 - 2\sin x$

Now, $\quad f(x_1) = f(x_2)$

$\Rightarrow \quad 3 - 2\sin x_1 = 3 - 2\sin x_2$

$\Rightarrow \quad \sin x_1 = \sin x_2$

$\Rightarrow \quad x_1 = x_2$

Hence, $f(x)$ is one-one.

Let $\quad y = 3 - 2\sin x$

$\Rightarrow \quad 2\sin x = 3 - y$

$\Rightarrow \quad x = \sin^{-1}\left(\frac{3-y}{2}\right)$

$\{\because \text{range of } \sin^{-1} x \text{ is } [-1, 1]\}$

$\therefore \quad -1 \leq \frac{3-y}{2} \leq 1$

$\Rightarrow \quad -2 \leq 3 - y \leq 2$

$\Rightarrow \quad -5 \leq -y \leq -1$

$\Rightarrow \quad 5 \geq y \geq 1$

$\Rightarrow \quad y \in (1, 5)$

Here, the range of $f(x)$ is (1, 5) which is the subset of codomain *i.e.*, R.

Hence, $f(x)$ is not onto function.

Consequently $f(x)$ is not bijective.

52. (d) Here, $\cos x$ and 2^{-x^2} are even funtions but 2^{x-x^4} is neither even nor odd function

For odd function, $f(-x) = -f(x)$

53. (c) Given $f(x) = x^3 + 5x + 1$, $x \in R$

Now, $f'(x) = 3x^2 + 5 > 0 \quad \forall x \in R$

$\therefore f(x)$ is strictly increasing function

$\therefore f(x)$ is one-one function.

Clearly, $f(x)$ is a continuous function and also increasing on R.

$$\lim_{x \to -\infty} f(x) = -\infty \text{ and } \lim_{x \to \infty} = \infty$$

So, $f(x)$ takes every value between $-\infty$ and ∞.

Thus, $f(x)$ is onto function.

54. (a) Let $z = (i)^i$

Taking log on both sides, we get

$$\Rightarrow \quad \log z = i \log i$$

$$\Rightarrow \quad \log z = i\left[\log \sqrt{0^2 + 1^2} + i \tan^{-1}\left(\frac{1}{0}\right)\right]$$

$$\left[\because \log(x + iy) = \left\{\log \sqrt{x^2 + y^2} + i \tan^{-1} \frac{y}{x}\right\}\right]$$

$$\Rightarrow \log z = i\{\log 1 + i \tan^{-1}(\infty)\}$$

$$\Rightarrow \quad = i\left\{0 + i \cdot \frac{\pi}{2}\right\} = i^2 \cdot \frac{\pi}{2}$$

$$\Rightarrow \quad \log z = -\frac{\pi}{2}$$

$$\Rightarrow \quad z = e^{-\pi/2} \quad \text{(positive and real)}$$

55. (a) Given, $\arg(\bar{z}) + \arg(-z) = \begin{cases} \pi, & \text{if } \arg(z) < 0 \\ -\pi, & \text{if } \arg(z) > 0 \end{cases}$

Now, $\sqrt{\{\arg(z) + \arg(-\bar{z}) - 2z\}\{\arg(-\bar{z}) + \arg(\bar{z})\}}$

$$= \sqrt{(\pi - 2\pi)(-\pi)}$$

$$= \sqrt{(-\pi) \times (-\pi)} = \sqrt{\pi^2} = \pi$$

$$\left[\because \arg(-\bar{z}) + \arg(z) = \begin{cases} \pi, & \text{if } \arg(-z) < 0 \\ -\pi, & \text{if } \arg(-z) > 0 \end{cases}\right.$$
and $(-z)$ lies in IIIrd quadrand.]

56. (d) If $z = \cos\theta + i\sin\theta = e^{i\theta}$; $\text{Im}(z) = \sin\theta$

$$\sum_{m=1}^{15} \text{Im}(z^{2m-1}) = \sum_{m=1}^{15} \text{Im}\{(e^{i\theta})^{2m-1}\}$$

$$= \sum_{m=1}^{15} \text{Im}\{e^{(2m-1)i\theta}\}$$

$$= \sum_{m=1}^{15} \{\sin(2m-1)\theta\}$$

$$= \sin\theta + \sin 3\theta + \sin 5\theta + \ldots + \sin 29\theta$$

$$= \sin\theta + \sin(\theta + 2\theta) + \sin(\theta + 4\theta) + \ldots + \sin(\theta + 28\theta)$$

$$= \frac{\sin\left\{\dfrac{\theta + (\theta + 28\theta)}{2}\right\} \cdot \sin\left(15 \cdot \dfrac{2\theta}{2}\right)}{\sin\left(\dfrac{2\theta}{2}\right)}$$

$$= \frac{\sin 15\theta \cdot \sin 15\theta}{\sin\theta}$$

$$\left[\because \sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots + \sin(\alpha + (n-1)\beta) = \frac{\sin\left\{\dfrac{\alpha + (\alpha + (n-1)\beta)}{2}\right\} \sin\left\{\dfrac{n\beta}{2}\right\}}{\sin\left(\dfrac{\beta}{2}\right)}\right]$$

At $(\theta = 2°)$,

$$= \frac{\sin 30° \cdot \sin 30°}{\sin 2°} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\sin 2°}$$

$$= \frac{1}{4 \sin 2°}$$

57. (c) Given equation of parabola,

$$y^2 - Kx + 8 = 0$$

$$\Rightarrow \quad y^2 = Kx - 8$$

$$\Rightarrow \quad y^2 = K\left(x - \frac{8}{K}\right) \quad \ldots\text{(i)}$$

which is of the form $y^2 = 4ax$

have directrix $x = -a$

From Eq. (i), the equation of directrix

$$x - \frac{8}{K} = -\frac{K}{4}$$

$$\Rightarrow \quad x = \frac{8}{K} - \frac{K}{4} \quad \ldots\text{(ii)}$$

Also, given equation of directrix,

$$x = 1 \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii),

$$\frac{8}{K} - \frac{K}{4} = 1$$

$$\Rightarrow \quad 32 - K^2 = 4K$$

$$\Rightarrow \quad K^2 + 4K - 32 = 0$$

$$\Rightarrow \quad K^2 + 8K - 4K - 32 = 0$$

$$\Rightarrow \quad K(K + 8) - 4(K + 8) = 0$$

$$\Rightarrow \quad (K - 4)(K + 8) = 0$$

$$\Rightarrow \quad K = 4 \text{ or } -8$$

58. (a) Given, equations

$$|z|=2 \quad \text{and} \quad |z|=|z-1|$$

Let $z = x + iy,$

Then, $|z| = |x+iy| = 2$

$\Rightarrow \quad |x+iy|^2 = 4$

$$x^2 + y^2 = 4 \quad \text{...(i)}$$

and $|z| = |z-1|$

$\Rightarrow \quad |x+iy|^2 = |(x-1)+iy|^2$

$\Rightarrow \quad x^2 + y^2 = (x-1)^2 + y^2$

$\Rightarrow \quad x^2 + y^2 = x^2 + 1 - 2x + y^2$

$\Rightarrow \quad 2x - 1 = 0$

$\Rightarrow \quad x = \dfrac{1}{2}$

Put this value in Eq. (i), we get

$$x^2 + y^2 = 4$$

$\Rightarrow \quad \dfrac{1}{4} + y^2 = 4$

$\Rightarrow \quad y^2 = 4 - \dfrac{1}{4} = \dfrac{15}{4}$

$\Rightarrow \quad y = \pm \dfrac{\sqrt{15}}{2}$

$\therefore \quad z = x + iy = \dfrac{1 \pm i\sqrt{15}}{2}$

Hence, required number of complex numbers = 2.

59. (c) $\because f: R \to R$ and $f(1) = 3$, $f'(1) = 6$

Then, $\lim_{x \to 0} \left\{\dfrac{f(1+x)}{f(1)}\right\}^{1/x}$

$\Rightarrow \lim_{x \to 0} \left\{1 + \dfrac{f(1+x)}{f(1)} - 1\right\}^{1/x} \quad (\because \infty^{\infty} \text{ form})$

$$= e^{\lim_{x \to 0} \frac{\left\{\frac{f(1+x)}{f(1)} - 1\right\}}{x}} \quad \left(\text{form } \frac{0}{0}\right)$$

Use L'hospital rule,

$$= e^{\lim_{x \to 0} \frac{f'(1+x)}{f(1)} \cdot (0+1)}$$

$$= e^{\frac{f'(1)}{f(1)}} = e^{6/3} = e^2$$

60. (d) Given, $z = 4 + i\sqrt{7}$

$$z^3 = 64 - 7\sqrt{7}\, i + 48\sqrt{7}\, i - 84$$

$$= -20 + 41\sqrt{7}\, i \quad \text{...(i)}$$

$$z^2 = 16 - 7 + 8i\sqrt{7}$$

$\Rightarrow \quad -4z^2 = -64 + 28 - 32i\sqrt{7}$

$$= -36 - 32i\sqrt{7} \quad \text{...(ii)}$$

$$-9z = -36 - 9i\sqrt{7} \quad \text{...(iii)}$$

Now, $z^3 - 4z^2 - 9z + 91$

$$= -20 + 41\sqrt{7}\, i - 36 - 32i\sqrt{7} - 36 - 9i\sqrt{7} + 91$$

$$= -92 + 91 + 41\sqrt{7}\, i - 41\sqrt{7}\, i = -1$$

61. (b) The ALU has some special purpose registers and the necessary circuitry, to carry out all the arithmetic and logic operations, which are included in the instructions supported by the CPU.

62. (a) A device or system not directly under the control of a computer system.

63. (b) A proxy server is used to process client requests for web pages.

64. (a) Repetition of the same data items in more than one file.

65. (b) The CD-ROM stops functioning would most likely not be a system of a virus.

66. (c) Data and instruction entered into a computer for processing purposes.

67. (a) ASCII → American Standard Code for Information Interchange, which is a standard coding system for computers.

68. (d) The two broad categories of software are system software and application software

69. (c) A peripheral device used in a word processing system is CRT (Cathod Ray Tube).

70. (d) The memory of these computers was constructed using electromagnetic relays and all data and instructions were fed into the system from punched cards.

71. (a) The daily processing of corrections to customer accounts best exemplifies the processing made of time sharing.

72. (c) Interactive processing could be used to describe the concurrent processing of computer programs via CRT's on one computer system.

73. (b) The most widely used commercial programming computer language is COBOL (Common Business Oriented Language)

74. (a) URL (Uniform Resource Locator) An addressing scheme used by www browsers to locate sites on the internet.

75. (b) In excel, any set of characters containing a letter, hyphen or space is considered text.

76. (b) Computers can be classified in the following hierarchical orders.

Super Micro, Personal Computer (PC) Large and Super Computer.

77. (d) In the binary language each letter of the alphabet,each number and each special character is made up of a unique combination of eight bits.

78. (a) BIOS (Basic Input Output System) manages the essential peripherals, such as the keyboard, screen, disk drives, parallel and serial port.

79. (a) Word processing, spread sheet ,and photo editing are examples of application software.

80. (c) The ability to recover and read deleted or damaged files from a criminal's computer is an example of low enforcement speciality called computer forensics.

81. (d) The most ferquently used instructions of a computer program are likely to be fetched from registers. The number of registers available on a processor and the operations that can be performed using those registers has a significant impact on the efficiency of code generated by optimizing compilers.

82. (d) A network is a group of two or more computer systems linked together.

83. (a) Spam is most often considered to be electronic junk mail or junk news group postings. Some people define spam even more generally as any unsolicited e-mail.

84. (c) A protocol is the special set of rules that end points in a telecommunication connection use when they communicate. Protocols specity interactions between the communicating entities.

85. (a) In database management systems, a file that defines the basic organisation of a database. A data dictionary contains a list of all files in the database, the number of records in each file and the names and types of each field.

86. (d) The delete key is used to remove characters and other objects. On PC's the delete key generally removes the character immediately under the cursor (or the right of the insertion point) or the highlighted text or object.

87. (c) A cursor is an indicator used to show the position on a computer monitor or other display device that will respond to input from a text input or pointing device.

88. (d) Malware or malicious software refers to software designed specifically to damage or disrupt a system, such as a virus or a trojan horse.

89. (b) After a picture has been taken with a digital camera and processed appropriately, the actual print of the picture is considered output.

90. (a) The PC (Personal Computer) and the Apple Macintosh are examples of two different platforms.

91. (a) Let the number of children = x

then by given condition,

$$x \cdot \frac{x}{8} = 16 \cdot \frac{x}{2}$$

$$\Rightarrow \quad x^2 = 64x$$

$$\Rightarrow \quad x(x-64) = 0$$

$$\Rightarrow \quad x = 64, \quad x \neq 0$$

$\therefore$ Number of books $= \dfrac{x^2}{8} = \dfrac{64 \times 64}{8} = 512$

92. (c) Between Monday to Saturday, working days are Monday, Tuesday, Thursday and Friday *i.e.,* 4 days for working.

He earns ₹ 100 for 1 working day.

$\therefore$ Total money he earned = 4×100 = ₹ 400

93. (a) In 1 min, distance covered by monkey = 5 m

In next 1 min, distance slips by monkey = 2 m

$\therefore$ In 2 min distance covered by monkey = 3 m

Now, monkey covers 3 m in 2 min.

Then, monkey cover 1 m in $\frac{2}{3}$ min.

$\therefore$ Monkey cover 75 m in $\frac{2}{3} \times 75 = 50$ min.

$\therefore$ Monkey cover 80 m in $(50 + 1) = 51$ min.

94. (b) Here, total distance = 25 km

Relative speed of Ramesh and Kunal = $2 + 3 = 5$ km/h

$\therefore$ Required time $= \dfrac{25}{5} = 5$ h

95. (b) Here, principal, P = ₹ 1200

SI = ₹ 432

Let rate = time = r

Then, according to the formula,

$$SI = \frac{P \cdot r \cdot t}{100}$$

$$\Rightarrow \quad 432 = \frac{1200 \times r \times r}{100}$$

$$\Rightarrow \quad r^2 = \frac{432 \times 100}{1200} = 36$$

$$\therefore \quad r = 6$$

96. (c) Here, A_1 = ₹ 9800, A_2 = ₹ 12005, $t_1 = 5$ yr and $t_2 = 8$ yr

So, rate of simple interest is uniform.

$\therefore$ According to the formula,

$$r = \left(\frac{A_2 - A_1}{A_2 t_1 - A_1 t_2}\right) \times 100$$

$$= \left(\frac{12005 - 9800}{12005 \times 5 - 9800 \times 8}\right) \times 100$$

$$= \left(\frac{2205}{60025 - 78400}\right) \times 100$$

$$= \frac{2205 \times 100}{18375}$$

$\therefore \quad r = 12\%$

97. (b) $987 \times x = 559981$

$9 + 8 + 7 = 24$ which is divisible by 3.

$987 \div 3 = 329$

$= 3 + 2 + 9 = 14$ which is divisible by 7.

So, only 555681 is divisible by 3 and 7.

98. (c) $7^{71} \times 6^{59} \times 3^{65} = 7^2 \times 6 \times 3 = 9 \times 6 \times 3$

$= 2$ (unit digit)

99. (a) New buckets $= \frac{2}{5} \times 25$ (capacity)

$= 10$ capacity of old

$\therefore$ Number of buckets required $= \frac{25 \times 25}{10} = 62.5 = 62\frac{1}{2}$

100. (b) Let $x \cdot y = 551 = 19 \times 29$

Then, $yz = 1073$

$\Rightarrow \quad 29z = 1073$

$\Rightarrow \quad z = 37$

$\therefore \quad x + y + z = 19 + 29 + 37 = 85$

101. (d) 3 yr ago, the age of five members

$= 17 \times 5 + 15 = 100$

Present time, the age of six members $= 17 \times 6 = 102$

Baby's age $= 102 - 100 = 2$ yr

102. (b) According to the question,

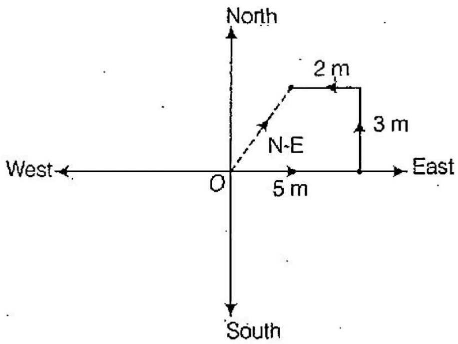
$40t + 50t = 300$

$\Rightarrow \quad 90t = 300$

$\therefore \quad t = \frac{30}{9} = \frac{10}{3} = 3\frac{1}{3}$

So, the required time is 3 h 20 min.

103. (d)



Hence, by figure we get he is now in North-East direction from his starting point.

104. (c) Number of white balls = 2

Black balls = 3

Red balls = 4

Required number of ways

$= {}^6C_2\, {}^3C_1 + {}^6C_1\, {}^3C_2 + {}^6C_0\, {}^3C_3$

$= 15 \times 3 + 6 \times 3 + 1 \times 1$

$= 45 + 18 + 1$

$= 64$

105. (d) Work done by machine P in 1 h $= \frac{1}{8}$

Work done by machine Q in 1 h $= \frac{1}{10}$

Work done by machine R in 1 h $= \frac{1}{12}$

$\therefore$ Work done by $(P + Q + R)$ in 1 h

$= \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = \frac{15 + 12 + 10}{120}$

$= \frac{37}{120}$

Work done by $(P + Q + R)$ in 2 h $= \frac{37}{120} \times 2 = \frac{37}{60}$

Remaining work $= 1 - \frac{37}{60} = \frac{23}{60}$

Now, work done by $(Q + R)$ in 1 h

$= \frac{1}{10} + \frac{1}{12} = \frac{6 + 5}{60}$

$= \frac{11}{60}$

$\because (Q + R)$ done $\frac{11}{60}$ work in 1h

Then, $(Q + R)$ done 1 work in $\frac{60}{11}$ h

$\therefore (Q + R)$ done $\frac{23}{60}$ work in $\frac{60}{11} \times \frac{23}{60} = \frac{23}{11}$ h

$= 2\frac{1}{11} \times 60 = 2$ h 5.45 min

$= 2$ h (approx)

Hence, at $11 + 2 = 1{:}00$ pm the work will be finished.

106. (c) Number of examinations = 3

Total marks in each exam = 500

Marks obtained in first exam = 45% of 500

$= \frac{45}{100} \times 500 = 225$

Marks obtained in second exam = 55% of 500

$= \frac{55}{100} \times 500 = 275$

Total marks obtained in three exams = 60% of 500

$= \frac{60}{100} \times 500 = 300$

Let marks obtained in third exam = x

Then, $\frac{225 + 275 + x}{3} = 300$

$\Rightarrow \quad 500 + x = 900$

$\therefore \quad x = 900 - 500 = 400$

107. (d) Given, number of children in 1 column = 30 and total number of columns = 16

$\therefore$ Total number of children = $16 \times 30 = 480$

Now, if number of children in 1 column = 24

Then, number of columns = $\frac{480}{24} = 20$

108. (c) Given, original price of the item = ₹ 250

After 10% discount,

Price of the item

$= 250 - 10\% \text{ of } 250$

$= 250 - \frac{10}{100} \times 250 =$ ₹ 225

If cash paid immediately, then

Discount = 12% of 225 = $\frac{12}{100} \times 225 =$ ₹ 27

Hence, price of the article = 225 − 27 = ₹ 198

109. (a) Radius $(r) = \frac{20}{100} = \frac{1}{2}$

Now, circumference $= 2\pi r = 2 \times \frac{22}{7} \times \frac{1}{5}$

$\therefore$ Required number of revolutions $= \frac{176}{2 \times 22} \times 7 \times 5$

$= 4 \times 7 \times 5 = 140$

110. (b) Given that, length of a rope is 14 m.

$\therefore$ Required area $= \frac{\theta}{360} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$= 154$

111. (d) Let the ten's digit = x

Then, its unit digit = $x + 2$

Now, by given condition,

$(10x + x + 2)(x + x + 2) = 144$

$(11x + 2)(2x + 2) = 144$

$\Rightarrow \quad 22x^2 + 22x + 4x + 4 = 144$

$\Rightarrow \quad 22x^2 + 26x - 140 = 0$

$\Rightarrow \quad 11x^2 + 13x - 70 = 0$

$\therefore \quad x = \frac{-13 \pm \sqrt{169 + 3080}}{22}$ (by Shridharacharya rule)

$= \frac{-13 \pm \sqrt{3249}}{22} = \frac{-13 \pm 57}{22}$

$= \frac{44}{22} \text{ or } -\frac{70}{22} = 2 \text{ or } -\frac{35}{11}$

$\therefore$ Required number is $x(x+2)$ *i.e.*, 24.

112. (b) Let the two parts of 48 be x and y.

Then, by given condition,

$7x + 5y = 246 \quad \ldots\text{(i)}$

and $\quad x + y = 48 \quad \ldots\text{(ii)}$

On multiplying Eq. (ii) by 5 and then subtracting it from Eq. (i), we get

$7x + 5y = 246$

$5x + 5y = 240$

$- \quad - \quad -$

$2x = 6 \Rightarrow x = 3$

and $\quad y = 45$

So, the smallest part is 3.

113. (b) $M \$ N \longrightarrow M \xrightarrow{\text{father}} N$

$M \neq N \longrightarrow M \xrightarrow{\text{sister}} N$

$H * N \longrightarrow M \xrightarrow{\text{brother}} N$

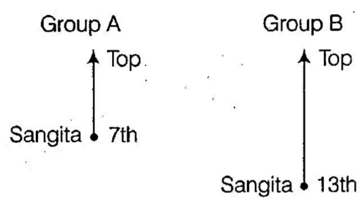
$\because \quad A \neq B \$ C * D$

i.e., $A \xrightarrow{\text{sister}} B \xrightarrow{\text{father}} C \xrightarrow{\text{brother}} D$

Nephew

Hence, C is nephew of A.

114. (b)



Students in group $A = 7$

Students in group $B = 13$

If students of both groups are brought together.

Then, required number of students in both groups = 20 (included Sangeeta in 2 times)

$\therefore$ Sangita's rank when both groups are brought together = 19

115. (c) By given condition,

(i) $M \ge P$ (ii) $Q > P \Rightarrow R > P$ (iii) $Q \not> R \Rightarrow R \ge Q$

Hence, R is greater than P.

116. (c) By given conditions,

(i) (Ram + 2 months) than Gagan

(ii) (Neeraj + 3 months) than Gagan

(iii) (Rehan + 1 Month) than Gagan

By above three conditions, we get required order

Neeraj, Ram, Rehan, Gagan

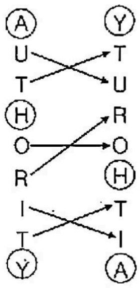
So, Neeraj should get the extra piece of pizza.

117. (b) Required number of ways

$= 5! \times 3! = 120 \times 6$

$= 720$

118. (c)



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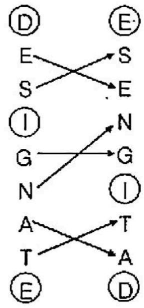

119. (a)

Alex	John	Herry	Marla
251	252	253	254
(Seat number)	(Seat number)	(Seat number)	(Seat number)

Alex sitting in 251 seat number.

120. (d) Given,

1001 ⟶ Pens

910 ⟶ Pencils

Hence, required number of students that each student gets the same number of pens and the same number of pencils

$= 1001 - 910 = 91$