JNU MCA
Solved Paper 2006

1. In a triangle with one angle $2\pi/3$, the lengths of the sides form an AP. If the length of the greatest side is 7cm, the radius of the circumcircle of the triangle is

(a) $\frac{7\sqrt{3}}{3}$ (b) $\frac{5\sqrt{3}}{3}$

(c) $\frac{2\sqrt{3}}{3}$ (d) $\frac{\sqrt{3}}{3}$

2. If in a ΔABC, $\sin A, \sin B, \sin C$ are in AP, then

(a) the altitudes are in AP

(b) the altitudes are in HP

(c) the altitudes are in GP

(d) None of the above

3. $\lim_{n \to \infty} (2k^{1/n} - 1)^n$ is equal to

(a) k^2 (b) $2k$

(c) $2 \ln(k)$ (d) None of these

4. The direction vector along which the function $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ decreases most rapidly at the point (1, 1) is given by

(a) $(1/\sqrt{2}, 1/\sqrt{2})$ (b) $(1/\sqrt{2}, -1/\sqrt{2})$

(c) $(-1/\sqrt{2}, -1/\sqrt{2})$ (d) $(-1/\sqrt{2}, 1/\sqrt{2})$

5. The function $f: R^2 \to R$ is defined by

$$f(x, y) = \begin{cases} \frac{\sin(xy^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(a) is differentiable at (0, 0)

(b) is continuous but not differentiable at (0,0)

(c) is not continuous at (0, 0)

(d) has continuous partial derivatives at (0,0)

6. Let $f(x) = x^3, x \in [a, b]$ and the value of the determinant

$$\begin{vmatrix} f(b) & b^2 & b & 1 \\ f(a) & a^2 & a & 1 \\ f'(a) & 2a & 1 & 0 \\ f''(a) & 2 & 0 & 0 \end{vmatrix}$$

is equal to (–16). Then, $b - a$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4

7. For the integral $\int_0^\infty \tan^n x \, dx$ is equal to $(-\pi)$, the least positive value of n is equal to

(a) 3/2 (b) 5/2 (c) 3 (d) None of these

8. Let y be an implicit function of x given by $x^4 - axy^2 - a^3y = 0$. If y is maximum, then

(a) $3xy + 4a^2 = 0$ (b) $3xy - 4a^2 = 0$

(c) $4x^4 + a^3y = 0$ (d) $3xy + 4a = 0$

9. Let $z = z(x, y)$ be an implicit function of x, y for all $x > 0, y > 0$, given by $xyz^2 + x^2y - xz^4 + y^2z^2 = 0$. Then, z is a homogeneous function of degree

(a) 1 (b) 2

(c) 1/2 (d) 1/4

10. The address lines required for a 256 k work memory are

(a) 8 (b) 10

(c) 18 (d) 20

11. A sequential circuit is one in which the state of the output is

(a) entirely determined by the states of the input

(b) determined by the present input as well as past state

(c) unpredictable

(d) not possible at all

12. If $\sin(\alpha + \beta) = 1$ and $\sin(\alpha - \beta) = 1/2$, where $\alpha, \beta \in [0, \pi/2]$, then $\frac{\tan(\alpha + 2\beta)}{\tan(2\alpha + \beta)}$ is equal to

(a) 1 (b) 2

(c) 3 (d) 4

13. Propositional formula $p \wedge (q \vee r) \to [(p \wedge q) \vee (p \wedge r)]$ is a

(a) tautology

(b) contradiction

(c) contingency

(d) None of the above

14. The solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$ is

(a) $x = C \exp[\cot^{-1}(y/x)]$

(b) $x = C \exp[\sin^{-1}(y/x)]$

(c) $x = C \exp[\tan^{-1}(y/x)]$

(d) None of the above

15. If the random variables X, Y and Z have the means $\mu_X = 2, \mu_Y = -3$ and $\mu_Z = 2$, the variances $\sigma_X^2 = 1, \sigma_Y^2 = 5$ and $\sigma_Z^2 = 2$ and covariances $\text{cov}(X, Y) = -2, \text{cov}(X, Z) = -1$ and $\text{cov}(Y, Z) = 1$, the variance of $W = 3X - Y + 2Z$ is

(a) 17 (b) 18

(c) 20 (d) None of these

16. The determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of

(a) n (b) a
(c) x (d) None of these

17. If a, b and c are three positive real numbers, then the minimum value of the expression $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c}$ is

(a) 1 (b) 2
(c) 3 (d) None of these

18. If p, q and r are any real numbers, then

(a) $\max(p, q) < \max(p, q, r)$
(b) $\min(p, q) = \frac{1}{2}(p + q - |p - q|)$
(c) $\min(p, q) < \min(p, q, r)$
(d) None of the above

19. A computationally efficient way to compute the sample mean of the data $x_1, x_2, \ldots, x_n$ is as follows

$\overline{x_{j+1}} = \overline{x_j} + \frac{x_{j+1} - \overline{x_j}}{k(j)}$, $j = 1, 2, \ldots, n$. Then, $k(j)$ is equal to

(a) j (b) $j+1$
(c) $j(j-1)$ (d) j^{-1}

20. A system composed of n separate components is said to be parallel system if it functions when atleast one of the components functions. For such a system, if a component i functions with probability p_i independent of other components, $i = 1, 2, \ldots, n$, what is the probability that the system functions?

(a) $p_1 p_2 \ldots p_n$
(b) $p_1 + p_2 + \ldots + p_n$
(c) $1 - (1 - p_1)(1 - p_2) \ldots (1 - p_n)$
(d) $(1 - p_1)(1 - p_2) \ldots (1 - p_n)$

21. Centre of mass of a half disc with radius a and uniform mass density is equal to

(a) $2a/3\pi$ (b) $4a/3\pi$
(c) $a/4\pi$ (d) $a/2\pi$

22. The value of the double integral $\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y}\, dy dx$ is equal to

(a) $\pi/2$ (b) -1
(c) 0 (d) 1

23. Let $w = x^2 + y^2$ and $y^3 - xy = -2$. Then, the value of $\partial w/\partial x$ at the point $(x, y) = (-1, -1)$ is equal to

(a) -2 (b) $-3/2$
(c) $2/3$ (d) None of these

24. The function $f(x) = \sum \frac{n^3}{n^4 + 1} \sin nx$ is

(a) continuous at $x = 0$ and differentiable in $(0, 2\pi)$
(b) discontinuous at $x = 0$ and non-differentiable in $(0, 2\pi)$
(c) continuous at $x = 0$ and non-differentiable in $(0, 2\pi)$
(d) discontinuous at $x = 0$ and differentiable in $(0, 2\pi)$

25. If the number $(z-1)/(z+1)$ is purely imaginary, then

(a) $|z| = 1$ (b) $|z| > 1$
(c) $|z| < 1$ (d) $|z| > 2$

26. If $F = (y^2 + z^3, 2xy - 5z, 3xz^2 - 5y)$, then a scalar function $\phi(x, y, z)$ such that $F = \text{grad}(\phi)$ given by

(a) $xy + xz^3 - yz + C$
(b) $y + xz^2 + 2xy + C$
(c) $xy^2 + xz^3 - 5yz + C$
(d) $xyz + xz^2 + yz + C$

27. A person walking along a straight road observes that at two points 1 km apart, the angles of elevation of a pole in front of him are 30° and 75°. The height of the pole is

(a) $250(\sqrt{3} + 1)$ m (b) $250(\sqrt{3} - 1)$ m
(c) $225(\sqrt{2} - 1)$ m (d) $225(\sqrt{2} + 1)$ m

28. X is a continuous random variable with probability function $f(x) = N \exp(-x^2 + 6x)$, $-\infty < x < \infty$, the value of N is

(a) $\frac{1}{\sqrt{2\pi}}$ (b) e^{-9}
(c) $\frac{e^{-9}}{\sqrt{\pi}}$ (d) None of these

29. The value of $\int_0^{\infty} \frac{1}{(4x^2 + \pi^2)\cosh x}\, dx$ is equal to

(a) $\frac{\ln 2}{2\pi}$ (b) $\frac{2 \ln 2}{2\pi}$
(c) $\frac{\pi}{2 \ln 2}$ (d) None of these

30. The value of $\oint_C x^2 y\, dx + (y^3 - xy^2)\, dy$, where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 4$, $x^2 + y^2 = 16$, is

(a) 2π (b) 12π
(c) 120π (d) None of these

31. Let **a**, **b** and **c** are three non-coplanar vectors and let **p**, **q** and **r** be the vectors defined by the relations $\mathbf{p} = \frac{\mathbf{b} \times \mathbf{c}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$, $\mathbf{q} = \frac{\mathbf{c} \times \mathbf{a}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$ and $\mathbf{r} = \frac{\mathbf{a} \times \mathbf{b}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$. Then, the value of the expression $(\mathbf{a} + \mathbf{b}) \cdot \mathbf{p} + (\mathbf{b} + \mathbf{c}) \cdot \mathbf{q} + (\mathbf{c} + \mathbf{a}) \cdot \mathbf{r}$ is equal to

(a) 0 (b) 1
(c) 2 (d) 3

32. Consider a complete binary tree. The number of nodes at level k is

(a) $2^k - 1$ (b) 2^k (c) $2^{k-1} - 1$ (d) 2^{k-1}

33. Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

(a) -2 (b) -1 (c) 1 (d) 2

34. $\lim_{n \to \infty} \left\{ \frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots + \frac{n}{1-n^2} \right\}$ is equal to

(a) 0 (b) $-\frac{1}{2}$
(c) $\frac{1}{2}$ (d) None of these

35. Backward Euler's method for solving differential equation $\frac{dy}{dx} = f(x, y)$ is

(a) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$
(b) $y_{n-1} = y_{n-1} + 2hf(x_n, y_n)$
(c) $y_{n+1} = y_n + hf(x_n, y_n)$
(d) $y_{n+1} = (1+h)\, f(x_{n-1}, y_{n+1})$

36. The value of integral $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}\, dx$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) None of these

37. If $y = ae^{-kt}\cos(pt + c)$ and $\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + n^2 y = 0$, then n^2 equals to

(a) $p^2 + k^2$ (b) p^2
(c) k^2 (d) $p^2 - k^2$

38. What is the meaning of following declaration?

(a) f is a function returning integer value.
(b) f is a function returning pointer to integer.
(c) f is a pointer to a function returning integer.
(d) It is not a valid declaration.

39. Program counter PC is used to store

(a) the number of statements in a program
(b) the number of instructions in a process
(c) the address of the next instruction to be executed
(d) the address of the first instruction of process

40. $(Z + X)(Z + \overline{X} + Y)$ is equal to

(a) $(Z + X)(Z + Y)$ (b) $Z(X + Y)$
(c) $X \cdot Z + Y$ (d) $ZX + ZY + XY$

41. Let $b_n = \int_0^1 \min(x, a_{n-1})dx$ and $a_n = \int_0^1 \max(x, b_{n-1})\, dx, C_n = a_n + b_n$. Then, the sequence (C_n) converges to

(a) $\sqrt{2}$ (b) 1
(c) 2 (d) None of these

42. The value of the integral $\int_0^1 y^2 \left(\ln \frac{1}{y^3}\right)^{-1/2} dy$ is equal to

(a) 1/3 (b) $\sqrt{\pi}$ (c) $\sqrt{\pi}/3$ (d) $\pi/3$

43. The number of solutions to the equation $z^2 + \bar{z} = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

44. If $\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$, then $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ is equal to

(a) $1/y$ (b) y
(c) $1 - y$ (d) $1 + y$

45. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation

(a) $a^2 + b^2 + 2ac = 0$
(b) $a^2 - b^2 + 2ac = 0$
(c) $a^2 + c^2 + 2ab = 0$
(d) $a^2 - b^2 - 2ac = 0$

46. The number of solutions of the equation $\sin 5x \cos 3x = \sin 6x \cos 2x$ in the interval $[0, \pi]$ is

(a) 3 (b) 4
(c) 5 (d) 6

47. If sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is

(a) a square
(b) a circle
(c) a straight line
(d) two intersecting lines

48. X is an exponential random variable with parameter λ with p.d.f.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; \text{if } x \ge 0 \\ 0 & ; \text{if } x < 0 \end{cases}$$

identify the correct one.

(a) $P(X > s + t) = P(X > s)\, P(X > t)$
(b) $P(X > s + t) = P(X > s) + P(X > t)$
(c) $P(X > s + t) = 1 - P(X = s)\, P(X = t)$
(d) $P(X > s + t) = \lambda st P(X > s)\, P(X > t)$

49. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then, the equation of the circle through their points of intersection and the point (1, 1) is

(a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$
(c) $x^2 + y^2 - 4y + 2 = 0$ (d) None of these

50. There exists a function $f(x)$ satisfying $f(0) = 1, f'(0) = -1, f(x) > 0$ for all x, and

(a) $f''(x) > 0$ for all x
(b) $-1 < f''(x) < 0$ for all x
(c) $-2 < f''(x) < -1$ for all x
(d) $f''(x) < -$ for all x

51. The value of $\int_{-\pi}^{\pi} \frac{\sin 3\theta}{(5 - 3\cos\theta)}\, d\theta$ is equal to

(a) infinity (b) 2/3
(c) 1/3 (d) None of these

52. The number of vectors of unit length perpendicular to the vectors $\mathbf{a} = (1, 1, 0)$ and $\mathbf{b} = (0, 1, 1)$ is

(a) one (b) two
(c) three (d) None of these

53. Given the following truth table: (R is the result)

A	B	R
0	0	1
0	1	0
1	0	1
1	1	1

Above TT corresponds to following formula

(a) $A \to B$ (b) $B \to A$
(c) $A \to B \vee B \to A$ (d) None of these

54. What will be the output of following program segment?

```
int array [5],i, * p;
for (i = 0; i < 5; i ++)
array [i] = i;
ip = array;
printf("%d \ n", * (ip + 3 * size of (int)))
```

(a) 3 (b) 6
(c) Garbage (d) None of these

55. If the vectors $(a, 1, 1)$, $(1, b, 1)$ and $(1, 1, c)$ $(a \neq b \neq c \neq 1)$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

(a) 3 (b) 2
(c) 1 (d) 0

56. The number of terms in the exponential series such that their sum gives the value of e^x correct to six decimal places at $x = 1$ is

(a) 6 (b) 8
(c) 10 (d) 14

57. Newton's iterative formula to find $\sqrt{N}$ is

(a) $x_{n+1} = x_n (2 - Nx_n)$
(b) $x_{n+1} = x_n (2 + Nx_n)$
(c) $x_{n+1} = 2\left(x_n + \frac{N}{x_n}\right)$
(d) None of the above

58. The equations $2x + 3y + 5z = 9; 7x + 3y - 2z = 8;$ $2x + 3y + \lambda z = \mu$ have infinite number of solutions, if

(a) $\lambda = 5$ (b) $\mu = 5$
(c) $\lambda = \mu = 5$ (d) None of these

59. Determine the value of k for which the function given by $f(x, y) = kxy$ for $x = 1, 2, 3$ and $y = 1, 2, 3$ can serve as a joint probability distribution

(a) $\frac{1}{36}$ (b) 1
(c) $\frac{1}{9}$ (d) 8

60. S is defined as

$$S = |x-1| + \left|x - \frac{1}{2}\right| + \left|x - \frac{1}{3}\right| + \left|x - \frac{1}{4}\right| + \left|x - \frac{1}{5}\right|.$$ Find the value of x for which S is minimum.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{78}{80}$

61. The centre of a circle passing through the point $(0, 1)$ and touching the curve $y = x^2$ at $(2, 4)$ is

(a) $\left(\frac{-16}{5}, \frac{27}{10}\right)$ (b) $\left(\frac{-16}{7}, \frac{5}{10}\right)$
(c) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (d) None of these

62. If $u = \cos(x + y) + \cos(x - y)$, then which of the following is/are true?

I. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$ II. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x \partial y}$

III. $\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial y \partial x}$

(a) I only (b) II only
(c) I and II only (d) I and III only

63. The TEST instruction for 8086 microprocessor performs the function of

(a) destructive AND
(b) non-destructive AND
(c) wait for an event
(d) None of the above

64. Let $S_n = \sum_{k=0}^{n} f_k^2, f_k$ is the kth Fibonacci number, $f_0 = f_1 = 1, f_{n+1} = f_n + f_{n-1}$. Then, the value $\sum_{n=0}^{\infty} (-1)^n S_n$ is equal to

(a) 1/2 (b) $\sqrt{5}/2$
(c) $(\sqrt{5} - 1)/2$ (d) None of these

65. The real value of θ for which the expression $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is a real number is

(a) $2n\pi$ (b) $(2n + 1)\pi$
(c) $2n\pi \pm \pi/2$ (d) None of these

66. If $\cos\alpha + \cos\beta + \cos\lambda = \sin\alpha + \sin\beta + \sin\gamma = 0$, then which of the following are true?

I. $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$
II. $\cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\gamma + \alpha) = 0$
III. $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

(a) I and II only
(b) II and III only
(c) III and I only
(d) I, II and III

67. The set of real x such that $\frac{2x - 1}{2x^3 + 3x^2 + x} > 0$ is

(a) $(-\infty, -1)$ (b) $(-\infty, 0)$
(c) $(-\infty, \infty)$ (d) None of these

68. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$ is equal to

(a) 0 (b) 1
(c) -1 (d) None of these

69. A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. The locus of the centre of the circle drawn on this chord as diameter is

(a) $x^2 + y^2 + ax = 0$ (b) $x^2 + y^2 + ay = 0$
(c) $x^2 + y^2 - ax = 0$ (d) $x^2 + y^2 - ay = 0$

70. The value of $\int_0^{\infty} \frac{\sin x}{x} dx$ is

(a) infinity (b) $\frac{\pi}{2}$
(c) π (d) None of these

71. Which of the following pairs is logically equivalent?

(a) $A \rightarrow B$ and $\neg A \vee B$
(b) $\neg(A \vee B)$ and $\neg A \wedge \neg B$
(c) $(A \vee \neg B) \rightarrow C$ and $(\neg A \wedge B \vee C)$
(d) All of the above

72. What will be the output of following program segment?

```
int i j;
j = 0;
for (i = 1; i < 10; i ++)
{
continue;
+ + j;
}
printf ("% d",j);
```

(a) 0 (b) 55
(c) 10 (d) None of these

73. Hexadecimal D9 is equivalent to octal
(a) 113 (b) 331
(c) 131 (d) 313

74. DMA is responsible for
(a) data movement in registers
(b) data movement in ALU
(c) data movement in I/O devices
(d) data movement from I/O to memory and *vice-versa*

75. If $|z^2 - 1| = |z|^2 + 1$, then z lies on a/an
(a) straight line (b) circle
(c) ellipse (d) None of these

76. The equation $3^{x-1} + 5^{x-1} = 34$ has
(a) no solution
(b) one solution
(c) two solutions
(d) more than two solutions

77. Given a statement

If it rains I am not going.

Converse of the statement is
(a) If I don't go, it rains.
(b) If I go, it doesn't rain.
(c) If I don't go, it doesn't rain.
(d) None of the above

78. If $x = 6$, $y = 11$, $z = -2$, find the value of statement $((x/2) > y) \| (x > z)$ in C language.
(a) 0 (b) 1
(c) 3 (d) None of these

79. If the lines $2(\sin A + \sin B)x - 2\sin(A - B)y = 3$ and $2(\cos A + \cos B)x + 2\cos(A - B)y = 5$ are perpendicular, then $\sin 2A + \sin 2B$ is equal to
(a) $\sin(A - B) - 2\sin(A + B)$
(b) $2\sin(A - B) - \sin(A + B)$
(c) $\sin(2(A - B)) - \sin(A + B)$
(d) $\sin(2(A - B)) - 2\sin(A + B)$

80. If G is the centroid and I is the incentre of the triangle with vertices $A(-36, 7)$, $B(20, 7)$ and $C(0, -8)$, then GI is equal to
(a) $\frac{\sqrt{250}}{3}$ (b) $\frac{\sqrt{205}}{3}$
(c) $\frac{\sqrt{181}}{3}$ (d) None of these

81. Locus of the mid-points of the chords of the circle $x^2 + y^2 = 4$ which subtends a right angle at the centre is
(a) $x + y = 2$ (b) $x^2 + y^2 = 1$
(c) $x^2 + y^2 = 2$ (d) $x - y = 0$

82. The value of $\int_{-\infty}^{\infty} \frac{1}{(5 + 4x + x^2)^2} dx$ is equal to
(a) π (b) $\frac{\pi}{2}$
(c) infinity (d) None of these

83. The value of k for which the points $A(1, 0, 3)$, $B(-1, 3, 4)$, $C(1, 2, 1)$ and $D(k, 2, 5)$ are coplanar is
(a) 1 (b) 2
(c) 0 (d) -1

84. If the equation of one tangent of the circle with centre at $(2, -1)$ from the origin is $3x + y = 0$, then the equation of the other tangent through the origin is
(a) $3x - y = 0$
(b) $x + 3y = 0$
(c) $x - 3y = 0$
(d) $x + 2y = 0$

85. The value of $\int_0^{\infty} \frac{x^{p-1}}{1 + x} dx, 0 < p < 1$ is equal to
(a) π (b) infinity
(c) $\frac{\pi}{2}$ (d) None of these

86. The value of $[\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{c} - \mathbf{a}]$ is
(a) 0 (b) 1
(c) 2 (d) 3

87. If $z = x + iy$, $z^{1/3} = a - ib$, $a \neq \pm ba$, $b \neq 0$, then $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$, where k is equal to
(a) 0 (b) 2
(c) 4 (d) None of these

88. The inequality $n! > 2^{n-1}$ is true for
(a) all $n \in N$ (b) $n > 2$
(c) $n > 1$ (d) $n \notin N$

89. The equation $3\sin^2 x + 10\cos x - 6 = 0$ is satisfied for $n \in I$, if
(a) $x = n\pi + \cos^{-1}(1/3)$
(b) $x = n\pi - \cos^{-1}(1/3)$
(c) $x = 2n\pi \pm \cos^{-1}(1/3)$
(d) None of the above

90. If $\mathbf{a}$ and $\mathbf{b}$ are two unit vectors, then the vector $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} \times \mathbf{b})$ is parallel to the vector
(a) $\mathbf{a} - \mathbf{b}$ (b) $\mathbf{a} + \mathbf{b}$
(c) $2\mathbf{a} - \mathbf{b}$ (d) $2\mathbf{a} + \mathbf{b}$

91. The solution set of the inequality $||x| - 1| < 1 - x$ is
(a) $(1, 1)$ (b) $(0, \infty)$
(c) $(-1, \infty)$ (d) None of these

92. The solution set of the inequality $4^{-x + 0.5} - 7 \cdot 2^{-x} - 4 < 0$ $(x \in R)$ is
(a) $(-\infty, \infty)$ (b) $(-2, \infty)$
(c) $(2, \infty)$ (d) $(2, 3.5)$

93. If $x\cos\alpha + y\sin\alpha = x\cos\beta + y\sin\beta = 2a$ $(0 < \alpha, \beta < \pi/2)$, then it is also true that

I. $\cos\alpha + \cos\beta = \frac{4ax}{x^2 + y^2}$

II. $\cos\alpha\cos\beta = \frac{4a^2 - y^2}{x^2 + y^2}$

III. $\cos\alpha\cos\beta = \frac{4ax}{x^2 + y^2}$

IV. $\cos\alpha + \cos\beta = \frac{4a^2 - y^2}{x^2 + y^2}$

The correct possibilities are
(a) I and II only (b) III and IV only
(c) I and III only (d) II and IV only

94. The coordinates (x, y) of a moving point P satisfy the equation $\frac{dx}{dt} = x$ and $\frac{dy}{dt} = -x^2$ for all $t \geq 0$. Find an equation of the curve in rectangular coordinates if it passes through $(1, -4)$ when $t = 0$

(a) $y = \frac{x^2 + 7}{7}$ (b) $y = \frac{-x^2 - 7}{2}$
(c) $y = x^2 + 7$ (d) $x^2 - y^2 = 7$

95. The number of flip-flops used to construct a ring counter which counts from decimal one to decimal eight will be

(a) 1 (b) 2
(c) 3 (d) 4

96. The straight line $y = 4x + c$ is tangent to the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$. Then, c is equal to

(a) ± 4 (b) $\pm \sqrt{6}$
(c) ± 1 (d) $\pm \sqrt{132}$

97. The area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = 1$ and $x = b$ is $(b-1)\sin(3b+4)$. Then $f(x)$ is

(a) $(x-1)\cos(3x+4)$
(b) $\sin(3x+4)$
(c) $\sin(3x+4) + 3(x-1)\cos(3x+4)$
(d) None of the above

98. The least value of the expression $2\log_{10} x - \log_x 0.01$, for $x > 1$ is

(a) 10 (b) -0.01
(c) 2 (d) None of these

99. If $y = \frac{1 + \sqrt{1 - \sin 4A}}{\sqrt{1 + \sin 4A} - 1}$, then one of the values of y is

(a) $\tan A$ (b) $\cot A$
(c) $-\tan(2A)$ (d) $-\cot A$

100. The expression $(2\sqrt{3} + 4)\sin x + 4\cos x$ lies in the interval

(a) $(-4, 4)$
(b) $(-2\sqrt{5}, 2\sqrt{5})$
(c) $(-2 + \sqrt{5}, 2 + \sqrt{5})$
(d) $(-2(2 + \sqrt{5}), 2(2 + \sqrt{5}))$

101. Let $a, b, c > 0$. The series $\sum_{n=1}^{\infty} \left\{ a^{1/n} - \left(\frac{b^{1/n} + c^{1/n}}{2} \right) \right\}$ is convergent, if

(a) $a = bc$ (b) $a = \sqrt{bc}$
(c) $a = \sqrt{bc}$ (d) $a = b\sqrt{c}$

102. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. The volume of the solid is equal to

(a) $8\pi/3$ (b) $32\pi/3$
(c) $4\pi/3$ (d) $16\pi/3$

103. Let $f(t) = \int_t^{t^2} \frac{\sin tx}{x} dx, t \neq 0$. Then, the value of $f'(1)$ is equal to

(a) $\sin(1)$ (b) 0
(c) $-\sin(1)$ (d) $2\sin(1)$

104. If the area of a triangle on the complex plane formed by the point $z, z + iz$ and iz is 50, then $|z|$ is

(a) 1 (b) 5
(c) 10 (d) 15

105. If A lies in the second quadrant and $3\tan A + 4 = 0$, the value of $2\cot A - 5\cos A + \sin A$ is equal to

(a) $-53/10$ (b) $23/10$
(c) $37/10$ (d) $7/10$

106. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then $\cos(\theta - \pi/4)$ is equal to

(a) $\pm \frac{1}{2\sqrt{2}}$ (b) $\pm \frac{1}{\sqrt{2}}$
(c) $\pm \sqrt{2}$ (d) $\pm 2\sqrt{2}$

107. The value of $\tan 1^\circ \tan 2^\circ \ldots\ldots \tan 89^\circ$ is

(a) -1 (b) 0
(c) 1 (d) None of these

108. If $\mathbf{A} = (1, 1, 1)$ and $\mathbf{C} = (0, 1, -1)$ are given vectors, then a vector $\mathbf{B}$ satisfying $\mathbf{A} \times \mathbf{B} = \mathbf{C}$ and $\mathbf{A} \cdot \mathbf{B} = 3$ is

(a) $(5/3, 2/3, 2/3)$ (b) $(2/3, 5/3, 2/3)$
(c) $(2/3, 2/3, 5/3)$ (d) None of these

109. For a real number y, let $[y]$ denote the greatest integer less than or equal to y. Function $f(x)$ is given by $\frac{\tan(\pi[x - \pi])}{1 + [x]^2}$.

(a) Then the function $f(x)$ is discontinuous at some x;
(b) Then the function $f(x)$ is continuous at all x, but the derivative $f'(x)$ does not exist for some x.
(c) Then for the function $f(x)$, $f''(x)$ exists for all x.
(d) Then for the function $f(x)$, $f'(x)$ exists for all x but the second derivative $f''(x)$ does not exist for some x.

110. Let a, b, c be non-zero real numbers such that

$$\int_0^1 (1 + \cos^2 x)(ax^2 + bx + c)\, dx$$

$$= \int_0^2 (1 + \cos^2 x)(ax^2 + bx + c)\, dx$$

Then, the quadratic equation $ax^2 + bx + c = 0$ has

(a) no root in $(0, 2)$
(b) a double root in $(0, 2)$
(c) two imaginary roots
(d) atleast one root in $(0, 2)$

111. The solution of the differential equation $y(2xy + e^x)\, dx - e^x dy = 0$ is

(a) $x^2 + C + ye^x = 0$
(b) $x(y^2 + C) + e^x = 0$
(c) $y(x^2 + C) + e^x = 0$
(d) None of the above

112. The propagation delay encountered in a ripple carry adder of four-bit size, with delay of a single flip-flop as t_p will be

(a) 0 (b) $t_p * 4$
(c) $t_p / 2$ (d) $\exp(t_p)$

113. The Gray code equivalent of 1010_2 will be

(a) 1111 (b) 0101
(c) 0011 (d) 1001

114. The 2's complement of N in n bit is

(a) 2^n (b) $2^n - N$
(c) 2^N (d) $N - 2$

115. What is the output of following program?

```
# include < stdio.h>
main()
{
int a, b, funct (int * a, int b);
a = 20;
b = 20;
funct (& a, b);
printf (" a = % d b = % d", a, b);
}
funct (int * a, int b)
{
* a = 10;
b = b + 10;
return;
}
```

(a) a = 10, b = 20 (b) a = 20, b = 10
(c) a = 20, b = 30 (d) None of these

116. What is the output of following program?

```
# include <stdio. h>
main()
{
int n,a, sum (int n);
int (* ptr) (int n);
n = 100;
ptr =& sum;
a = (* ptr) (n);
printf ("Sum = %d / n", a);
}
int sum (int n)
{
Int i, j;
j = 0;
for (i = 1; i < = n; i + +)
j + = i;
return (j);
}
```

(a) Sum =5050
(b) Sum =5000
(c) Produces compile time error
(d) Produces run time error

117. If $z = (\lambda + 3) + i(5 - \lambda^2)^{1/2}$, then the locus of z is a/an

(a) ellipse (b) circle
(c) plane (d) None of these

118. If $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$, then x is equal to

(a) 4 (b) 2
(c) 3.14 (d) None of these

119. The number of real solutions of $\sin(e^x) = 5^x + 5^{-x}$ is

(a) infinite (b) 5
(c) 0 (d) None of these

120. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{xy + y}{xy + x}$ is

(a) $y + x = \ln \dfrac{kx}{y}$ (b) $y - x = \ln \dfrac{ky}{x}$
(c) $y - x = \ln \dfrac{kx}{y}$ (d) None of these

Answers *with* Solutions

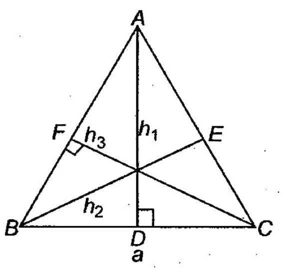
1. (a) From sine rule, we know that

$$\frac{a}{\sin A} = 2R$$

$$\Rightarrow \quad R = \frac{a}{2\sin A} = \frac{7}{2\sin\frac{2\pi}{3}} = \frac{7}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

($\because$ The greatest angle is $\frac{2\pi}{3}$ corresponding to the greatest side 7.)

2. (b) Area of triangle, $\Delta = \frac{1}{2} a h_1$ but $a = 2R \sin A$



$\Rightarrow \Delta = (R\sin A)\, h_1 \Rightarrow h_1 = \dfrac{\Delta}{R \sin A}$

Similarly, $h_2 = \dfrac{\Delta}{R\sin B}$, $h_3 = \dfrac{\Delta}{R \sin C}$

Given $\sin A, \sin B, \sin C$ are in AP.

$\Rightarrow \dfrac{\Delta}{h_1 R}, \dfrac{\Delta}{h_2 R}, \dfrac{\Delta}{h_3 R}$ are in AP.

$\Rightarrow \dfrac{1}{h_1}, \dfrac{1}{h_2}, \dfrac{1}{h_3}$ are in AP.

$\Rightarrow h_1, h_2, h_3$ are in HP.

3. (a) Put $n = \dfrac{1}{h}$

As, $n \to \infty, h \to 0$

$$\therefore \quad \lim_{n\to\infty} (2k^{\frac{1}{n}} - 1)^n = \lim_{h\to 0} (2k^h - 1)^{\frac{1}{h}}$$

$$= e^{\lim_{h\to 0} \frac{1}{h}(2k^h - 2)}$$

$$= e^{2\lim_{h\to 0} \frac{k^h - 1}{h}} = e^{2\ln k}$$

$$= k^2$$

4. (c) Since, $f(x, y) = \dfrac{x^2}{2} + \dfrac{y^2}{2}$

$$\Rightarrow \quad \nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} = x\mathbf{i} + y\mathbf{j} \text{ at the point } (1, 1)$$

$\nabla f = \mathbf{i} + \mathbf{j}$

Now, direction cosine of vector $\mathbf{i} + \mathbf{j}$ is $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ in increasing sense.

$\Rightarrow$ In decreasing sense, direction cosine is $\left(-\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}\right)$.

5. (b) Taking limit along $y = mx$

$$\lim_{\substack{x\to 0\\ y\to 0}} f(x,y) = \lim_{x\to 0} f(x, mx)$$

$$= \lim_{x\to 0} \frac{\sin[x(mx)^2]}{x^2 + (mx)^2}$$

$$= \lim_{x\to 0} \frac{\sin(m^2 x^3)}{x^2(1+m^2)}$$

$$= \lim_{x\to 0} \frac{\sin(m^2x^3)}{m^2 x^3} \times \frac{m^2 x}{(1+m^2)}$$

$$= 1 \times \lim_{x\to 0} \frac{m^2}{1+m^2} \times x$$

$$= 0$$

$$= f(0,0) \quad \text{(given)}$$

$\Rightarrow$ $f(x,y)$ is continuous at (0,0).

Again,

$$f'(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(h^3m^2)}{h^2(1+m^2)} - 0}{h}$$

$$= \lim_{h\to 0} \frac{\sin(m^2h^3)}{h^3(1+m^2)}$$

$$= \lim_{h\to 0} \frac{\sin(m^2h^3)}{m^2h^3} \times \frac{m^2}{(1+m^2)}$$

$$= \frac{m^2}{1+m^2}$$

which is not independent of m.

$\Rightarrow$ $f(x,y)$ is not differentiable at (0, 0).

6. (c) $\because f(x) = x^3$

$\therefore$ $f(a) = a^3$ and $f'(a) = 3a^2$ and $f(b) = b^3$ and $f''(a) = 6a$

Given, $$\begin{vmatrix} f(b) & b^2 & b & 1 \\ f(a) & a^2 & a & 1 \\ f'(a) & 2a & 1 & 0 \\ f''(a) & 2 & 0 & 0 \end{vmatrix} = -16$$

$$\Rightarrow \begin{vmatrix} b^3 & b^2 & b & 1 \\ a^3 & a^2 & a & 1 \\ 3a^2 & 2a & 1 & 0 \\ 6a & 2 & 0 & 0 \end{vmatrix} = -16$$

Applying $R_2 \to R_2 - R_1$

$$\begin{vmatrix} b^3 & b^2 & b & 1 \\ a^3 - b^3 & a^2 - b^2 & a-b & 0 \\ 3a^2 & 2a & 1 & 0 \\ 6a & 2 & 0 & 0 \end{vmatrix} = -16$$

$$\Rightarrow \begin{vmatrix} a^3 - b^3 & a^2 - b^2 & a - b \\ 3a^2 & 2a & 1 \\ 6a & 2 & 0 \end{vmatrix} = -16$$

$$\Rightarrow -2(b-a)^3 = -16$$

$$\Rightarrow (b-a)^3 = 8$$

$$\Rightarrow b - a = 2$$

7. (d) No correct option is given.

As, $\int_0^\infty \tan^n x\, dx \neq -\pi$ for any positive n

8. (a) $$f(x) = x^4 - axy^2 - a^3y = 0 \quad \text{...(i)}$$

At maxima $\frac{\partial f}{\partial x} = 0$

$$\Rightarrow 4x^3 - ay^2 = 0$$

$$\Rightarrow 4x^4 - axy^2 = 0$$

$$\Rightarrow 4(axy^2 + a^3y) - axy^2 = 0 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow 3axy^2 + 4a^3y = 0$$

$$\Rightarrow 3xy + 4a^2 = 0$$

9. (c) $xyz^2 + x^2y - xz^4 + y^2z^2 = 0$

$$\Rightarrow xz^4 - (xy + y^2)z^2 - x^2y = 0$$

$$\Rightarrow z^2 = \frac{xy + y^2 \pm \sqrt{(xy+y^2)^2 + 4x^3y}}{2x}$$

= Equation of degree $2 - 1 = 1$

$\Rightarrow$ z will be equation of degree $\frac{1}{2}$.

10. (c) The address lines for $= 256\ k$

$$= 2^8 \times 2^{10}$$
$$= 2^{18}$$

Here, 18 address lines are required.

So, option (c) is correct.

11. (b) A sequential circuit is one in which the state of the output is determined by the present input as well as past state.

12. (c) $\sin(\alpha + \beta) = 1 \Rightarrow \alpha + \beta = \frac{\pi}{2}$ as $\alpha, \beta \in \left[0, \frac{\pi}{2}\right]$

$$\sin(\alpha - \beta) = \frac{1}{2} \Rightarrow \alpha - \beta = \frac{\pi}{6}$$

$$\Rightarrow \alpha = \frac{\pi}{3} \text{ and } \beta = \frac{\pi}{6}$$

Now, $$\frac{\tan(\alpha + 2\beta)}{\tan(2\alpha + \beta)} = \frac{\tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right)}{\tan\left(\frac{2\pi}{3} + \frac{\pi}{6}\right)} = \frac{-\sqrt{3}}{-\frac{1}{\sqrt{3}}} = 3$$

13. (a) $p \wedge (q \vee r) \to [(p \wedge q) \vee (p \wedge r)]$

$\Rightarrow$ $(p \wedge q) \vee (p \wedge r) \to [(p \wedge q) \vee (p \wedge r)]$

LHS and RHS are same, so it is always true. Hence, it is tautology.

14. (c) $$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2} \quad \text{...(i)}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From Eq. (i), we get

$$v + x\frac{dv}{dx} = 1 + v + v^2$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v^2$$

$$\Rightarrow \frac{dv}{1+v^2} = \frac{dx}{x}$$

$$\Rightarrow \tan^{-1} v = \ln x - \ln C$$

$$\Rightarrow \tan^{-1}\frac{y}{x} = \ln\frac{x}{C}$$

$$\Rightarrow \frac{x}{C} = e^{\tan^{-1}\frac{y}{x}}$$

$$\Rightarrow x = Ce^{\tan^{-1}\frac{y}{x}}$$

i.e., $$x = C\exp\left(\tan^{-1}\frac{y}{x}\right)$$

15. (b) $W = 3X - Y + 2Z$

$\Rightarrow$ $\text{Var}(W) = 9\sigma_X^2 + \sigma_Y^2 + 4\sigma_Z^2$

$- 6\,\text{cov}(X, Y) - 4\,\text{cov}(Y, Z) + 12\,\text{cov}(Z, X)$

$= 9(1) + 5 + 4(2) - 6(-2) - 4(1) + 12(-1) = 18$

16. (a) $\Delta = \begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$

$= a^2 (\sin x) - a(-\sin 2x) + \sin x$

It is independent of n.

Alternate method

$$\frac{d\Delta}{dn} = 0 - x(0) + x(0) = 0$$

$\Rightarrow$ Δ is independent of n.

17. (d) $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c}$

$= \left(\frac{b}{a} + \frac{a}{b}\right) + \left(\frac{c}{a} + \frac{a}{c}\right) + \left(\frac{c}{b} + \frac{b}{c}\right) \geq 2 + 2 + 2 = 6$

(since $x + \frac{1}{x} \geq 2$ by using AM $\geq$ GM)

18. (b) Min $(p, q) = \frac{p + q - |p - q|}{2}$

and Max $(p, q) = \frac{p + q + |p - q|}{2}$

19. (b) $\bar{X}_{n+1} = \frac{x_1 + x_2 + \ldots + x_n + x_{n+1}}{n+1}$

$\Rightarrow$ $(n+1)\bar{X}_{n+1} = (x_1 + x_2 + \ldots + x_n) + x_{n+1}$

$\Rightarrow$ $(n+1)\bar{X}_{n+1} = n\bar{X}_n + x_{n+1} = (n+1)\bar{X}_n - \bar{X}_n + x_{n+1}$

$\Rightarrow$ $\bar{X}_{n+1} = \bar{X}_n + \frac{x_{n+1} - \bar{X}_n}{n+1}$

$\Rightarrow$ $k(n) = n + 1$

$\Rightarrow$ $k(j) = j + 1$

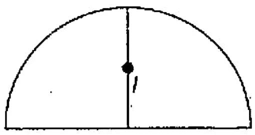
20. (c) Probability that the system function

= probability that atleast none component functions

= 1 − probability that none of the component functions

$= 1 - (1 - p_1)(1 - p_2) \ldots (1 - p_n)$

21. (b)



If the half disc is revolved about x-axis it will form sphere.

Let l be the distance of centre of mass from x-axis.

By Pappu's theorem

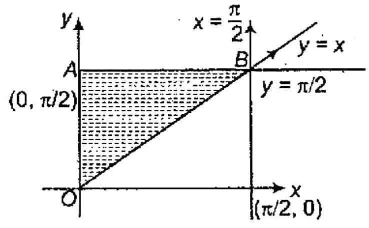
2π (distance of central of mass of half disc) × (area of half disc)

= volume of the sphere

$\Rightarrow$ $2\pi l \times \left(\frac{\pi a^2}{2}\right) = \frac{4}{3}\pi a^3$

$\Rightarrow$ $l = \frac{4a}{3\pi}$

22. (d) Let $I = \int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin y}{y} \, dydx$



By change of order of integral

$I = \int_0^{\frac{\pi}{2}} \int_0^y \frac{\sin y}{y} \, dxdy$

$= \int_0^{\frac{\pi}{2}} \frac{\sin y}{y} \, dy \int_0^y dx$

$= \int_0^{\frac{\pi}{2}} \sin dy = [-\cos y]_0^{\frac{\pi}{2}}$

$= -\left[\cos \frac{\pi}{2} - \cos 0\right] = 1$

23. (b) $w = x^2 + y^2 \Rightarrow$ and $y^3 - xy = -2$

$\Rightarrow$ $y(y^2 - x) = -2 \Rightarrow y^2 (y^2 - x)^2 = 4$

$\Rightarrow$ $(w - x^2)(w - x^2 - x)^2 = 4$

Partially differentiating both sides w.r.t. x, we get

$\left(\frac{\partial w}{\partial x} - 2x\right)(w - x^2 - x)^2$

$+ 2(w - x^2 - x)\left(\frac{\partial w}{\partial x} - 2x - 1\right)(w - x^2) = 0$ ...(i)

At $(x, y) = (-1, -1)$ we have $w = 2$

Putting these values in Eq. (i), we get

$\left(\frac{\partial w}{\partial x} + 2\right)(2)^2 + 2(2)\left(\frac{\partial w}{\partial x} + 1\right)(1) = 0$

$\Rightarrow$ $\frac{\partial w}{\partial x} + 2 + \left(\frac{\partial w}{\partial x} + 1\right)(1) = 0$

$\Rightarrow$ $2\frac{\partial w}{\partial x} + 3 = 0$

$\Rightarrow$ $\frac{\partial w}{\partial x} = -\frac{3}{2}$

24. (a) $f(x) = \sum \frac{n^3}{n^4 + 1} \sin n\, x$

$f(0) = \sum \frac{n^3}{n^4 + 1} \sin n \cdot 0 = 0$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \sum \frac{n^3}{n^4 + 1} \sin nx$

$= \lim_{x \to 0} \sum \frac{n^3}{n^4 + 1} \cdot \frac{\sin nx}{nx} \cdot nx$

$= \lim_{x \to 0} \sum \frac{n^4}{n^4 + 1} \left(\frac{\sin nx}{nx}\right) . x = 0$

As, $\lim_{x \to 0} f(x) = f(0)$

So, $f(x)$ is continuous at $x = 0$

At 6 $x = \frac{\pi}{2}$

$f(x) = \sum \frac{n^3}{n^4 + 1} \sin \frac{n\pi}{2}$

$= \sum \frac{n^3}{n^4 + 1} (1, 0 \text{ or } -1)$

Hence, it becomes alternate series which is convergent. Hence, it is also differentiable.

25. (a) $w = \frac{(z - 1)}{(z + 1)} = \frac{x - 1 + iy}{x + 1 + iy}$

$= \frac{(x - 1) + iy}{(x + 1)^2 + y^2} \times (x + 1) - iy$

$= \frac{(x^2 - 1 + y^2) + iy^2}{(x + 1)^2 + y^2}$

w is purely imaginary $\Rightarrow$ Re$(w) = 0$

$\Rightarrow$ $x^2 + y^2 - 1 = 0 \Rightarrow x^2 + y^2 = 1$

$\Rightarrow$ $|z|^2 = 1 \Rightarrow |z| = 1$

26. (c) $F = \text{grad}\,(\phi)$

$$= \nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial x}\mathbf{j} + \frac{\partial\phi}{\partial x}\mathbf{k}$$

$$= (y^2 + z^3)\,\mathbf{i} + (2xy - 5z)\,\mathbf{j} + (3xz^2 - 5y)\mathbf{k}$$

$$\Rightarrow \quad \frac{\partial\phi}{\partial x} = y^2 + z^3, \frac{\partial\phi}{\partial y} = 2xy - 5z, \frac{\partial\phi}{\partial z} = 3xz^2 - 5y$$

Now,

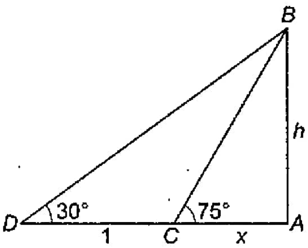
$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$$

$$\Rightarrow \quad \phi = \int (y^2 + z^3)\,dx + \int (2xy - 5z)\,dy + \int (3xz^2 - 5y)\,dz$$

$$= \int (y^2\,dx + 2xy\,dy) - 5\int (y\,dz + z\,dy) + \int (z^3\cdot dx + 3xz^2 dz)$$

$$= \int d(xy^2) - 5\int d(yz) + \int d(xz^3) = xy^2 - 5yz + xz^3 + C$$

27. (a) Let AB be the pole and C, D be the two points.



Let $AB = h$ and $AC = x$

In $\Delta\, ABC$,

$$\tan 75° = \frac{AB}{AC} = \frac{h}{x} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \quad \text{...(i)}$$

In $\Delta\, ABD$, $\tan 30° = \dfrac{AB}{AD} = \dfrac{h}{x+1} = \dfrac{1}{\sqrt{3}}$ ...(ii)

$x = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}h$ from Eq. (i) and $x + 1 = \sqrt{3}\,h$ from Eq. (ii),

$$\Rightarrow \quad \frac{\sqrt{3}-1}{\sqrt{3}+1}h + 1 = \sqrt{3}h$$

$$\Rightarrow \quad h\left(\sqrt{3} - \frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = 1 \Rightarrow h\left(\frac{4}{\sqrt{3}+1}\right) = 1$$

$$\Rightarrow \quad h = \frac{\sqrt{3}+1}{4}\text{ km} = 250\,(\sqrt{3}+1)\text{ m}$$

28. (c) $f(x) = N\exp(-x^2 + 6x)$ $\quad \infty < x < \infty$

$$= N e^{-[(x^2 - 6x + 9) - 9]} = N e^9 e^{-(x-3)^2}$$

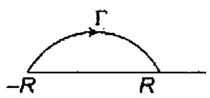
Probability density function of normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

$$\Rightarrow \quad 2\sigma^2 = 1 \text{ and } N e^9 = \frac{1}{\sqrt{2\pi}\,\sigma}$$

$$\Rightarrow \quad N = \frac{1}{\sqrt{2\pi}\,\sigma}e^{-9} = \frac{1}{\sqrt{2\pi}\cdot\frac{1}{\sqrt{2}}}e^{-9} = \frac{e^{-9}}{\sqrt{\pi}}$$

29. (d) $I = \displaystyle\int_0^{\infty} \frac{1}{(4x^2 + \pi^2)\cosh x}dx = f(x)$



$$\Rightarrow \quad I = \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{(4z^2 + \pi^2)\cosh z}dz$$

Let $\quad f(z) = \dfrac{1}{(4z^2 + \pi^2)\cosh z}$

Poles of $f(z)$ are given by

$$z = \pm\frac{i\pi}{2} \text{ and } \cosh z = 0$$

For $z = \dfrac{i\pi}{2}$, $\cosh z = \cosh\dfrac{i\pi}{2} = \dfrac{e^{\frac{i\pi}{2}} + e^{-\frac{i\pi}{2}}}{2} = 0$

$\therefore$ Res $f(z)$ at $z = \dfrac{i\pi}{2}$

$$= \frac{1}{(4z^2 + \pi^2)\sinh z + 8z\cosh z}$$

$$= \frac{1}{16\,z\sinh z + 8\cosh z + (4z^2 + \pi^2)\cosh z}$$

$$= \frac{1}{-8\pi}$$

$$\therefore \quad I = \frac{1}{2}2\pi i \times \frac{1}{-8\pi} = -\frac{i}{8}$$

30. (c) $I = \oint_C x^2y\,dx + (y^3 - xy^2)\,dy$

$$= \iint_R \left[\frac{\partial}{\partial x}(y^3 - xy^2) - \frac{\partial}{\partial y}(x^2y)\right]dxdy$$

(by Green's theorem)

$$= \iint_R (-y^2 - x^2)\,dxdy$$

$$= -\iint_R (x^2 + y^2)\,dxdy$$

Putting $\quad x = r\cos\theta, y = r\sin\theta$

$\Rightarrow \quad dxdy = r\,d\theta dr$

Also, $\quad r = 2$ to 4 and $\theta = 0$ to 2π

$$\therefore \quad I = -\int_0^{2\pi}\int_2^4 r^2\cdot r\,dr\,d\theta$$

$$= -\int_0^{2\pi}d\theta\cdot\int_2^4 r^3\,dr$$

$$= -\frac{1}{4}(4^4 - 2^4)(2\pi - 0)$$

$$= 120\,\pi \text{ (neglect '–' sign)}$$

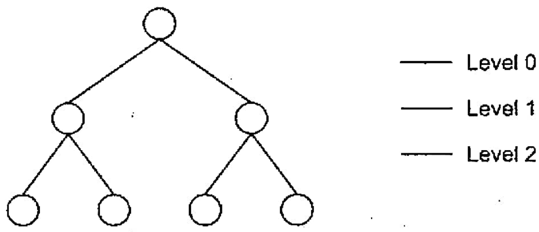
31. (d) $(\mathbf{a} + \mathbf{b})\cdot\mathbf{p} = (\mathbf{a} + \mathbf{b})\cdot\dfrac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\;\mathbf{b}\;\mathbf{c}]}$

$$= \frac{\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]} + \frac{\mathbf{b}\cdot(\mathbf{b}\times\mathbf{c})}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$$

$$= 1 + 0 = 1$$

Similarly, $(\mathbf{b} + \mathbf{c})\cdot\mathbf{q} = 1$ and $(\mathbf{c} + \mathbf{a})\cdot\mathbf{r} = 1$

32. (b) For complete binary tree



At level $0 = 2^0 = 1$ node

At level $1 = 2^1 = 2$ nodes

At level $2 = 2^2 = 4$ nodes

At level $k = 2^k$ nodes

33. (c) Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$

and $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$

$\Rightarrow \quad u = v \quad \therefore \frac{du}{dv} = 1$

34. (b) $\lim_{n\to\infty}\left[\frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots\ldots + \frac{n}{1-n^2}\right]$

$= \lim_{n\to\infty} \frac{1+2+\ldots\ldots+n}{1-n^2}$

$= \lim_{n\to\infty} \frac{n(n+1)}{2(1-n^2)} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2(\frac{1}{n^2}-1)} = -\frac{1}{2}$

35. (c)

36. (a) $I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}\, dx$ ...(i)

By using $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}}\, dx$ ...(ii)

Adding Eqs. (i) and (ii), we get

$2I = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2}$

$\Rightarrow \quad I = \frac{\pi}{4}$

37. (a) Auxiliary equation of $\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2 y = 0$ is

$m^2 + 2km + n^2 = 0$

$\Rightarrow \quad m = \frac{-2k \pm \sqrt{4k^2 - 4n^2}}{2}$

$\Rightarrow \quad m = -k \pm \sqrt{n^2 - k^2}\, i$

$\therefore$ CF is

$y = C_1 e^{(-k+\sqrt{n^2-k^2}\, i)t} + C_2 e^{(-k-\sqrt{n^2-k^2}\, i)t}$

$= e^{-kt}[A e^{ipt} + B e^{-ipt}]$

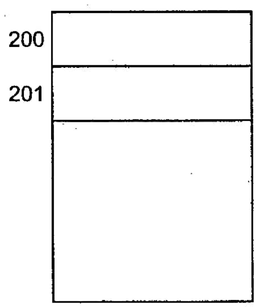
which will transform into

$y = ae^{-kt}\cos(pt + C)$

$\Rightarrow \quad p^2 = n^2 - k^2 \quad \Rightarrow \quad n^2 = p^2 + k^2$

38. (*) The given question is *incomplete*.
'*' means no option is correct.

39. (c) Program counter is used to store the address of the next instruction to be executed. If instruction which is executing currently is at the address of 200. So, the program counter (PC) stores the value of 201, if at 201, there is a next instruction to be executed.



40. (a) Given, $(Z+X)(Z+\overline{X}+Y)$

$\Rightarrow \quad Z + ZX + Z\overline{X} + X\overline{X} + ZY + XY$

$\Rightarrow \quad Z + Z(X+\overline{X}) + 0 + ZY + XY$

$(\because A\cdot A = A,\ A\cdot\overline{A} = 0 \text{ and } A + \overline{A} = 1)$

$\Rightarrow \quad Z + Z + ZY + XY$ $\quad (\because A + A = A)$

$\Rightarrow \quad Z + ZY + XY$

$\Rightarrow \quad Z(1+Y) + XY$ $\quad (\because 1 + A = 1)$

$\Rightarrow \quad Z + XY$

$\{\because A + BC = (A+B)(A+C)\}$

$\Rightarrow \quad (Z+X)(Z+Y)$

41. (b) Given sequence $< C_n >$ converges.

If $\min(x, a_{n-1}) = \max(x, b_{n-1}) = x$

$\Rightarrow \quad C_n$ converges to $\int_0^1 x\, dx + \int_0^1 x\, dx = 2\left[\frac{x^2}{2}\right]_0^1 = 1$

42. (c) $I = \int_0^1 y^2\left[\ln\left(\frac{1}{y^3}\right)\right]^{-\frac{1}{2}} dy$

Put $\ln\left(\frac{1}{y^3}\right) = z \quad \Rightarrow \quad -\ln y^3 = z$

$\Rightarrow \quad y^3 = e^{-z} \Rightarrow y = e^{-\frac{z}{3}}$

$\Rightarrow \quad dy = e^{-\frac{z}{3}}\cdot\left(-\frac{1}{3}\right)dz$

At $\quad y = 0, z = \infty, y = 1, z = 0$

$\therefore \quad I = \int_\infty^0 e^{-\frac{2z}{3}} \cdot z^{-\frac{1}{2}} \cdot e^{-\frac{z}{3}}\left(-\frac{1}{3}\right)dz$

$= \frac{1}{3}\int_0^\infty z^{-\frac{1}{2}} e^{-z}\, dz = \frac{1}{3}\int_0^\infty z^{\frac{1}{2}-1}\cdot e^{-z}\, dz$

$= \frac{1}{3}\Gamma\frac{1}{2} = \frac{1}{3}\sqrt{\pi}$

43. (d) $z^2 + \bar{z} = 0$

Let $\quad z = x + iy$

$\Rightarrow \quad x^2 - y^2 + 2ixy + x - iy = 0$

$\Rightarrow \quad (x^2 - y^2 + x) + iy(2x - 1) = 0$

$\Rightarrow \quad y(2x-1) = 0$

and $\quad x^2 + x - y^2 = 0$

$\Rightarrow \quad y = 0 \text{ or } x = \frac{1}{2}$

At $\quad y = 0$

$x^2 + x - y^2 = 0$

$\Rightarrow \quad x^2 + x = x(x+1) = 0$

$\Rightarrow \quad x = 0, -1 \Rightarrow z = 0, -1$

At $x = \frac{1}{2}, y^2 = \left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{3}{4}$

$\Rightarrow \quad y = \frac{\pm\sqrt{3}}{2} \Rightarrow z = \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\Rightarrow$ There are four solutions

$z = 0, -1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

44. (b) $\left(\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}\right)\times\left(\frac{1+\sin\alpha}{1-\cos\alpha+\sin\alpha}\right)$

$= \frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)^2 - \cos^2\alpha} = \frac{2\sin\alpha(1+\sin\alpha)}{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha}$

$= \frac{2\sin\alpha(1+\sin\alpha)}{2\sin\alpha(1+\sin\alpha)} = 1$

$\Rightarrow \quad \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = y$

45. (b) $\sin\theta$ and $\cos\theta$ are the roots of $ax^2 - bx + c = 0$

$$\Rightarrow \quad \sin\theta + \cos\theta = \frac{b}{a} \quad \ldots\text{(i)}$$

$$\text{and} \quad \sin\theta\cos\theta = \frac{c}{a} \quad \ldots\text{(ii)}$$

On squaring Eq. (i), we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$$

$$\Rightarrow \quad 1 + \frac{2c}{a} = \frac{b^2}{a^2} \quad \text{[using Eq. (ii)]}$$

$$\Rightarrow \quad a^2 + 2ac = b^2$$

$$\Rightarrow \quad a^2 - b^2 + 2ac = 0$$

46. (c) $\sin 5x \cos 3x = \sin 6x \cos 2x$

$\Rightarrow$ $2\sin 5x \cos 3x = 2\sin 6x \cos 2x$

$\Rightarrow$ $\sin 8x + \sin 2x = \sin 8x + \sin 4x$

$\Rightarrow$ $\sin 2x = \sin 4x = 2\sin 2x \cos 2x$

$\Rightarrow$ $\cos 2x = \frac{1}{2}$ or $\sin 2x = 0$

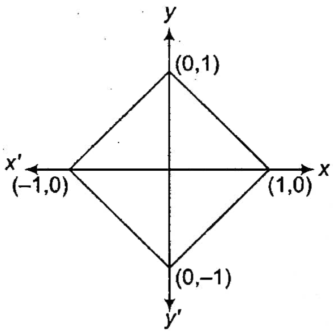
$\Rightarrow$ $2x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ or $2x = 0, \pi, 2\pi$

$\Rightarrow$ $x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $x = 0, \frac{\pi}{2}, \pi$

$\Rightarrow$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, 0, \frac{\pi}{2}, \pi$

Hence, 5 solutions are there.

47. (a) Taking x and y-axes as two lines.



The sum of distance of (x, y) from the two lines will be $|x| + |y| = 1$, which is a square.

48. (a) $f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$

$$P(X > x) = \int_x^{\infty} \lambda e^{-\lambda x}\, dx$$

$$= \left[\frac{\lambda e^{-\lambda x}}{-\lambda}\right]_x^{\infty} = e^{-\lambda x}$$

$$P(X > s + t) = e^{-\lambda(s+t)} = e^{-\lambda s}\, e^{-\lambda t}$$

$$= P(X > s)\, P(X > t)$$

49. (b) Equation of circle through intersection of circles

$x^2 + y^2 - 6 = 0$ and $x^2 + y^2 - 6x + 8 = 0$ is

$(x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6x + 8) = 0$

It passes through (1,1).

$\therefore$ $(1 + 1 - 6) + \lambda(1 + 1 - 6 + 8) = 0$

$\Rightarrow$ $-4 + 4\lambda = 0 \Rightarrow \lambda = 1$

Equation of required circle is

$$x^2 + y^2 - 6 + (x^2 + y^2 - 6x + 8) = 0$$

$$\Rightarrow \quad 2x^2 + 2y^2 - 6x + 2 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 3x + 1 = 0$$

50. (a) Let $f(x) = x^2 - x + 1 \Rightarrow f'(x) = 2x - 1$

$\Rightarrow$ $f''(x) = 2$

$\Rightarrow$ $f(0) = 1$; $f'(0) = -1$

and $f''(x) > 0, \forall x$

51. (d) Let $I = \int_{-\pi}^{\pi} \frac{\sin 3\theta}{5 - 3\cos\theta}\, d\theta$

$$\sin 3(2\pi - \theta) = \sin(6\pi - 3\theta) = -\sin 3\theta$$

$$\cos(2\pi - \theta) = \cos\theta$$

$$\text{If} \quad f(\theta) = \frac{\sin 3\theta}{5 - 3\cos\theta}$$

$$f(2\pi - \theta) = \frac{-\sin 3\theta}{5 - 3\cos\theta} = -f(\theta)$$

Hence, $I = 0$

52. (b) Required vectors are

$$\pm \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}$$

53. (b) This is the truth table of conditional statements *i.e.*, $B \to A$.

54. (d) In the question array size limit is 0 to 4 and array $[i] = i$; statement is executed 5 times that means

0	1	2	3	4
array [0]	array [1]	array [2]	array [3]	array [4]

Now, next statement is $ip =$ array; but pointer which is declared initially that is p. So, one error will come. Hence, option (d) is correct.

55. (c) $(a, 1, 1), (1, b, 1)$ and $(1, 1, c)$ are coplanar.

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

Using $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$\Rightarrow \quad \begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$$

$\Rightarrow$ $(a - 1)(bc - c - 1 + c) - (1 - b)(0 - 1 + c) = 0$

$\Rightarrow$ $(a - 1)(bc - 1) - (1 - b)(c - 1) = 0$

$\Rightarrow$ $(1 - a)(1 - bc) + (1 - b)(1 - c) = 0$

Dividing by $(1 - a)(1 - b)(1 - c)$, we get

$$\frac{1 - bc}{(1 - b)(1 - c)} + \frac{1}{1 - a} = 0$$

$$\Rightarrow \quad \frac{1}{1 - a} + \frac{1 - bc}{(1 - b)(1 - c)} + 1 = 0 + 1$$

$$\Rightarrow \quad \frac{1}{1 - a} + \frac{1 - b + 1 - c}{(1 - b)(1 - c)} = 1$$

$$\Rightarrow \quad \frac{1}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = 1$$

56. (c) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$$\frac{x^n}{n!} < \frac{1}{2} \times 10^{-6} \quad \text{at } x = 1$$

$$\Rightarrow \quad 2 \times 10^6 < n!$$

$$\Rightarrow \quad n! > 2000000$$

At $n = 10$, first time this inequality is satisfied.

57. (d) $x = \sqrt{N} \Rightarrow x^2 = N$

$\Rightarrow$ $x^2 - N = 0$

Let $f(x) = x^2 - N$

$\Rightarrow$ $f'(x) = 2x$

$$\Rightarrow \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 - N}{2x_n} = \frac{x_n^2 + N}{2x_n}$$

$$\Rightarrow \quad x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right).$$

58. (d) $2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

have infinite number of solutions, if $r(A:B) = r(A)$

$$\Rightarrow \left(\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -\frac{15}{2} & -\frac{39}{4} & -\frac{47}{2} \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{array}\right)$$

By $R_2 \to R_2 - \frac{7}{2} R_1$

$R_3 \to R_3 - R_1$

For infinite number of solutions

$\lambda - 5 = 0$ and $\mu - 9 = 0$

$\Rightarrow \quad \lambda = 5$ and $\mu = 9$

59. (a) $f(x, y) = kxy$; $x = 1, 2, 3$ and $y = 1, 2, 3$

will be joint probability distribution, if

$$\sum_y \sum_x f(x, y) = 1$$

$$\sum_{x=1}^{3} \sum_{y=1}^{3} kxy = 1$$

$$\Rightarrow \quad k(1+2+3)(1+2+3) = 1 \Rightarrow k = \frac{1}{36}$$

60. (b) $S = |x-1| + \left|x - \frac{1}{2}\right| + \left|x - \frac{1}{3}\right| + \left|x - \frac{1}{4}\right| + \left|x - \frac{1}{5}\right|$

is minimum at the mid value of the roots of the values under modulus.

Mid value of $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1$ is $\frac{1}{3}$.

61. (c) $y = x^2$

$$\Rightarrow \quad \frac{dy}{dx} = 2x$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(2,\,4)} = 4$$

Equation of tangent line is

$y - 4 = 4(x - 2)$

$\Rightarrow \quad 4x - y - 4 = 0$

Now, equation of circle is

$(x-2)^2 + (y-4)^2 + \lambda(4x - y - 4) = 0$

It is passing through $(0, 1)$, then

$4 + 9 + \lambda(0 - 1 - 4) = 0$

$$\Rightarrow \quad \lambda = \frac{13}{5}$$

Equation of circle is

$5\{(x-2)^2 + (y-4)^2\} + 13(4x - y - 4) = 0$

$\Rightarrow \quad 5(x^2 + y^2) + 32x - 53y + 48 = 0$

$$\Rightarrow \quad x^2 + y^2 + \frac{32}{5}x - \frac{53}{5}y + \frac{48}{5} = 0$$

$\therefore$ Centre of circle is $\left(-\frac{16}{5}, \frac{53}{10}\right)$.

62. (a) $u = \cos(x + y) + \cos(x - y)$

$$\Rightarrow \quad \frac{\partial^2 u}{\partial x^2} = -[\cos(x+y) + \cos(x-y)] \quad \ldots\text{(i)}$$

$$\frac{\partial^2 u}{\partial y^2} = -[\cos(x+y) + \cos(x-y)] \quad \ldots\text{(ii)}$$

$$\text{and} \quad \frac{\partial^2 u}{\partial x \partial y} = -\cos(x+y) + \cos(x-y) \quad \ldots\text{(iii)}$$

From Eqs. (i), (ii) and (iii) only (a) is true.

63. (b) The TEST instruction for 8086 microprocessor performs the function of non-destructive AND.

64. (d) $S_n = \sum_{k=0}^{\infty} f_k^2$

Let $S = \sum_{n=0}^{\infty} (-1)^n S_n$

$= S_0 - S_1 + S_2 - S_3 + S_4 - S_5 + \ldots\ldots$

$= f_0^2 - (f_0^2 + f_1^2) + (f_0^2 + f_1^2 + f_2^2) - \ldots\ldots$

$= -(f_1^2 + f_3^2 + f_5^2 + \ldots\ldots)$

or $(f_0^2 + f_2^2 + f_4^2 + \ldots\ldots)$

which varies between $-\infty$ and ∞, so none is true.

65. (c) $z = \frac{1 + i\cos\theta}{1 - 2i\cos\theta} = \frac{1 + i\cos\theta}{1 - 2i\cos\theta} \times \frac{1 + 2i\cos\theta}{1 + 2i\cos\theta}$

$$= \frac{(1 - 2\cos^2\theta) + 3i\cos\theta}{1 + 4\cos^2\theta}$$

It is purely real, if $\cos\theta = 0$

$$\Rightarrow \quad \theta = 2n\pi \pm \frac{\pi}{2}$$

66. (a) $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$

$(\cos\alpha + \cos\beta + \cos\gamma) + i(\sin\alpha + \sin\beta + \sin\gamma) = 0$

$\Rightarrow \quad e^{i\alpha} + e^{i\beta} + e^{i\gamma} = 0$

Squaring both sides, we get

$[e^{i\alpha} + e^{i\beta} + e^{i\gamma}]^2 = 0$

$\Rightarrow \quad e^{i2\alpha} + e^{i2\beta} + e^{i2\gamma} + 2[e^{i(\alpha+\beta)} + e^{i(\beta+\gamma)} + e^{i(\gamma+\alpha)}]$

$\Rightarrow \quad [\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$+ 2\{\cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\gamma + \alpha)\}]$

$+ i[\sin 2\alpha + \sin 2\beta + \sin 2\gamma + 2\{\sin(\alpha + \beta)$

$+ \sin(\beta + \gamma) + \sin(\gamma + \alpha)\}] = 0$

Hence, (I) and (II) are true.

$\Rightarrow$ Real part is zero.

67. (d)

$$\frac{2x - 1}{2x^3 + 3x^2 + x} > 0$$

$$\Rightarrow \quad \frac{2x - 1}{x(2x^2 + 3x + 1)} > 0$$

$$\Rightarrow \quad \frac{2x - 1}{x(2x + 1)(x + 1)} > 0 \quad \ldots\text{(i)}$$

+ | − | + | − | +

at -1, $-\frac{1}{2}$, 0, $\frac{1}{2}$

$\Rightarrow$ Eq. (i) holds for

$x \in (-\infty, -1) \cup \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

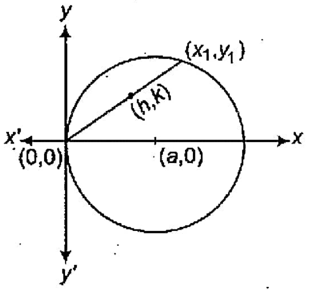
68. (a) $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$

$= (\cos^4 x + \cos^2 x)^3 - 1$

$= (\sin^2 x + \cos^2 x)^3 - 1 \qquad (\because \sin x = \cos^2 x)$

$= 1 - 1 = 0$

69. (c) Circle is



$$x^2 + y^2 - 2ax = 0$$

Let (x_1, y_1) be the other end of chord.

$\therefore \quad x_1^2 + y_1^2 - 2ax_1 = 0 \quad \ldots$(i)

Again, let (h, k) be the mid-point of this chord.

$\Rightarrow \quad h = \frac{0 + x_1}{2}, k = \frac{0 + y_1}{2}$

$\Rightarrow \quad x_1 = 2h, y_1 = 2k$

From Eq. (i), $\quad 4h^2 + 4k^2 - 4ah = 0$

$\Rightarrow \quad h^2 + k^2 - ah = 0$

Hence, locus is $x^2 + y^2 - ax = 0$

70. (b) Since, $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \pi$

$\Rightarrow \quad 2\int_0^{\infty} \frac{\sin x}{x} dx = \pi$

$\Rightarrow \quad \int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$

71. (d) $A \to B \leftrightarrow \neg A \vee B$ (by conditional formula)

$\neg (A \vee B) \to \neg A \wedge \neg B$ (by De Morgan law)

$(A \vee \neg B) \to C \leftrightarrow \neg (A \vee \neg B) \vee C$

$\leftrightarrow (\neg A \wedge B \vee C)$ (by conditional and De Morgan law).

So, all three are logically equivalent.

72. (a) The continue statement is used to skip the rest statement in the loop and sends control to the starting of the for loop.

So, every time in the for loop $++j$ statement is not executed so j will not be incremented and j remains 0.

73. (b) $(D9)_{16} = (\_\_\_)_8$

First of all convert the given hexadecimal to binary $(11011001)_2$

Now, this binary form is converted into octal by making a pair of 3 binary digits from right side.

$(11\ 011\ 001)_2 = (331)_8$

74. (d) DMA (Direct Memory Access) is responsible for data movement from I/O to memory and *vice-versa*.

75. (a) $|z^2 - 1| = |z|^2 + 1$

Let $z = x + iy$

$\Rightarrow |x^2 - y^2 - 1 + 2ixy| = x^2 + y^2 + 1$

$= \sqrt{(x^2 - y^2 - 1)^2 + 4x^2y^2} = \sqrt{(x^2 + y^2 + 1)}$

$\Rightarrow (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)$

$\Rightarrow 4x^2y^2 = [x^2 + (y^2 + 1)]^2 - [x^2 - (y^2 + 1)]^2 = 4(y^2 + 1)x^2$

$\Rightarrow \quad x^2y^2 = (y^2 + 1)x^2$

$\Rightarrow \quad x^2 = 0$

$\Rightarrow \quad x = 0$

which is a straight line.

76. (b) $3^{x-1} + 5^{x-1} = 34$

$\Rightarrow \quad x = 3$ is the only solution.

77. (a) Let p = It rains.

q = I am going.

$\sim q$ = I am not going.

$p \to \sim q$ represent "If I don't go, it rains."

Converse of $p \to \sim q$ is $\sim q \to p$ represent "If I don't go, it rains."

78. (b) Given, $\quad x = 6, y = 11, z = -2$ and the statement is

$((x/2) > y) \,||\, (x > z)$

In the given statement OR operator ($||$) is used , so, this type of statement is conditional statement. If any one side of OR operator becomes true, then the result of the statement will be 1 otherwise 0.

$((6/2) > 11) \,||\, (6 > -2)$

$\Rightarrow \quad (3 > 11) \,||\, (6 > -2)$

$\Rightarrow \quad 0 \,||\, 1$

$\Rightarrow \quad 1$

79. (d) Lines,

$2(\sin A + \sin B)\, x - 2\sin(A - B)\, y = 3$ and

$2(\cos A + \cos B)x + 2\cos(A - B)\, y = 5$

are perpendicular, if

$$\frac{\sin A + \sin B}{\sin(A - B)} \times \left(\frac{-(\cos A + \cos B)}{\cos(A - B)}\right) = -1$$

$\Rightarrow \quad \sin(A - B)\cos(A - B) = (\sin A + \sin B)(\cos A + \cos B)$

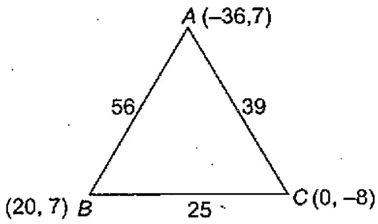
$\Rightarrow \quad 2\sin(A - B)\cos(A - B) = 2[\sin A \cos A + \sin A \cos B + \sin B \cos A + \sin B \cos B]$

$\Rightarrow \quad \sin(2(A - B)) = \sin 2A + \sin(A + B) + \sin(A - B) + \sin(A + B) - \sin(A - B) + \sin 2B$

$\Rightarrow \quad \sin(2(A - B)) = \sin 2A + \sin 2B + 2\sin(A + B)$

$\Rightarrow \quad \sin 2A + \sin 2B = \sin(2(A - B)) - 2\sin(A + B)$

80. (b) Centroid of $\Delta\, ABC$ is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.



$\Rightarrow \quad G$ is $\left(-\frac{16}{3}, 2\right)$.

By using distance formula, we get

$AB = 56, BC = 25, CA = 39$

$\therefore \quad$ Incentre of $\Delta\, ABC$ is

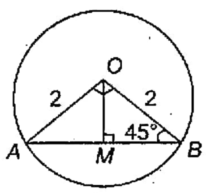
$$\left(\frac{-36 \times 25 + 20 \times 39 + 0}{56 + 25 + 39}, \frac{7 \times 25 + 7 \times 39 - 8 \times 56}{56 + 25 + 39}\right) \Rightarrow$$

$I(-1, 0)$

$\Rightarrow \quad GI = \sqrt{\left(\frac{-13}{3}\right)^2 + (2)^2}$

$= \sqrt{\frac{169 + 36}{9}} = \frac{\sqrt{205}}{3}$

81. (c) Let M be (h, k).



$\sin \angle OBM = \frac{OM}{OB}$

$\Rightarrow \quad \sin 45° = \frac{\sqrt{h^2 + k^2}}{2} \Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{h^2 + k^2}}{2}$

$\Rightarrow \quad h^2 + k^2 = 2$

Hence, locus is $x^2 + y^2 = 2$

82. (d) $I = \int_{-\infty}^{\infty} \frac{1}{(x^2 + 4x + 5)^2} dx$

$= 2\int_0^{\infty} \frac{1}{[1 + (x+2)^2]^2} dx$

Put $x + 2 = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta \, d\theta$

$\therefore \quad I = 2\int_0^{\infty} \frac{1}{[1 + (x+2)^2]^2} dx$

$= 2\int_{\tan^{-1}2}^{\frac{\pi}{2}} \cos^4\theta \cdot \sec^2\theta \, d\theta$

$= 2\int_{\tan^{-1}2}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$

$= \int_{\tan^{-1}2}^{\frac{\pi}{2}} (1 + \cos 2\theta) \, d\theta = \left[\theta + \frac{\sin 2\theta}{2}\right]_{\tan^{-1}2}^{\frac{\pi}{2}}$

$= \left(\frac{\pi}{2} + 0\right) - \left(\tan^{-1}2 - \frac{2}{5}\right)$

$\Rightarrow \quad I = \cot^{-1}2 + \frac{2}{5}$

83. (d) $A(1, 0, 3)$, $B(-1, 3, 4)$, $C(1, 2, 1)$, $D(k, 2, 5)$ are coplanar.

If $\quad [\text{AB BC CD}] = 0$

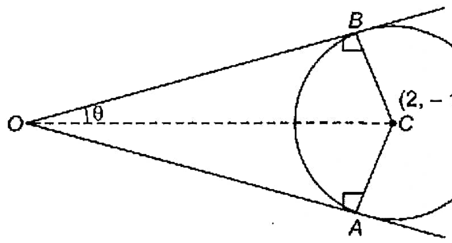
$\Rightarrow \begin{vmatrix} -2 & 3 & 1 \\ 2 & -1 & -3 \\ k-1 & 0 & 4 \end{vmatrix} = 0$

$\Rightarrow \quad -2(-4 - 0) - 3(8 + 3k - 3) + 1(0 + k - 1) = 0$

$\Rightarrow \quad 8 - 15 - 9k + k - 1 = 0$

$\Rightarrow \quad -8 = 8k \Rightarrow k = -1$

84. (c) Radius of circle is



$\left|\frac{6-1}{\sqrt{9+1}}\right| = \left|\frac{5}{\sqrt{10}}\right| = \sqrt{\frac{5}{2}}$

$OC = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$\Rightarrow \quad \sin\theta = \frac{BC}{OC} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = 45°$

$\Rightarrow$ OA is perpendicular to OB.

$\Rightarrow$ Other tangent through origin is $x - 3y = 0$.

85. (d) Since, $\int_0^{\infty} \frac{x^{p-1}}{1+x} dx = \frac{\pi}{\sin p\pi} = \Gamma p \, \Gamma(1-p)$

$(0 < p < 1)$

86. (a) $[\mathbf{a} - \mathbf{b}, \ \mathbf{b} - \mathbf{c}, \ \mathbf{c} - \mathbf{a}]$

$= [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] - [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

$= 0$

87. (c) Given, $z^{\frac{1}{3}} = a - ib$

$\Rightarrow \quad z = (a - ib)^3$

$\Rightarrow \quad x + iy = a^3 - 3a^2 ib + 3a(ib)^2 - (ib)^3$

$\Rightarrow \quad x + iy = (a^3 - 3ab^2) - i(3a^2 b - b^3)$

$\Rightarrow \quad x = a^3 - 3ab^2, y = b^3 - 3a^2 b$

$\Rightarrow \quad \frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2 = 4(a^2 - b^2)$

$\Rightarrow \quad k = 4$

88. (b) For $n > 2$ given inequality holds.

89. (c) $\quad 3\sin^2 x + 10\cos x - 6 = 0$

$\Rightarrow \quad 3 - 3\cos^2 x + 10\cos x - 6 = 0$

$\Rightarrow \quad 3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow \quad (3\cos x - 1)(\cos x - 3) = 0$

$\cos x = \frac{1}{3} \qquad (\cos x \neq 3)$

$\Rightarrow \quad x = 2n\pi \pm \cos^{-1}\frac{1}{3}$

90. (a) $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} \times \mathbf{b})$

$= \mathbf{a} \times (\mathbf{a} \times \mathbf{b}) + \mathbf{b} \times (\mathbf{a} \times \mathbf{b})$

$= (\mathbf{a} \cdot \mathbf{b})\,\mathbf{a} - (\mathbf{a} \cdot \mathbf{a})\,\mathbf{b} + (\mathbf{b} \cdot \mathbf{b})\,\mathbf{a} - (\mathbf{b} \cdot \mathbf{a})\,\mathbf{b}$

$= (\mathbf{a} \cdot \mathbf{b} + 1)(\mathbf{a} - \mathbf{b})$

$=$ scalar multiple of $(\mathbf{a} - \mathbf{b})$

$\Rightarrow$ $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} \times \mathbf{b})$ is parallel to $\mathbf{a} - \mathbf{b}$.

91. (d) $||x| - 1| < 1 - x$

which holds for all $x < 0$

i.e., inequality satisfies, if $x \in (-\infty, 0)$

92. (b) $\quad 4^{-x + 0.5} - 7 \cdot 2^{-x} - 4 < 0$

$\Rightarrow \quad 4^{-x} \cdot 4^{0.5} - 7 \cdot 2^{-x} - 4 < 0$

$\Rightarrow \quad (2^{-x})^2 \cdot 2 - 7 \cdot 2^{-x} - 4 < 0$

Put $2^{-x} = t$

$\Rightarrow \quad 2t^2 - 7t - 4 < 0$

$\Rightarrow \quad (2t + 1)(t - 4) < 0$

$\Rightarrow \quad t < 4 \qquad (\because 2t + 1 > 0)$

$\Rightarrow \quad 2^{-x} < 4$

$\Rightarrow \quad 2^{-x} < 2^2 \Rightarrow -x < 2$

$\Rightarrow \quad x > -2$

i.e., $\quad x \in (-2, \infty)$

93. (a) $x\cos\alpha + y\sin\alpha = 2a$

and $x\cos\beta + y\sin\beta = 2a$

$\Rightarrow$ α and β are roots of

$x\cos\theta + y\sin\theta = 2a$

Now,

$x\cos\theta + y\sin\theta = 2a$

$\Rightarrow \quad y\sin\theta = 2a - x\cos\theta$

$\Rightarrow \quad y^2 - y^2\cos^2\theta = 4a^2 + x^2\cos^2\theta - 4ax\cos\theta$

$\Rightarrow \quad (x^2 + y^2)\cos^2\theta - 4ax\cos\theta + 4a^2 - y^2 = 0$

$\Rightarrow \quad \cos\alpha + \cos\beta = \frac{4ax}{x^2 + y^2}$

and $\quad \cos\alpha\cos\beta = \frac{4a^2 - y^2}{x^2 + y^2}$

94. (b) $\frac{dy}{dt} = -x^2; \frac{dx}{dt} = x$

$\Rightarrow \quad \frac{dy}{dx} = -x$

$\Rightarrow \quad dy = -x\,dx$

$\Rightarrow \quad y = -\frac{x^2}{2} + C$

It passes through $(1, -4)$

$\Rightarrow \quad -4 = -\frac{1}{2} + C \Rightarrow C = -\frac{7}{2}$

$\Rightarrow \quad y = \frac{-x^2 - 7}{2}$

96. (d) $y = mx + c$ is tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$c^2 = a^2m^2 + b^2$$

$\Rightarrow$ $c^2 = 8(4)^2 + 4 = 132$

$\Rightarrow$ $c = \pm\sqrt{132}$

97. (c) $A = \int_1^b f(x)\,dx = (b-1)\sin(3b+4)$ By Leibnitz's rule.

$\Rightarrow$ $\frac{dA}{db} = f(b) = 3(b-1)\cos(3b+4) + \sin(3b+4)$

$\Rightarrow$ $f(x) = \sin(3x+4) + 3(x-1)\cos(3x+4)$

98. (d) $2\log_{10} x - \log_x 0.01$

$= 2\log_{10} x - \log_x 10^{-2}$

$= 2\log_{10} x + 2\log_x 10$

$= 2[\log_{10} x + \log_x 10] \ge 2.2\sqrt{\log_{10} x \log_x 10} \ge 4$

As AM $\ge$ GM of two positive values.

99. (b) $y = \frac{1 + \sqrt{1 - \sin 4A}}{\sqrt{1 + \sin 4A} - 1}$

$= \frac{1 + (\cos 2A - \sin 2A)}{\cos 2A + \sin 2A - 1}$

$= \frac{2\cos^2 A - 2\sin A\cos A}{2\sin A\cos A - 2\sin^2 A}$

$= \frac{\cos A(2\cos A - 2\sin A)}{\sin A(2\cos A - \sin A)} = \cot A$

100. (d) $(2\sqrt{3} + 4)\sin x + 4\cos x$

It's range is from $[-a, a]$, then

$a = \sqrt{(2\sqrt{3} + 4)^2 + (4)^2}$

$= \sqrt{44 + 16\sqrt{3}}$

$= 2\sqrt{11 + 4\sqrt{3}} > 4 + 2\sqrt{5}$

101. (b) $\sum_{n=1}^{\infty} \left\{a^{1/n} - \left(\frac{b^{1/n} + c^{1/n}}{2}\right)\right\}$ is convergent, if

$\lim_{n\to\infty} a^{1/n} - \frac{b^{1/n} + c^{1/n}}{2} = 0$

$\Rightarrow$ $\lim_{n\to\infty} a^{1/n} = \lim_{n\to\infty} \frac{b^{1/n} + c^{1/n}}{2} = \lim_{n\to\infty} (\sqrt{bc})^{1/n}$

AM $\approx$ GM

$\Rightarrow$ $a = \sqrt{bc}$

102. (a) $y = x^2$ and $y = 2x$

$\Rightarrow$ $2x = x^2 \Rightarrow x(2 - x) = 0$

$\Rightarrow$ $x = 0, 2 \Rightarrow y = 0, 4$

Volume of bounded region

$V = \int_{y=a}^{b} \pi(x_1^2 - x_2^2)dy$

Here, $a = 0, b = 4, x_1^2 = y, x_2^2 = \frac{y^2}{4}$

$\Rightarrow$ $V = \int_0^4 \pi\left(y - \frac{y^2}{4}\right)dy$

$= \left[\pi\left(\frac{y^2}{2} - \frac{y^3}{12}\right)\right]_0^4 = \frac{8\pi}{3}$

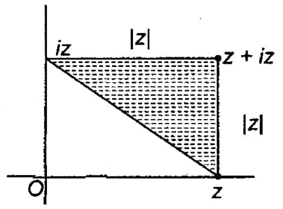
103. (a) $f(t) = \int_t^{t^2} \frac{\sin tx}{x}\,dx$

By using Newton's Leibnitz rule, we get

$f'(t) = 2t\frac{\sin t^3}{t} - \frac{\sin t^2}{t}$

$\Rightarrow$ $f'(1) = 2\sin 1 - \sin 1 = \sin(1)$

104. (c) Area of triangle $= \frac{1}{2}|z||z| = 50$



$\Rightarrow$ $|z| = 10$

105. (b) $3\tan A + 4 = 0$

$\Rightarrow$ $\tan A = -4/3$

Given angle A lies in the second quadrant

$\Rightarrow$ $\sin A = 4/5, \cos A = -3/5, \cot A = -3/4$

$\therefore$ $2\cot A - 5\cos A + \sin A$

$= \frac{-6}{4} + 3 + \frac{4}{5} = \frac{23}{10}$

106. (a) $\tan(\pi\cos\theta) = \cot(\pi\sin\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\Rightarrow$ $\pi\cos\theta = \frac{\pi}{2} - \pi\sin\theta$

$\Rightarrow$ $\cos\theta + \sin\theta = \frac{1}{2}$

Now, $\cos\left(\theta - \frac{\pi}{4}\right) = \cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta) = \frac{1}{2\sqrt{2}}$

107. (c) $\tan 1^\circ \tan 2^\circ \ldots \tan 89^\circ$

$= (\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ) \ldots \tan 45^\circ$

$= 1.1 \ldots 1 = 1$

$\left[\begin{array}{l}\because \tan(90^\circ - \theta) = \cot\theta \\ \text{and } \tan\theta\cot\theta = 1\end{array}\right]$

108. (a) Given, $\mathbf{A} \times \mathbf{B} = \mathbf{C}$

$\mathbf{A} \cdot \mathbf{B} = 3$

$\mathbf{A} = \mathbf{i} + \mathbf{j} + \mathbf{k}$; $\mathbf{C} = \mathbf{j} - \mathbf{k}$

Let $\mathbf{B} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

$\Rightarrow$ $\mathbf{A} \times \mathbf{B} = \mathbf{C}$ gives

$\Rightarrow$ $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \mathbf{j} - \mathbf{k}$

$(z - y)\mathbf{i} + (x - z)\mathbf{j} + (y - x)\mathbf{k} = \mathbf{j} - \mathbf{k}$

$y = z, x - z = 1, y - x = -1$ ...(i)

$\mathbf{A} \cdot \mathbf{B} = x + y + z = 3$ ...(ii)

Soving Eqs. (i) and (ii),

$x + 2y = 3 \Rightarrow y = \frac{2}{3}$

and $x - y = 1$

$\Rightarrow$ $x = \frac{5}{3}$

$z = \frac{2}{3}$

$\mathbf{B}$ is $\left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right)$.

109. (c) $f(x) = \frac{\tan(\pi[x - \pi])}{1 + [x]^2} = 0$

As at integral multiple of π, $\tan\theta$ is zero, so it is differentiable everywhere.

110. (d) $\int_0^1 (1+\cos^2 x)(ax^2+bx+c)\,dx$

$$= \int_0^2 (1+\cos^2 x)(ax^2+bx+c)\,dx$$

$$\Rightarrow \quad \int_1^2 (1+\cos^2 x)(ax^2+bx+c)\,dx = 0$$

$\Rightarrow$ $ax^2+bx+c=0$ has atleast one root in (1, 2) and hence in (0, 2).

111. (c) $y(2xy+e^x)\,dx - e^x\,dy = 0$ ...(i)

$\Rightarrow \quad M\,dx + N\,dy = 0$

where, $M = 2xy^2 + ye^x$

$N = -e^x$

$$\frac{\partial M}{\partial y} = 4xy + e^x$$

and $$\frac{\partial N}{\partial x} = -e^x$$

$$\therefore \quad \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-e^{-x} - 4xy - e^x}{y(2xy+e^x)}$$

$$= -\frac{2}{y} = \text{function of } y \text{ alone}$$

$\Rightarrow$ Integrating factor is

$$e^{-\int \frac{2}{y} dy} = e^{-2\ln y} = \frac{1}{y^2}$$

Multiplying Eq. (i) by $\frac{1}{y^2}$, we get

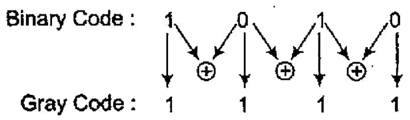
$$\left(2x + \frac{e^x}{y}\right)dx - \frac{e^x}{y^2}\,dy = 0$$

It is exact, so solution is $x^2 + \frac{e^x}{y} = -C$

$$\Rightarrow \quad y(x^2+C) + e^x = 0$$

112. (b) The propagation delay encountered in ripple carry adder of 4-bit size is 4* $t_p$where t_p is the delay of a single flip-flop and adder is 4-bit. So, we have 4 flip-flop is used.

113. (a) We have binary code = $(1010)_2$



Gray Code is $(1111)_2$.

114. (b) Given a positive number N in base 2 with an integer part of n digits. The 2's complement of N is defined as $2^n - N$ for $N \neq 0$ and 0 for $N = 0$

115. (a) In the given program, call by value and call by reference are used. Variable a is called by reference while variable b is called by value. So, a would change but b remain same

$$a = 10, b = 20$$

116. (a) In this, we have given $n = 100$. So, this n is passed to function sum through pointer variable ptr. We pass the address of function sum to ptr and call the function through ptr.

In function sum, there is a loop which will be going on from 1 to 100 that means 100 times and each time variable j calculates the sum of digits.

So, $j = 5050\ (1+2+3+4+5+6+\ldots\ldots+100)$

So, sum = 5050

117. (b) $z = (\lambda+3) + i(5-\lambda^2)^{\frac{1}{2}}$

$$\Rightarrow \quad x = \lambda + 3, y = (5-\lambda^2)^{\frac{1}{2}}$$

$$\Rightarrow \quad (x-3)^2 = \lambda^2, y^2 = 5 - \lambda^2$$

$$\Rightarrow \quad (x-3)^2 + y^2 = 5$$

which is a circle.

118. (b) $x = \sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots\infty}}}$

$$\Rightarrow \quad x^2 = 2 + x$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad (x+1)(x-2) = 0$$

$$\Rightarrow \quad x = 2 \qquad (x \neq -1)$$

119. (c) Using AM $\geq$ GM

$$5^x + 5^{-x} \geq 2\sqrt{5^x \cdot 5^{-x}}$$

$$\Rightarrow \quad 5^x + 5^{-x} \geq 2$$

But $|\sin(e^x)| \leq 1$

$\Rightarrow$ Given equation has no real solution.

120. (c) $\frac{dy}{dx} = \frac{xy+y}{xy+x} = \frac{y(x+1)}{x(y+1)}$

$$\Rightarrow \quad \frac{y+1}{y}\,dy = \frac{x+1}{x}\,dx$$

$$\Rightarrow \quad \int\left(1+\frac{1}{y}\right)dy = \int\left(1+\frac{1}{x}\right)dx$$

$$\Rightarrow \quad y + \ln y = x + \ln x + \ln k$$

$$\Rightarrow \quad y - x = (\ln x - \ln y) + \ln k$$

$$= \ln\frac{x}{y} + \ln k$$

$$= \ln\frac{kx}{y}$$