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ENTRANCE EXAMINATION, 2012 **414**

MASTER OF COMPUTER APPLICATIONS

[**Field of Study Code : MCAM (224)**]

Time Allowed : 3 hours

Maximum Marks : 480
Weightage : 100

INSTRUCTIONS FOR CANDIDATES

Candidates must read carefully the following instructions before attempting the Question Paper :

(i) Write your Name and Registration Number in the space provided for the purpose on the top of this Question Paper and in the Answer Sheet.

(ii) **Please darken the appropriate Circle of Question Paper Series Code on the Answer Sheet.**

(iii) All questions are compulsory.

(iv) Answer all the 120 questions in the Answer Sheet provided for the purpose by darkening the correct choice, i.e., (a) or (b) or (c) or (d) with BALLPOINT PEN only against the corresponding circle. Any overwriting or alteration will be treated as wrong answer.

(v) Each correct answer carries 4 marks. **There will be negative marking and 1 mark will be deducted for each wrong answer.**

(vi) Answer written by the candidates inside the Question Paper will not be evaluated.

(vii) Calculators and Log Tables may be used.

(viii) Pages at the end have been provided for Rough Work.

(ix) Return the Question Paper and Answer Sheet to the Invigilator at the end of the Entrance Examination. **DO NOT FOLD THE ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS

1. Use only Blue/Black Ballpoint Pen (do not use pencil) to darken the appropriate Circle.
2. Please darken the whole Circle.
3. Darken <u>ONLY ONE CIRCLE</u> for each question as shown in the example below :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	Ø ⓑ ⓒ ⓓ	Ø ⓑ ⓒ Ø	⊙ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

4. Once marked, no change in the answer is allowed.
5. Please do not make any stray marks on the Answer Sheet.
6. Please do not do any rough work on the Answer Sheet.
7. Mark your answer only in the appropriate space against the number corresponding to the question.
8. **Ensure that you have darkened the appropriate Circle of Question Paper Series Code on the Answer Sheet.**

1. Among the following statements, identify the number of correct statements :

(i) The function defined by

$$f(x) = \frac{ax+b}{cx+d}$$

always has maxima and minima for whatever values of the real numbers a, b, c and d.

(ii) $\log(x)$ is a convex function in the real line.

(iii) The function defined by $f(x) = x - \sin x$ is a decreasing function throughout in any interval of values of the variable x.

(a) 0

(b) 1

(c) 2

(d) 3

2. For arbitrary constants c_1 and c_2, the solution space of the differential equation $y'' - 8y' + 16y = 0$ will be

(a) $y(x) = c_1e^{-4x} + c_2xe^{-4x}$

(b) $y(x) = c_1e^{4x} + c_2e^{-4x}$

(c) $y(x) = c_1e^{4x} + c_2xe^{4x}$

(d) None of the above

3. Evaluate the integral

$$\int_{-1}^{1} |2x-1|\, dx$$

where $|\cdot|$ denotes the absolute value.

(a) $\frac{5}{2}$

(b) $\frac{3}{2}$

(c) 0

(d) None of the above

4. How many committees of five people can be chosen from 20 men and 12 women if at least 4 women must be chosen on each committee?

(a) 9872

(b) 10012

(c) 10692

(d) None of the above

5. There are five different houses, A to E, in a row. A is to the right of B and E is to the left of C and right of A. Further, B is to the right of D. Which house will be in the middle?

(a) A

(b) B

(c) D

(d) None of the above

6. If a matrix A is invertible, then which property/properties of A remains/remain true?

(i) A is symmetric.

(ii) A is triangular.

(iii) All entries are integers.

(a) Only (i)

(b) Only (i) and (ii)

(c) All the properties (i), (ii) and (iii)

(d) None of the above

7. The number of diagonals that can be drawn by joining the vertices of an octagon is

(a) 28

(b) 20

(c) 24

(d) 48

8. Consider the function $(x+2)\cos^2 x$ for $x \geq 2$. Determine its order in terms of big-O notation.

(a) $O(x)$

(b) $O(x^2)$

(c) $O(\log(x))$

(d) None of the above

9. A particle acted by constant forces $4\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ is displaced from the point (1, 2, 3) to the point (5, 4, 1), where **i**, **j** and **k** are unit vectors along the X-, Y- and Z-axis respectively. Then the total work done by the forces is

(a) 20 units

(b) 30 units

(c) 40 units

(d) None of the above

10. The maximum value of the function defined by

$$f(x) = 2\sin x + \sin 2x$$

in the interval $\left[0, \frac{3\pi}{2}\right]$ is

(a) $\frac{5}{2}$

(b) $\frac{3\sqrt{5}}{2}$

(c) $\frac{3\sqrt{3}}{2}$

(d) None of the above

11. In how many ways can the letters of the word 'attention' be rearranged?

(a) 28220

(b) 30240

(c) 32120

(d) None of the above

12. In a certain code language

(i) 'mxy das zci' means 'good little frock'

(ii) 'jmx cos zci' means 'girl behaves good'

(iii) 'nug drs cos' means 'girl makes mischief'

(iv) 'das ajp cos' means 'little girl fell'.

Which word in that language stands for 'frock'?

(a) zci

(b) das

(c) mxy

(d) Insufficient information

13. The number of solutions of the equation $\sqrt{3x^2 + x + 5} = x - 3$ is

(a) ∞

(b) 1

(c) 0

(d) None of the above

14. The radius of the circle in which the sphere $x^2 + y^2 + z^2 = 5$ is cut by the plane $x + y + z = 3\sqrt{3}$ is

(a) $\sqrt{3}$

(b) $3\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$

(d) None of the above

15. Suppose A, B and C are sets. Consider the following statements :

(i) $A \in B$, $B \subseteq C$. Then $A \subseteq C$ is true.

(ii) $A \not\subset B$. Then $B \subset C$ is true.

(iii) $C \in \wp(A)$ if and only if $C \subseteq A$, where $\wp(A)$ denotes the power set of A.

The number of correct statements among (i)–(iii) is

(a) 1

(b) 2

(c) 3

(d) None of the above

16. Of 30 personal computers (PCs) owned by faculty members in a university department, 20 run Windows, 8 have 21 inch monitors, 25 have CD-ROM drives, 20 have at least two of these features and 6 have all the three features. How many PCs have at least one of these features?

(a) 22

(b) 24

(c) 27

(d) None of the above

17. In the complex plane, consider the following statements :

(i) If $\left|e^{z}\right| = 1$, then z is a pure imaginary number.

(ii) There are complex numbers z such that $|\sin z| > 1$.

(iii) The function $\sin\bar{z}$ is nowhere analytic, where $\bar{z}$ is the complex conjugate of the number z.

Identify the number of correct statements.

(a) 0 (b) 1

(c) 2 (d) 3

18. Among the six students A, B, C, D, E and F, it is given that—

(i) D and F are tall, while the others are short

(ii) A, C and D are wearing glasses, while the others are not

Identify the short students who are not wearing glass.

(a) B, E, F

(b) B, E

(c) B, C

(d) None of the above

19. For the matrix $A = \begin{pmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{pmatrix}$, find the number of c values in which the matrix A is not invertible.

(a) 0 (b) 1

(c) 2 (d) 3

20. Transform the well-formed formula $P \rightarrow Q \wedge R$ into a disjunction normal form (DNF) and conjunction normal form (CNF) respectively.

(a) $\neg P \vee (Q \wedge R)$ and $(\neg P \vee Q) \wedge (\neg P \vee R)$

(b) $P \vee \neg(Q \wedge R)$ and $(P \wedge \neg Q) \wedge (\neg P \vee R)$

(c) $\neg P \wedge (Q \vee R)$ and $(P \vee \neg Q) \wedge (\neg P \vee R)$

(d) None of the above

21. What is the probability that the sum of two numbers x and y randomly chosen on the interval (0, 1) is greater than 1, while the sum of their squares is less than 1?

(a) $\frac{\pi}{2} - \frac{1}{4}$

(b) $\frac{\pi}{4} - \frac{1}{2}$

(c) $\frac{\pi}{6} - \frac{1}{3}$

(d) None of the above

22. The subtraction of $2A_{16}$ from 84_{16} results in

(a) 68_{16}

(b) $A6_{16}$

(c) $5A_{16}$

(d) $5B_{16}$

23. 'Joule' is related to energy and in the same way 'Pascal' is related to

(a) volume

(b) pressure

(c) purity

(d) beauty

24. If $x = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$, then the value of $\frac{\cos\alpha}{1+\sin\alpha}$ is equal to

(a) $1-x$

(b) $1+x$

(c) $\frac{1}{x}$

(d) None of the above

25. Suppose a matrix A of order 3 has eigenvalues 1, – 1, 3. What is the determinant of A^{-1}, where A^{-1} is the inverse of the matrix A?

(a) -3

(b) 3

(c) $2/3$

(d) None of the above

26. If $P(x, y)$ is a point on the line $y = -3x$ such that P and the point (3, 4) are the opposite sides of the line $3x - 4y = 8$, then

(a) $x > \frac{8}{15}, \; y < -\left(\frac{8}{5}\right)$

(b) $x > \frac{8}{5}, \; y < -\left(\frac{8}{15}\right)$

(c) $x = \frac{8}{15}, \; y = -\left(\frac{8}{5}\right)$

(d) None of the above

27. The value of the integral

$$\int_0^\infty \frac{x\log x}{(1+x^2)^2}\,dx$$

is

(a) 0

(b) $\log 2$

(c) $2\log 5$

(d) ∞

28. Find the global minimizers of the following function :

$$f(x, y) = e^{x-y} + e^{y-x}$$

(a) All points along the X-axis

(b) All points along the Y-axis

(c) Global minimum of the function $f(\cdot)$ does not exist

(d) None of the above

29. From the two statements—

(i) some cubs are tigers

(ii) some tigers are goats

we can conclude that

(a) some cubs are goats

(b) no cub is a goat

(c) all cubs are goats

(d) None of the above

30. Let X equal -1, 0 or 1 with equal probability and let $Y = |X|$. A simple calculation shows $\text{cov}(X, Y)$ equals

(a) 1

(b) -1

(c) 0

(d) None of the above

31. Let A and B be the matrices of the same order. Consider the following statements :

(i) The eigenvalues of A are equal to the eigenvalues of A^t, where A^t is the transpose of A.

(ii) The eigenvalues of AB are the product of the eigenvalues of A and B.

(iii) The eigenvalues of $(A + B)$ are the sum of the individual eigenvalues of A and B.

Identify the correct statements.

(a) Only (i) and (ii)

(b) Only (i) and (iii)

(c) (i), (ii) and (iii)

(d) None of the above

32. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that

$$\min f(x) > \max g(x)$$

then we will have

(a) $c^2 > 2b^2$

(b) $2c^2 < b^2$

(c) $b^2 + c^2 < 2$

(d) None of the above

33. Find the matrix A^{50}, when the matrix A is

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$$

(a) $\begin{pmatrix} 2^{50} & (-1)^{50-1} \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 2^{50} & -3+2^{50} \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2^{50} & -1 \\ 0 & 1 \end{pmatrix}$

(d) None of the above

34. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then its inverse is

(a) $\frac{1}{2}(1 + \sqrt{1 - 2\log_2 x})$

(b) $\frac{1}{2}(1 + \sqrt{1 + 2\log_2 x})$

(c) $\frac{1}{2}(1 + \sqrt{1 + 4\log_2 x})$

(d) None of the above

35. For a given real-valued function $h(t)$, $t \geq 0$, the Laplace transform denoted by $\bar{h}(s)$ is defined by

$$\bar{h}(s) = \int_0^{\infty} e^{-st} h(t)\, dt$$

The Laplace transform of $e^{-at}h(t)$ is

(a) $\bar{h}(s+a)$

(b) $\dfrac{\bar{h}(s)}{s+a}$

(c) $a\bar{h}(s)$

(d) None of the above

36. Among the four groups of letters from (a) to (d) given, three of them are alike in a certain way, while one is different. Identify the one that is different.

(a) ALMZ

(b) BTUY

(c) CPQX

(d) DEFY

37. How many ways can k distinguishable balls be distributed into n urns so that there are k_i balls in urn i?

(a) $\dfrac{k!}{(k_1 + k_2 + \ldots + k_n)!}$

(b) $\dfrac{k!}{k_1!k_2!\ldots k_n!}$

(c) $k_1!k_2!\ldots k_n!$

(d) None of the above

38. $\lim\limits_{n \to \infty} \left(\dfrac{1+i}{\sqrt{\pi}}\right)^n$ is equal to

(a) 0

(b) i

(c) ∞

(d) None of the above

39. AB is a chord of the parabola $y^2 = 4ax$ with the end A at the vertex of the given parabola. BC is drawn perpendicular to AB meeting the axis of the parabola at C. The projection of BC on this axis is

(a) a

(b) $2a$

(c) $4a$

(d) None of the above

40. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

(a) 18/25 (b) 16/25

(c) 729/1000 (d) 27/75

41. If the product of the roots of the equation $x^2 - 5kx + 2e^{4 \quad c} - 1 = 0$ is 31, then the sum of the roots is .

(a) 10

(b) 8

(c) 5

(d) None of the above

42. Let $f : Z \to Z$ be a function defined by $f(x) = 3x^3 - x$, where Z is the set of integers. Then the function f is

(a) injective only

(b) surjective only

(c) bijective

(d) None of the above

43. An equation of a tangent to the hyperbola

$$16x^2 - 25y^2 - 96x + 100y - 356 = 0$$

which makes an angle $\pi/4$ with the transverse axis is

(a) $y = x + 2$

(b) $y = 2x - 3$

(c) $y = x + 6$

(d) $x = 2y - 3$

44. If $s_n = \frac{1}{2}(1 - (-1)^n)$ for $n \geq 1$, then as $n \to \infty$

$$\frac{s_1 + s_2 + \ldots + s_n}{n}$$

converges to

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) None of the above

45. A triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates (3, 4) and (–4, 3) respectively, then $\angle QPR$ is equal to

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

46. If $y = \int_1^{t^3} \sqrt[3]{z} \log z \, dz$ and $x = \int_{\sqrt{t}}^{3} z^2 \log z \, dz$, then $\frac{dy}{dx}$ is

(a) $-4t^{5/2}$

(b) $35t^{5/2}$

(c) $-36t^{5/2}$

(d) None of the above

47. February 29, 1952 occurred on which day of the week?

(a) Sunday

(b) Wednesday

(c) Friday

(d) None of the above

48. Let $f(x)$ be a polynomial function and satisfy the conditions

$$f(x)f(1/x) = f(x) + f(1/x) \text{ and } f(3) = 28$$

Then the value of $f(4)$ is given by

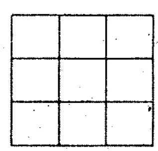
(a) 65

(b) 62

(c) 60

(d) None of the above

49. How many squares are there in the given figure?



(a) 12

(b) 14

(c) 16

(d) None of the above

50. For any three vectors **a**, **b**, **c** if $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ and $|\mathbf{a}| = 3$, $|\mathbf{b}| = 5$ and $|\mathbf{c}| = 7$, then the angle between **a** and **b** is

(a) $\frac{5\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{6}$

51. Ganesh appeared for mathematics examination. He tried to solve correctly all the 100 problems given but some of them went wrong and scored 85. The score was calculated by subtracting two times the number of wrong answers from the correct answers. Then the number of problems solved correctly is

(a) 95

(b) 92

(c) 90

(d) None of the above

52. If the sum of the lengths of the hypotenuse and another side of a right-angled triangle is given, then the area of the triangle is maximum when the angle between those sides is

(a) 30 degrees

(b) 60 degrees

(c) 90 degrees

(d) None of the above

53. Determine the probability that after $2n$ tosses of a fair coin, there have been the same number of heads as tails.

(a) $\binom{2n}{n}\frac{1}{2^{2n}}$

(b) $\binom{2n}{n}\frac{1}{2^{n}}$

(c) $\frac{1}{2^{2n}}$

(d) None of the above

54. Let a, b be positive integers and let p be a prime number such that $\gcd(a, p^2) = p$ and $\gcd(b, p^3) = p^2$ are satisfied, where $\gcd(.,.)$ denotes the greatest common divisor. Then $\gcd(ab, p^4)$ will be equal to

(a) p

(b) p^2

(c) p^3

(d) None of the above

55. Let n be a positive integer such that $(1+i)^n = 4096$ is true, where $i^2 = -1$. Then the value of n is

(a) 20

(b) 24

(c) 28

(d) None of the above

56. Identify the correct statements from the following :

(i) The diagonal entries of a skew-symmetric matrix are zero.

(ii) The determinant of a skew-symmetric matrix of order 3 will be always equal to zero.

(iii) The determinant of an orthogonal matrix of order 3 will be always equal to zero.

(a) (i) and (ii) only

(b) (ii) and (iii) only

(c) (i) and (iii) only

(d) None of the above

57. By the transformation

$$u = x - ct, \quad v = x + ct$$

the partial differential equation

$$\frac{\partial^2 z(x, t)}{\partial t^2} = c \frac{\partial^2 z(x, t)}{\partial x^2}$$

will reduce to

(a) $\frac{\partial^2 z(u, v)}{\partial u \partial v} = u^2 + v^2$

(b) $\frac{\partial^2 z(u, v)}{\partial u \partial v} = uv$

(c) $\frac{\partial^2 z(u, v)}{\partial u \partial v} = 0$

(d) None of the above

58. Imagine that you have two empty stacks of integers, s1 and s2. Draw a picture of each stack after the execution of the following pseudocode :

```
pushStack(s1, 3);
pushStack(s1, 5);
pushStack(s1, 7);
pushStack(s1, 9);
pushStack(s1, 11);
while(!emptyStack(s1))
{
      popStack(s1, x);
      x = x + 1;
      pushStack(s2, x);
}
```

(a)

3	4
5	6
7	8
9	10
s1	s2

(b)

3	
5	
7	
9	
s1	s2

(c)

	4
	6
	8
	10
s1	s2

(d) None of the above

59. Let X denote a random variable that takes on any of the values -1, 0, 1 with respective probabilities

$$P\{X = -1\} = 0 \cdot 2, \quad P\{X = 0\} = 0 \cdot 5 \text{ and } P\{X = 1\} = 0 \cdot 3$$

Compute the expected value of $E(X^2)$.

(a) 0·35

(b) 0·5

(c) 0·625

(d) None of the above

60. Suppose a matrix A of order 3 has eigenvalues 1, 2, 4. What is the trace of A^2?

(a) 8

(b) 7

(c) 21

(d) 64

61. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then we have $\cos^{-1} x + \cos^{-1} y =$

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{8}$

(d) None of the above

62. Find a third equation that can be solved if $x + y + z = 0$ and $x - 2y - z = 1$.

(a) $3x + z = 2$

(b) $3y + 2z = 4$

(c) $2x - y = 1$

(d) None of the above

63. For any real number a, $\lim_{x \to \infty} \sqrt{x}\{\sqrt{x+a} - \sqrt{x}\}$ is equal to

(a) ∞

(b) 0

(c) a

(d) None of the above

64. In a group of cows and hens, the number of legs are 14 more than twice the number of heads. Then the number of cows will be

(a) 5

(b) 7

(c) 10

(d) None of the above

65. Evaluate the following integral :

$$\int_0^\infty \frac{dx}{(1+x)^2}$$

(a) 0

(b) 1

(c) Integral does not exist

(d) None of the above

66. With a 100 kHz clock frequency, eight bits can be serially entered into a shift register in

(a) 8 ms

(b) 80 ms

(c) 8 μs

(d) 80 μs

67. The probability that a man who is 85 years old will die before attaining the age of 90 is 1/3. Four persons A_1, A_2, A_3 and A_4 are 85 years old. The probability that A_1 will die before attaining the age of 90 and will be the first to die is

(a) $\frac{31}{228}$

(b) $\frac{13}{282}$

(c) $\frac{65}{324}$

(d) None of the above

68. Which one of the following formats of a digital image is odd-one-out?

(a) BMP

(b) JPEG

(c) RLE

(d) TIFF

69. In a triangle ABC, line BP is drawn perpendicular to BC to meet CA in P such that $CA = AP$. Then $\frac{BP}{AB}$ is equal to

(a) $2\sin A$

(b) $2\sin B$

(c) $2\sin C$

(d) None of the above

70. Suppose a matrix A is invertible and by exchanging its first two rows, you get the matrix B. Then B is invertible and is obtained from the inverse of A by

(a) exchanging the first two rows of the inverse of A and keeping its remaining entries fixed

(b) exchanging the first two columns of the inverse of A and keeping its remaining entries fixed

(c) exchanging the first two rows and columns of the inverse of A and keeping its remaining entries fixed

(d) None of the above

71. What is the decimal representation of the octal number $(51735)_8$?

(a) 21469

(b) 21220

(c) 21008

(d) None of the above

72. Find the shortest distance from the origin to the surface defined by

$$x^2 + 8xy + 7y^2 = 225$$

(a) 0

(b) 12

(c) 22

(d) None of the above

73. *A* and *B* are brothers. *C* and *D* are sisters. *A*'s son is *D*'s brother. How is *B* related to *C*?

(a) Father

(b) Brother

(c) Grandfather

(d) Uncle

74. If *A* and *B* are two events such that

$$P(A \cup B) = \frac{3}{4}, \; P(A \cap B) = \frac{1}{4} \; \text{and} \; P(A^c) = \frac{2}{3}$$

where $P(A^c)$ denotes the probability of the complement of *A*, then $P(A^c \cup B)$ is

(a) $\frac{5}{12}$

(b) $\frac{5}{9}$

(c) $\frac{8}{11}$

(d) None of the above

75. In a 4-variable Karnaugh map, a 2-variable product term is produced by

(a) a 2-cell group of 1^s

(b) an 8-cell group of 1^s

(c) a 4-cell group of 1^s

(d) a 4-cell group of 0^s

76. If z is a complex number and lies in the second quadrant, then in which quadrant of the complex plane, the complex number $i\bar{z}$ lies, where $\bar{z}$ is the complex conjugate of z and $i^2 = -1$?

(a) First quadrant

(b) Second quadrant

(c) Third quadrant

(d) Fourth quadrant

77. The sum of the roots of the equation $4^x - 3(2^{x+3}) + 128 = 0$ is

(a) 0

(b) 5

(c) 8

(d) None of the above

78. In Gauss elimination method, the coefficient matrix is reduced into a

(a) diagonal matrix

(b) triangular matrix

(c) unit matrix

(d) null matrix

79. Suppose $f : A \to B$ and $g : B \to C$ are functions. Consider the following statements :

(i) If $(g \circ f)$ is one-to-one and the function f is onto, then the function g is one-to-one.

(ii) If $(g \circ f)$ is one-to-one, then the function f is one-to-one.

(iii) If $(g \circ f)$ is onto and the function g is one-to-one, then the function f is onto.

Among the above statements, identify the correct statements.

(a) (i) and (ii) only

(b) (ii) and (iii) only

(c) (i), (ii) and (iii)

(d) None of the above

80. Arrange the following numbers in ascending order :

$$\log(2+4),\ \log 2+\log 4,\ \log(6-3),\ \log 6-\log 3$$

(a) $\log(2+4),\ \log 2+\log 4,\ \log 6-\log 3,\ \log(6-3)$

(b) $\log 2+\log 4,\ \log(2+4),\ \log(6-3),\ \log 6-\log 3$

(c) $\log 6-\log 3,\ \log(6-3),\ \log 2+\log 4,\ \log(2+4)$

(d) None of the above

81. Consider the limit $\lim_{z \to 0} \left(\frac{z}{\bar{z}}\right)^2$ in the complex plane, where $\bar{z}$ is the complex conjugate of z.

Then the values of the limit as z approaches zero along the real axis, along the imaginary axis and along the line $y = x$ will be

(a) 1, 1, − 1

(b) 1, 1, 0

(c) −1, − 1, 1

(d) None of the above

82. A computer science class consists of 13 females and 12 males. Six class members are to be chosen at random to plan a picnic. What is the probability that exactly 4 females and 2 males are chosen?

(a) 0·1

(b) 0·2

(c) 0·3

(d) 0·4

83. Suppose a random variable X is uniformly distributed between 0 and 1 whose pdf (probability density function) is

$$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{else} \end{cases}$$

Then its mean and variance become

(a) 1/2, 1/12

(b) 1/4, 1/16

(c) 1/6, 1/17

(d) None of the above

84. If a circle passes through the point (3, 4) and cuts the circle $x^2 + y^2 = a^2$ orthogonally, the equation of the locus of its centre is

(a) $3x + 4y = a^2 + 25$

(b) $x + 8y = a^2 + 25$

(c) $6x + 8y = a^2 + 25$

(d) None of the above

85. A vector $\mathbf{c}$ perpendicular to the vectors $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ satisfying the condition $\mathbf{c} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) = -6$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the X-, Y- and Z-axis respectively, is

(a) $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$

(b) $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

(c) $-3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$

(d) None of the above

86. Let $R = \{(x, y) : x, y \in A, \ x + y = 4\}$ be a relation, where $A = \{1, 2, 3, 4, 5\}$. Then R is

(a) reflexive, symmetric but not transitive

(b) symmetric but not reflexive and not transitive

(c) not reflexive, not symmetric and not transitive

(d) None of the above

87. Which of the following operators in C++ can be overloaded?

(a) Conditional operator (? :)

(b) Scope resolution operator (::)

(c) Member access operator (.*)

(d) Relational operator (<=)

88. Let $r \neq 0$ be a real number. Then the sum of the series

$$r^2 + \frac{r^2}{1+r^2} + \frac{r^2}{(1+r^2)^2} + \ldots$$

is equal to

(a) ∞

(b) $1 + r^2$

(c) $\frac{1}{1+r^2}$

(d) None of the above

89. How many even numbers in the range of 100–999 have no repeated digits?

(a) 298

(b) 328

(c) 368

(d) None of the above

90. A frog starts climbing a 30 ft wall. Each hour it climbs 3 ft and slips back 2 ft. How many hours does it take to reach the top and get out?

(a) 30

(b) 29

(c) 28

(d) None of the above

91. A continuous random variate X has the probability density function (pdf)

$$f(x) = \frac{c}{1+x^2}, \quad -\infty < x < \infty$$

Then the value of c is

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{\pi}$

(d) None of the above

92. If an integer needs two bytes of storage, then the maximum value of an unsigned integer is

(a) $2^{16} - 1$

(b) $2^{15} - 1$

(c) 2^{16}

(d) 2^{15}

93. The expression $X = (A + B + C)(A + B + C')(A + B' + C)(A + B' + C')(A' + B' + C)$ is equivalent to

(a) $A(B + C) + BC$

(b) $A(B' + C)$

(c) $AB' + BC'$

(d) None of the above

94. The output of the program

```
main( )
{int i=5; i=(++i)/(i++); printf("% d",i);}
```

is

(a) 5

(b) 1

(c) 6

(d) 2

95. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 12 more than the total number of subsets of the second set. Then the values of m and n respectively are

(a) 5, 3

(b) 6, 4

(c) 4, 2

(d) None of the above

96. Among the following statements, identify the number of correct statements :

(i) Let A be a set and suppose that $x \in A$. Then $x \subseteq A$ is possible.

(ii) $\phi \in \{x, y, \phi\}$ and $\phi \subseteq \{x, y, \phi\}$, where ϕ is the empty set.

(iii) The number of elements of the power set of the power set of the empty set is 2.

(a) 1

(b) 2

(c) 3

(d) None of the above

97. Consider the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & \frac{1}{2} \end{pmatrix}$. Then the matrix A is

(a) positive definite

(b) positive semi-definite only

(c) negative definite

(d) indefinite

98. Suppose u_n and v_n are sequences defined recursively by

$$u_1 = 0,\ v_1 = 1 \text{ and for } n > 1,\ u_{n+1} = (u_n + v_n)/2,\ v_{n+1} = (u_n + 3v_n)/4$$

Then the sequences $\{u_n\}$ and $\{v_n\}$ will become

(a) both increasing

(b) both decreasing

(c) one increasing and the other decreasing

(d) None of the above

99. Consider the function $f(x) = \dfrac{e^{1/x}}{1 + e^{1/x}}$ for $x \neq 0$. Then the values of the limit of the function $f(x)$ when $x \to 0^+$ and $x \to 0^-$ will be

(a) Both the limits do not exist

(b) 0, 0 respectively

(c) 0, 1 respectively

(d) None of the above

100. If $u = \arctan x$, then

$$(1 + x^2)\frac{d^2u}{dx^2} + 2x\frac{du}{dx}$$

will be equal to

(a) x

(b) u

(c) 1

(d) None of the above

101. The period of the function $f(x) = \cos^2 3x + \tan 4x$ is

(a) π

(b) $\pi / 3$

(c) $\pi / 6$

(d) None of the above

102. Find the binary representation of the number 2159.

(a) 100 001 101 101

(b) 110 011 101 111

(c) 101 101 001 100

(d) None of the above

103. The error quantity which must be added to the true representation of the quantity in order that the result is exactly equal to the quantity we are seeking to generate is called

(a) truncation error

(b) round-off error

(c) relative error

(d) absolute error

104. Identify the types of singularity of the following complex functions, both at $z = 0$:

(i) $f(z) = \dfrac{e^{2z} - 1}{z}$

(ii) $g(z) = z^3 \sin\left(\dfrac{1}{z}\right)$

(a) Both are removable singularities

(b) Both are essential singularities

(c) Essential and removable singularities

(d) Removable and essential singularities

105. Find the sum of all the numbers between 100 and 1000 which are divisible by 14.

(a) 32388

(b) 35392

(c) 38396

(d) None of the above

106. Let $n > 3$ be an integer and let $A = \{1, 2, 3, \ldots, n\}$. How many subsets B of A have the property that $B \cup \{1, 2\} = A$?

(a) 1 (b) 2

(c) 3 (d) 4

107. Let $\{s_n\}$ be a sequence defined by the recurrence relation

$$s_n = \sqrt{\frac{ab^2 + s_n^2}{a+1}}, \text{ for } n \geq 1$$

where $b > a$ and $s_1 = a > 0$.

Then $\lim_{n \to \infty} s_n$ is equal to

(a) ∞

(b) b

(c) $a + b$

(d) None of the above

108. The age of a father is twice that of the elder son. Ten years hence the age of the father will be three times that of the younger son. If the difference of ages of the two sons is 15 years, the age of the father will be

(a) 50 years

(b) 60 years

(c) 65 years

(d) None of the above

109. A five-figure number is formed by the digits 0, 1, 2, 3, 4 without repetition. The probability that the number formed is divisible by 4 is

(a) 9/16 (b) 5/16

(c) 7/16 (d) None of the above

110. Consider the following statements :

(i) Suppose A is a matrix such that $\det(A) = 0$. Then at least one of the cofactors must be zero.

(ii) Suppose A is a matrix in which all its entries are either 0 or 1. Then $\det(A)$ will be equal to 1, 0 or −1.

(iii) Suppose A is a matrix in which $\det(A) = 0$. Then all its principal minors will be zero.

Identify the wrong statements.

(a) Only (i) and (ii)

(b) Only (i) and (iii)

(c) (i), (ii) and (iii)

(d) None of the above

111. One of the disadvantages of raster scan display is

(a) it cannot display colour images

(b) lines may appear jaggy

(c) it cannot take advantages of technological research and mass production of the television industry

(d) None of the above

112. What are the next two terms in the sequence 17, 15, 26, 22, 35, 29, ..., ...?

(a) 42, 50

(b) 48, 40

(c) 46, 38

(d) None of the above

113. A ball is drawn from an urn containing three white and three black balls. After the ball is drawn, it is then replaced and another ball is drawn. This goes on indefinitely. What is the probability that of the first four balls drawn, exactly two are white?

(a) $\frac{3}{8}$

(b) $\frac{2}{9}$

(c) $\frac{5}{11}$

(d) None of the above

114. If $ab \neq 0$, the equation

$$ax^2 + 2xy + by^2 + 2ax + 2by = 0$$

represents a pair of straight lines, if

(a) $a^2 + b^2 = 2$

(b) $ab = 2$

(c) $a + b = 2$

(d) None of the above

115. Let $g(x) = \int_0^x f(t)\, dt$, where the function $f(\cdot)$ is such that

$$\frac{1}{2} \leq f(t) \leq 1 \text{ for } 0 \leq t \leq 1 \text{ and } 0 \leq f(t) \leq \frac{1}{2} \text{ for } 1 \leq t \leq 2$$

Then $g(2)$ satisfies the inequality

(a) $-\frac{1}{2} \leq g(2) < \frac{1}{2}$

(b) $0 \leq g(2) < 2$

(c) $\frac{3}{2} < g(2) \leq 3$

(d) None of the above

116. Bill and Gates go target shooting together. Both shoot at a target at the same time. Suppose, Bill hits the target with probability $0 \cdot 7$, whereas Gates, independently, hits the target with probability 0 [illegible] Given that the target is hit, what is the probability that Gates hits it?

(a) $\frac{19}{45}$

(b) $\frac{11}{21}$

(c) $\frac{13}{27}$

(d) None of the above

117. If $\cos\theta = \cos\alpha\cos\beta$, then the product

$$\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right)$$

is equal to

(a) $\tan^2\left(\frac{\alpha}{2}\right)$

(b) $\tan^2\left(\frac{\beta}{2}\right)$

(c) $\tan^2\left(\frac{\theta}{2}\right)$

(d) None of the above

118. Suppose the roots of a quadratic equation are (8/5) and –(7/3). What is the value of the coefficient of the x-term, if the equation is written in the standard form $ax^2 + bx + c = 0$ with $a = 1$?

(a) 2/5

(b) 7/5

(c) 11/5

(d) None of the above

119. Find the number of ways a postman can deliver four letters, each to the wrong address.

(a) 7

(b) 8

(c) 9

(d) 10

120. Find the length of the 3-D curve defined in parametric form as

$$x = at^2, \quad y = 2at \text{ and } z = at \text{ in } 0 \le t \le 1$$

(a) $\frac{a}{8}(5\log 5 + 12)$

(b) $a(5\log 7 + 8)$

(c) $\frac{a}{4}(2\log 5 + 7)$

(d) None of the above

★★★